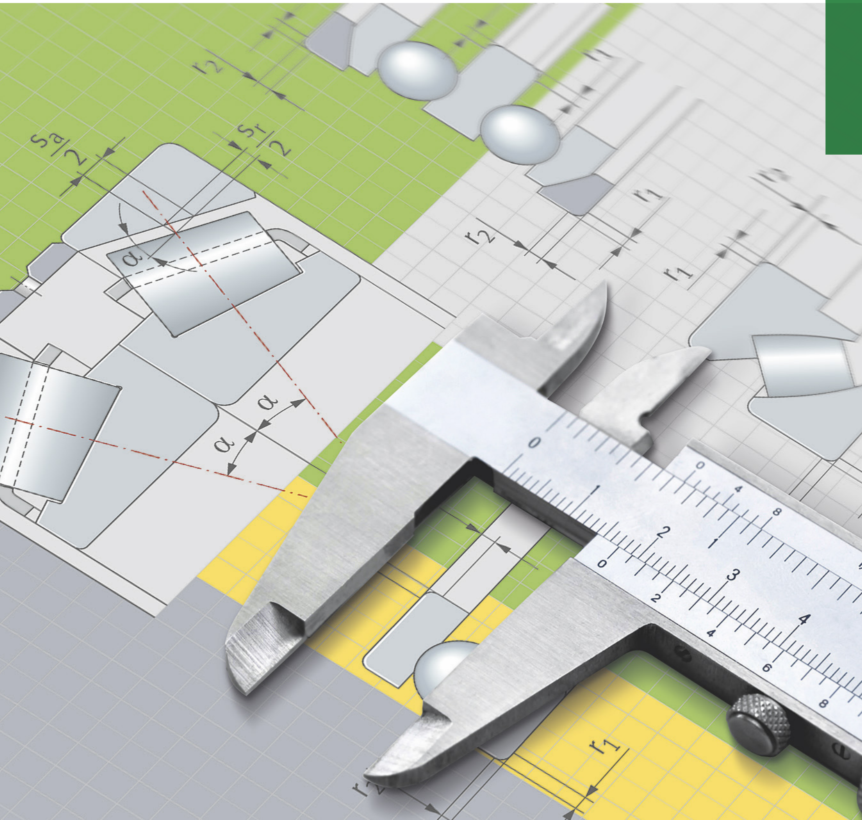
SCHAEFFLER

Technical Pocket Guide



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PDF only

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Foreword

Schaeffler has always and on principle been committed to its responsibility for sustainable training and global transfer of knowledge. Even if the transition from an industry-based to a knowledge-based society was not so clearly defined in the mid-1980s, it was already clear to the attentive observer that education was becoming an extremely valuable commodity in which it was worth investing. From this notion came the idea for the INA Technical Pocket Guide ITT, which was produced under the technical direction of Prof. Dr.-Ing. Ernst-Günter Paland, presented for the first time at the Hannover Messe (English: Hanover Fair) in 1985 and contained 96 pages.

Over the years, this reference work with its equations and laws has been increased to 370 pages and published in several editions. More than 750 000 copies have been distributed since its initial publication. In addition to its function as a reference work, it has also proven itself as a work book for use in training and further education, by designers, engineers, technicians, and students of technical and scientific disciplines.

As confirmed by the high demand for this publication, it was completely revised by Schaeffler in 2013 and has been published since then under the name Schaeffler Technical Pocket Guide STT. For the purposes of technical coordination, it was possible to obtain the services of Prof. Dr.-Ing. Harald Meerkamm of the Friedrich-Alexander University of Erlangen-Nuremberg and formerly Head of the Chair for Engineering Design. Without his exceptional support, experience, diligence and patience, it would not have been possible to produce the Schaeffler Technical Pocket Guide in this form.

Fortunately, Prof. Dr.-Ing. Sandro Wartzack, Head of the Chair for Engineering Design at the Friedrich-Alexander University of Erlangen-Nuremberg, has now agreed to take on this task and thus continue technical coordination of the Schaeffler Technical Pocket Guide. We would like to thank Prof. Dr.-Ing. Sandro Wartzack and his team of technical personnel, who contributed to this 3rd edition, for their outstanding commitment, trusting cooperation and providing an array of fresh ideas for the future, which we look forward to implementing in upcoming joint projects. We would also like to thank all of the readers who pointed out corrections and amendments.

Interested parties can request copies of the Schaeffler Technical Pocket Guide STT via ➔ *http://www.schaeffler.de/std/1D50* in our Media Library. This 3rd edition can be downloaded for the first time in PDF format. For users of electronic media, the Schaeffler Technical Pocket Guide is also available as an “app” at ➔ *apps.schaeffler.com*.

We are convinced that this new edition will meet with the same positive response as the previous versions.

Herzogenaurach, October 2021
The Publisher

Preface

For over 25 years, the INA Technical Pocket Guide ITT from Schaeffler has been a standard work among apprentices in metal-processing and electrical engineering professions, as well as designers, technicians, engineers and also students involved in technical and scientific disciplines. With its brief, compact and concise layout, it successfully bridges the gap between comprehensive textbook and purely tabular work.

As a result of the huge demand for this guide – over three quarters of a million copies have been distributed since its initial publication more than two and a half decades ago – Schaeffler decided to produce a fully revised version and reissue this as the Schaeffler Technical Pocket Guide STT. The key priority in this process was to retain the former character of the guide as a reference work which provides rapid access to detailed information and to ensure scientific relevance, intelligibility, clarity and presentability, plus the addition of topical subjects. As a result, all of the contents have been updated, the data on standards, tolerances and fits have been brought up to date and the current subject of mechatronics has been added.

The chapter Design elements – which has been expanded to include rotatory and translatory bearings and examples of applications which use these products – has been completely reconfigured and significantly extended in accordance with the scientifically founded "function-oriented approach". The technical principles of rolling bearings, where were previously covered in two chapters of the guide, have been consolidated in this chapter. The "function-oriented approach" follows the concept of structuring the field of machine elements according to their respective function.

As it was not the intention to produce extensive descriptions of the subject areas in the manner found in textbooks, the statements have been kept deliberately concise, in the sense of a reference work, with the focus on fast, practical usability. This will allow the reader to locate the required information in condensed form and quickly familiarise himself or herself with the specialist knowledge. The layout of the chapters, the use of colour as a breakdown and control parameter, a reader-friendly typography and the well-structured tables and formulae all help to provide easy access to information.

As drawings are often involved in understanding technical and scientific relationships, the pictures have been prepared in such a way that their contents are clearer and more readily accessible. This opens up new possibilities in the transfer of information involving diagrammatic elements. This all follows the principles of modern didactic textbook design and supports the uptake of information.

I would like to thank the publisher's employees who have worked on this edition for their comments, suggestions, tips, amendments and corrections and for the trusting and stimulating working relationship we have shared, the excellent support they have provided and the conscientious, operative implementation of this project. I would also like to sincerely thank my STT Team at the Chair for Engineering Design, who have given me excellent support throughout all stages of this project.

Preface

I hope that this proven work book and reference work, in its new edition as the Schaeffler Technical Pocket Guide STT, will assist all readers in tackling their everyday tasks.

Erlangen, March 2014
Prof. Dr.-Ing. Harald Meerkamm
Friedrich-Alexander University of Erlangen-Nuremberg

Preface to the 3rd edition

Following the involvement of my predecessor, Prof. Dr.-Ing. Harald Meerkamm, in the Schaeffler Technical Pocket Guide STT over many years, I am delighted to have been granted this opportunity to resume coordination of the Pocket Guide, with the support of my STT team, and continue the established tradition of my Chair. Although my aim is to maintain the established, successful structure, my team and I also have numerous new ideas for the future. For the purposes of this edition, however, I will only be addressing urgent updates and would therefore like to thank you, the reader, for your feedback.

I would like to express my heartfelt gratitude to the team at Schaeffler and my STT team at the Chair for Engineering Design for their support.

Erlangen, October 2021
Prof. Dr.-Ing. Sandro Wartzack
Friedrich-Alexander University of Erlangen-Nuremberg

Schaeffler Group – We pioneer motion

As a leading global automotive and industrial supplier, the Schaeffler Group has been driving ground-breaking inventions and developments in the fields of motion and mobility for over 70 years. With innovative technologies, products and services in the fields of CO_2efficient drives, electric mobility, Industry 4.0, digitalisation and renewable energies, the company is a reliable partner for producing motion and mobility solutions that are more efficient, intelligent and sustainable. The technology company produces precision components and systems for drive trains and chassis applications as well as rolling and plain bearing solutions for a multitude of industrial applications.

Figure 1
Headquarters in Herzogenaurach

Schaeffler at a glance

Sales in 2020: around 12,6 billion euros

Employees worldwide: approx. 83 300

Employees in Germany: almost 30 000

Locations: around 200 locations in more than 50 countries

Patent statistics for 2020 according to the German Patent and Trade Mark Office DPMA:
Ranked as the second most innovative company in Germany with more than 1 900 patent applications.

Further information on Schaeffler can be found at:
www.schaeffler.com

Training and studying – Professional future at Schaeffler

Training and studying have a long tradition at Schaeffler and are regarded as a top priority. Approximately 2 800 young people around the globe – from Brazil to Vietnam – are training for their dream job at Schaeffler, including roles as engineers in Development as well as specialists in IT, Production or Administration.

Schaeffler's range of around 25 training programs and 15 study programs offered at 50 locations across 15 countries caters for a wide range of professional interests, focussing on industrial/technical subjects such as mechanical engineering, electronics and mechatronics.

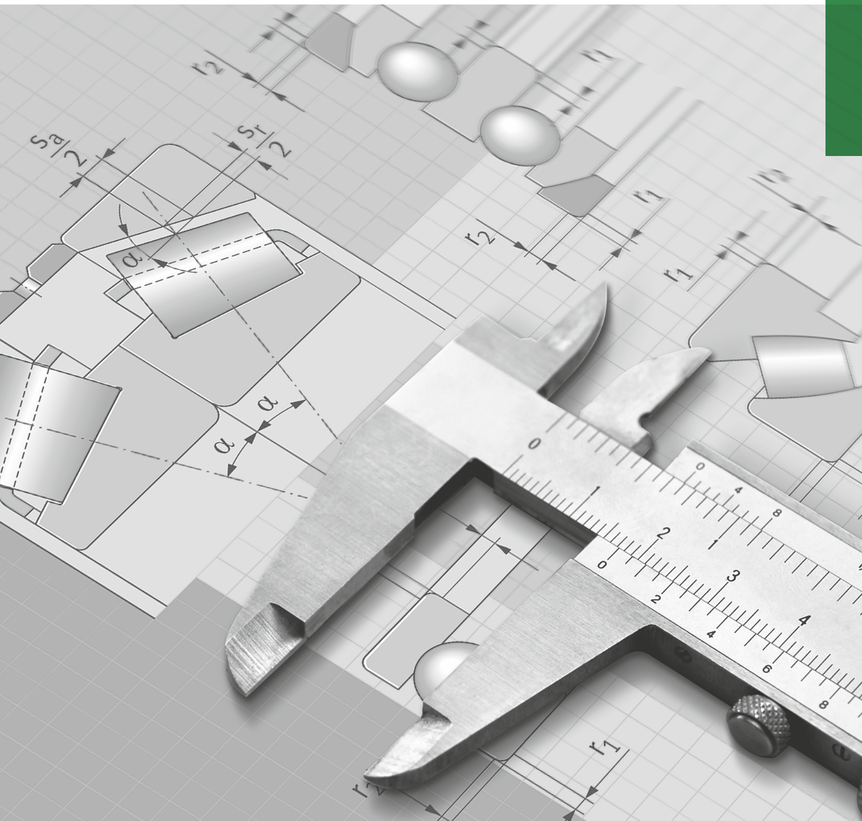
In Germany, around 1 000 apprentices complete their training with a certificate from the Chamber of Commerce and Industry (IHK) after two to three and a half years. In addition, around 300 students complete a bachelor degree course in three to four and a half years in combination with an intensive practical qualification, which is conducted in the form of intensive practical phases or as an accompanying training course certified by the Chamber of Commerce and Industry.

Schaeffler works with renowned vocational colleges and higher education institutions to impart theoretical knowledge and promote practical skills.

Further information can be found at:
www.schaeffler.de/careers

Figure 2
Training and studying at Schaeffler

Technical Pocket Guide



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Units of measurement and scripts

International system of units SI

Base units in SI

The "Law on Units of Measurement" of 2 July 1969 came into force in the Federal Republic of Germany on 2 July 1970. This law defined the legal units of measurement for commercial practice that were to be introduced no later than 31 December 1977.
Furthermore, this law defined the base values and base units of the International System of Units (Système International d'Unités, abbreviated to SI).

The following table shows the seven base values and base units of the SI (also known as MKS: "metre - kilogram - second").

Base value	Base unit		Definition
	Designation	Symbol	
Length	Metre	m	1 metre is the length of the path travelled by light during a time interval of 1/299 792 458 of a second 17th CGPM, 1983[1)]
Mass	Kilogram	kg	1 kilogram is the mass of the international prototype of the kilogram 1st CGPM, 1889 and 3rd CGPM, 1901
Time	Second	s	1 second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the ^{133}Cs atom 13th CGPM, 1967
Electric current	Ampere	A	1 ampere is that constant current which if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed 1 metre apart in a vacuum, would produce between these conductors a force equal to 1/5 000 000 newton per metre of length 9th CGPM, 1948

Continuation of table, see Page 17.

[1)] CGPM: Conférence Génerale des Poids et Mesures (General Conference on Weights and Measures).

Continuation of table, Base units in SI, from Page 16.

Base value	Base unit		Definition
	Designation	Symbol	
Thermo-dynamic temperature	Kelvin	K	1 kelvin is the 273,16th part of the thermodynamic temperature of the triple point[1] of water 13th CGPM, 1967
Amount of substance	Mole	mol	1 mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 12/1 000 kilogram of carbon ^{12}C. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles 14th CGPM, 1971
Luminous intensity	Candela	cd	1 candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \cdot 10^{12}$ Hertz ($\triangleq$ vacuum wavelength 555 nm) and that has a radiant intensity in that direction of 1/683 watt per steradian 16th CGPM – 1979 – Resolution 3

[1] Fixed point of the international temperature scale. The triple point is the only state at which all three states of aggregation (solid, liquid and gaseous) are in equilibrium (at 1 013,25 hPa). At 273,16 K, it is 0,01 K above the ice point of water (273,15).

Derived units

Further SI units can be derived from the base units. If this derivation leads only to the numerical factor 1, the derived units are **coherent** relative to the base units. Coherent units form a system of units.

Derivation of the unit “Newton”

In accordance with Newton’s basic law, force is a value derived from the base values of mass, time and length.

This coherent value has the unit name “Newton” and the unit symbol “N”:

- force = mass · acceleration
- $1 \text{ N} = 1 \text{ kg} \cdot 1 \text{ m/s}^2$.

SI units and derived units The following table shows an excerpt of common SI units and units derived therefrom (other derived values and units are given in the specific sections).

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Length	***l***	**Metre**	**–**	**m**	Micron 1 µ = 1 µm = 10^{-6} m
		–	Nautical mile	1 NM = 1852 m	
Area	*A*	Square metre	–	m^2	Ångström 1 Å = 10^{-10} m
		–	Are	1 a = 100 m^2	
		–	Hectare	1 ha = 10^4 m^2	X unit 1 xu = 10^{-13} m
Volume	*V*	Cubic metre	–	m^3	
		–	Litre	1 l = 10^{-3} m^3	
Elongation	ϵ	■	–	m/m	
Plane angle	α β γ	Radian	–	1 rad[4] = 1 m/m	Right angle 1 ∟ = (π/2) rad
		–	Degree	1° = π/180 rad	
		–	Minute	1′ = π/10800 rad	Gradian 1 g = 1 gon
		–	Second	1″ = π/648 000 rad	
		–	Gon	1 gon = π/200 rad	Centigon 1′ = 1 cgon
Solid angle	Ω	Steradian	–	1 sr = 1 m^2/m^2	
Mass	***m***	**Kilogram**	**–**	**kg**	Gamma 1 γ = 1 µg
		–	Gram	1 g = 10^{-3} kg	
		–	Tonne	1 t = 10^3 kg	Quintal 1 dz = 100 kg
Mass of precious stones		–	Metric carat	1 Kt = 0,2 · 10^{-3} kg	
Mass/length	*m′*	■	–	kg/m	
Mass of textile fibres	–	–	Tex	1 tex = 10^{-6} kg/m	
Mass/area	*m″*	■	–	kg/m^2	

Continuation of table, see Page 19.

■ SI units without special unit names.
The units are formed on the basis of the units pertaining to the base values.

1) Legal units as of 2 July 1970.
2) Units that legally may no longer be used as of 1 January 1978.
3) Formula symbol standardised in accordance with DIN 1304.
4) The unit rad can be replaced by “1” in calculation.

Continuation of table, SI units and derived units, from Page 18.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Density	ρ	■	–	kg/m^3	The numerical value of the specific gravity in kp/m^3 is not always equal to the numerical value of the density, but is dependent on location, see also Equation 4, Page 27
Specific volume	v	■	–	m^3/kg	
Time	t	**Second**	**–**	**s**	–
		–	Minute	1 min = 60 s	
		–	Hour	1 h = 3 600 s	
		–	Day	1 d = 86 400 s	
		–	Year	1 a ≈ 365,24 d	
Rotational speed	n	■	–	1/s	rpm is still permissible, however it should be replaced by min^{-1}
		–	Revolution/ minute	1 rpm = 1 min^{-1}	
Frequency	f	Hertz	–	1 Hz = 1/s	–
Angular frequency	ω	–	1/s		
Velocity	v	■	–	m/s	
		–	Kilometre/ hour	1 km/h = (1/3,6) m/s	
		–	Knot	1 kn = 1 NM/h	
Acceleration	a	■		m/s^2	
Angular velocity	ω	■	–	rad/s	
Angular acceleration	$\dot{\omega}$	■	–	rad/s^2	
Volume flow	$\dot{V}$	■	–	m^3/s	
Mass flow	$\dot{m}$	■	–	kg/s	

Continuation of table, see Page 20.

■ SI units without special unit names.
The units are formed on the basis of the units pertaining to the base values.

[1] Legal units as of 2 July 1970.

[2] Units that legally may no longer be used as of 1 January 1978.

[3] Formula symbol standardised in accordance with DIN 1304.

Continuation of table, SI units and derived units, from Page 19.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Force	F	Newton	–	1 N = 1 kg · m/s^2	Kilopond 1 kp = 9,806 65 N
Impulse	p	■	–	kg · m/s	
Angular momentum	L	■	–	kg · m^2/s	Technical atmosphere 1 at = 1 kp/cm^2
Pressure	p	Pascal	–	1 Pa = 1 N/m^2	Physical atmosphere 1 atm = 1,013 25 bar
Stress	σ τ	■	Newton/ square millimetre	1 N/mm^2 = 1 MPa	Water column 1 mm WC = 1 kp/m^2
		■	Bar	1 bar = 10^5 Pa	Mercury column 1 mm Hg = 1,333 2 hPa
Work Energy	W E	Joule	–	1 J = 1 N · m	Kilopond metre 1 kpm = 9,81 J
Heat quantity	Q	Watt second	–	1 W · s = 1 kg · m^2/s^2	Horsepower hour 1 hp · h = 0,735 5 kW · h
		–	Kilowatt hour	1 kW · h = 3,6 MJ	Kilocalorie 1 kcal = 4,186 8 kJ
Moment of force	M	Newton metre	–	N · m	Kilopond metre 1 kpm = 9,81 N · m
Power, energy flow Heat flow	P $\dot{Q}$	Watt	–	1 W = 1 J/s = 1 N · m/s	Horsepower 1 hp = 0,735 5 kW 1 kW = 1,36 PS 1 kcal/s = 4,186 8 kW
Dynamic viscosity	η	Pascal second	–	1 Pa · s = 1 N · s/m^2	Poise 1 P = 0,1 Pa · s Centipoise 1 cP = 1 mPa · s
Kinematic viscosity	ν	■	–	m^2/s	Stokes 1 St = 10^{-4} m^2/s Centistokes 1 cSt = 1 mm^2/s

Continuation of table, see Page 21.

■ SI units without special unit names.
The units are formed on the basis of the units pertaining to the base values.

[1] Legal units as of 2 July 1970.
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[3] Formula symbol standardised in accordance with DIN 1304.

Continuation of table, SI units and derived units, from Page 20.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Electric current	***I***	**Ampere**	**–**	**A**	–
Voltage (potential difference)	U	Volt	–	1 V = 1 W/A	
Electrical resistance	R	Ohm	–	1 Ω = 1 V/A	
Electrical conductance	G	Siemens	–	1 S = 1/Ω	
Apparent power	S	–	Volt-ampere	1 W = 1 V · A	
Reactive power	Q	–	Var	1 var = 1 W	
Quantity of electricity, electrical charge	Q	Coulomb	–	1 C = 1 A · s	
	–	–	Ampere-hour	1 A · h = 3 600 C	
Electrical capacitance	C	Farad	–	1 F = 1 C/V	
Electrical flux	ψ	–	–	C	
Electrical flux density	D	–	–	C/m^2	
Electrical field strength	E	–	–	V/m	
Magnetic flux	Φ	Weber	–	1 Wb = 1 V · s	Maxwell 1 M = 10^{-8} Wb
Magnetic flux density	B	Tesla	–	1 T = 1 Wb/m^2	Gauss 1 G = 10^{-4} T
Magnetic field strength	H	–	–	A/m	Oerstedt 1 Oe = 10^3/(4π) A/m = 79,58 A/m
Inductance	L	Henry	–	1 H = 1 Wb/A	–

Continuation of table, see Page 22.

[1] Legal units as of 2 July 1970.
[2] Units that legally may no longer be used as of 1 January 1978.
[3] Formula symbol standardised in accordance with DIN 1304.

Continuation of table, SI units and derived units, from Page 21.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Temperature	***T***	**Kelvin**	**–**	**K**	–
Celsius temperature	*t*	–	Degree Celsius	1 °C = 1 K[4]	
Thermal diffusivity	*a*	–	–	m^2/s	
Specific heat capacity	*c*	–	–	J/(kg · K)	1 kcal/(kg · grd) = 4,187 kJ/(kg · K)
Entropy	*S*	–	–	J/kg	–
Specific entropy	*s*	–	–	J/(kg · K)	
Enthalpy	*H*	Joule	–	J	
Thermal conductivity	λ	–	–	W/(m · K)	1 kcal/(m · h · grd) = 1,163 W/(m · K)
Heat transfer coefficient	α	–	–	$W/(m^2 \cdot K)$	–
Heat transition coefficient	*k*	–	–	$W/(m^2 \cdot K)$	

Continuation of table, see Page 23.

1) Legal units as of 2 July 1970.

2) Units that legally may no longer be used as of 1 January 1978.

3) Formula symbol standardised in accordance with DIN 1304.

4) The Celsius temperature t is the name for the particular difference between any thermodynamic temperature T and the temperature T_0 = 273,15 K. It is therefore $t = T - T_0 = T - 273{,}15$ K.
The degree Celsius is the name for the unit Kelvin when specifying Celsius temperatures.
In compound units, temperature differences must be specified in K, for example kJ/(m · s · K).
Tolerances for Celsius temperatures are presented, for example,
as t = (50 ±2) °C or t = 50 °C ±2 °C or t = 50 °C ±2 K.

Continuation of table, SI units and derived units, from Page 22.

Value	Formula symbol[3]	Units[1]			Units no longer to be used[2] and their conversion
		Name		Symbol and its conversion	
		SI unit	Derived unit		
Amount of substance	***n***	**Mole**	**–**	**mol**	–
Atomic unit of mass	u	–	–	$1{,}660\,6 \cdot 10^{-27}$ kg	
Energy	W	Electron volt	–	1 eV = $1{,}602\,2 \cdot 10^{-19}$ J	
Activity of a radioactive substance	A	Becquerel	–	1 Bq = 1/s	Curie 1 Ci = $3{,}7 \cdot 10^{10}$/s
Absorbed dose	D	Gray	–	1 Gy = 1 J/kg	Rem 1 rem = 10^{-2} J/kg
Energy dose rate	$\dot{D}$	–	–	W/kg	–
Ion dose	J	–	–	C/kg	Röntgen 1 R = $258 \cdot 10^{-6}$ C/kg
Ion dose rate	j	–	–	A/kg	–
Dose equivalent	H	Sievert	–	1 Sv = 1 J/kg	
Luminous intensity	***I***	**Candela**	**–**	**cd**	
Luminance	L	–	–	cd/m^2	Stilb 1 sb = 10^4 cd/m^2 Apostilb 1 asb = $(1/\pi)$ cd/m^2
Luminous flux	Φ	Lumen	–	1 lm = 1 cd · sr	–
Luminous energy	Q	–	–	1 lm · s	
Illuminance	E	Lux	–	1 lx = 1 lm/m^2	
Refractive power of lenses	D	–	Dioptre	1 dpt = 1/m	

[1] Legal units as of 2 July 1970.

[2] Units that legally may no longer be used as of 1 January 1978.

[3] Formula symbol standardised in accordance with DIN 1304.

Internationally defined prefixes

Decimal fractions or multiples of SI units are designated by prefixes before the names of the units or by prefix symbols before the unit symbols. This factor, by which the unit is multiplied, is generally a power of ten with a positive or negative exponent.

The prefix symbol is placed immediately in front of the unit symbol, without a space, to form a coherent unit, such as the millimetre (mm). Compound prefixes, such as millikilogram (mkg), must not be used.

Prefixes must not be used for the following units: the time units of the minute, hour, day, year; the temperature unit of the degree Celsius; the angular units of the degree, second, minute.

The following table shows a selection of the most important prefixes:

Power of ten		Prefix	Prefix symbol
10^{-18}	Trillionth	Atto	a
10^{-15}	Billiardth	Femto	f
10^{-12}	Billionth	Pico	p
10^{-9}	Milliardth	Nano	n
10^{-6}	Millionth	Micro	μ
10^{-3}	Thousandth	Milli	m
10^{-2}	Hundredth	Centi	c
10^{-1}	Tenth	Deci	d
10^{1}	Ten	Deca	da
10^{2}	Hundred	Hecto	h
10^{3}	Thousand	Kilo	k
10^{6}	Million	Mega	M
10^{9}	Milliard[1)]	Giga	G
10^{12}	Billion[1)]	Tera	T
10^{15}	Billiard[1)]	Peta	P
10^{18}	Trillion[1)]	Exa	E

[1)] In the USA: 10^9 = 1 billion; 10^{12} = 1 trillion; 10^{15} = 1 quadrillion; 10^{18} = 1 quintillion.

Definitions of terms according to DIN 1305

According to DIN 1305 (January 1988), the following definitions and terms are specified:

1 Scope

This standard applies to the field of classical physics and its application in engineering and economic usage.

2 Mass

The mass m describes the characteristic of a body, which manifests itself in inertia effects in relation to a change in its motion state and also in its attraction for other bodies.

The mass of a body is decisive for its inertial behaviour. In Newton's law of forces, the inert mass m must therefore be used, while the associated forces are inertia forces. However, the mass is also simultaneously the cause of gravity (weight force). In this case, the heavy mass m must be used. These are, in terms of phenomena, differing characteristics of mass but are equivalent in all relationships.

3 Measured value of weight

In the case of a weighing operation in a fluid (liquid or gas) of the density ρ_{fl}, the measured value of weight W is defined by the following relationship:

Equation 1

$$W = m \frac{1 - \frac{\rho_{fl}}{\rho}}{1 - \frac{\rho_{fl}}{\rho_G}}$$

where ρ is the density of the weighed bulk and ρ_G is the density of the units of weight.

4 Conventional measured weight value

The conventional measured weight value W_{std} is calculated on the basis of the equation from section 3 Measured value of weight with the standard conditions ρ_{fl} = 1,2 kg/m^{3}and ρ_G = 8 000 kg/m^3. In this case, the density of the weighed bulk at 20 °C must be inserted for ρ.

5 Force

The force F is the product of the mass m of a body and the acceleration a which it experiences or would experience as a result of the force F:

Equation 2

$$F = m \cdot a$$

6 Weight force

The weight force F_G of a body of mass m is the product of the mass m and the gravitational acceleration g:

Equation 3

$$F_G = m \cdot g$$

7 Weight

The word “weight” is predominantly used in three different senses:

- instead of “measured value of weight”
- as a shortened form of “weight force”
- as a shortened form of “unit of weight” (see DIN 8120-2).

If misunderstandings could arise, the word “weight” should be replaced by the relevant term “measured value of weight”, “weight force” or “unit of weight”.

8 Load

The word “load” is used in different senses in engineering (e.g. for power, force or for an object).

If misunderstandings could arise, the word “load” should be avoided.

Explanations

We live and have weight resting on the base of an ocean of air. In weighing, a correction is hardly ever made for the buoyancy of the air, although this would actually be necessary. We almost always accept the uncorrected measured value, which is also the basis for settlement of accounts in commerce where goods are sold by weight. It is necessary, however, to distinguish between the mass and the result of a weighing operation in air, namely the measured value of weight.
Where weighed items are of low density, as in the case of mineral oils, the relative difference between the mass and the measured value of weight is equivalent to approximately 1 per mil. It is smaller where weighed items are of high density. Air has a measured weight value of zero.
Bodies of the same mass but of differing density will have different measured weight values. The measured weight value of a body will also change if there is a change in the density of the surrounding air.
The measured weight value is also dependent on the weather.

Unit systems no longer to be used

The Physical System of Units

Like the SI System of Units, the Physical System of Units uses the base values of length, mass and time, but uses the following base units for these values:

Base value	Base unit	
	Designation	Symbol
Length	Centimetre	cm
Mass	Gram	g
Time	Second	s

The Technical System of Units

The Technical System of Units used the following base values and base units:

Base value	Base unit	
	Designation	Symbol
Length	Metre	m
Force	Kilopond	kp
Time	Second	s

In the Technical System of Units, force was defined as the base value with the unit name "kilopond" (kp). All forces were compared with the gravitational attraction of the Earth (weight). In contrast to mass, however, the acceleration of free fall (and therefore the weight) is dependent on the location.

The following definition was therefore given:
1 kilopond is the force with which the mass of 1 kilogram exerts pressure on its support surface at the location of the standard acceleration of free fall ($g_n = 9{,}806\,65\ m/s^2$).

Equation 4

$$1\ \text{kp} = 1\ \text{kg} \cdot 9{,}806\,65\ \text{m}/\text{s}^2 = 9{,}806\,65\ \frac{\text{kg m}}{\text{s}^2}$$

The relationship between the International System of Units SI and the Technical System of Units is given by virtue of the fact that, in the SI system, the derived, coherent quantity with the unit name "Newton" was defined for the force:

Equation 5

$$1\ \text{N} = 1\ \frac{\text{kg m}}{\text{s}^2}$$

Therefore, 1 kp = 9,806 65 N.

For technical conversions from one system to the other, it is normally sufficient to use 1 kp = 9,81 N.

Conversions into the International System of Units SI

Anglo-American systems

The Anglo-American units are based on older, English systems of units and are still in common use in the USA.
They include the units of the fps system ("foot-pound-second").

Units in the fps system

The following table shows the conversion of the most significant units in the fps system and SI:

	fps system		SI (MKS)	
Length	1 ft	= 1/3 yd = 12 in	1 ft	= 0,304 8 m
Area	1 ft^2	= 144 in^2	1 ft^2	= 0,092 903 m^2
Volume	1 ft^3 1 gal (US)	= 1728 in^3 = 6,228 2 gal (UK) = 0,832 68 gal (UK)	1 ft^3	= 0,028 316 9 m^3
Velocity	1 ft/s 1 knot	= 1,157 67 mile/h = 1,687 7 ft/s	1 ft/s	= 0,304 8 m/s
Acceleration	1 ft/s^2		1 ft/s^2	= 0,304 8 m/s^2
Mass	1 lb 1 slug	= cwt/112 = 32,174 lb	1 lb 1 slug	= 0,453 592 kg = 14,593 9 kg
Force	1 lbf 1 pdl	 = 0,031 081 lbf	1 lbf 1 pdl	= 4,448 22 N = 0,138 255 N
Work	1 ft · lb 1 btu	= 0,323 832 cal_{IT} = 252 cal_{IT} = 778,21 ft · lb	1 ft · lb 1 btu	= 1,355 82 J = 1,055 06 kJ
Pressure	1 lb/ft^2 1 lb/in^2 1 atm	= 6,944 4 · 10^{-3} lb/in^2 = 0,068 046 atm = 29,92 in Hg = 33,90 ft water	1 lb/ft^2 1 lb/in^2 1 atm	= 47,88 N/m^2 = 6 894,76 N/m^2 = 1,013 25 bar
Density	1 lb/ft^3 1 lb/gal	= 5,787 04 · 10^{-4} lb/in^3 = 6,228 2 lb/ft^3	1 lb/ft^3 1 lb/gal	= 16,018 5 kg/m^3 = 99,763 3 kg/m^3
Temperature	32 °F 212 °F	= 0 °C = 100 °C	1 °F	= 0,555 6 °C
Power	1 ft · lb/s	= 1,814 8 · 10^{-3} hp = 1,281 82 · 10^{-3} btu/s	1 ft · lb/s	= 1,353 34 W
Specific heat capacity	1 btu/(lb · deg F)		1 btu/(lb · deg F)	= 4,186 8 kJ/(kg · K)
Coefficient of thermal conductivity	1 btu/(ft · h · deg F)		1 btu/(ft · h · deg F)	= 1,730 6 W/(m · K)
Heat transfer (heat transition) coefficient	1 btu/(ft^2 · h · deg F)		1 btu/(ft^2 · h · deg F)	= 5,677 8 W/(m^2 · K)
Kinematic viscosity	1 ft^2/s		1 ft^2/s	= 0,092 903 m^2/s
Dynamic viscosity	1 lb/(ft · s)		1 lb/(ft · s)	= 1,488 16 kg/(m · s)

Source: Dubbel, 21st edition.

Length, area and volume dimensions The following table shows the conversion from German to English dimensions for length, area and volume:

German – English			English – German		
Length dimensions					
1 mm	= 0,039 370 14 inches		1 pt = 1/864 foot	= 1/72 inch	= 0,352 78 mm
1 cm	= 0,393 701 47 inches		1 inch		= 25,399 956 mm
1 m	= 3,280 851 feet		1 foot	= 12 inches	= 304,799 472 mm
1 m	= 1,093 616 yards				= 0,304 799 m
1 m	= 0,546 808 fathoms		1 yard	= 3 feet	
1 km	= 0,621 372 statute miles			= 36 inches	= 0,914 398 m
1 km	= 0,539 614 nautical miles		1 fathom	= 2 yards	
1 km	= 0,539 037 Admiralty miles			= 6 feet	
1 German statute mile		= 7,5 km		= 72 inches	= 1,828 797 m
1 German nautical mile		= 1,852 km	1 stat. mile	= 880 fathoms	
1 geographical mile		= 7,420 438 54 km (15 miles = 1 degree of arc)		= 1 760 yards	
				= 5 280 feet	
1 degree of arc		= 111,306 6 km		= 1 English mile	= 1,609 341 km
1 degree of meridian		= 111,120 6 km	1 common English mile	= 5 000 feet	= 1,523 995 km
			1 nautical mile	= 6 080 feet	= 1,853 178 km
			1 Admiralty mile	= 6 086,5 feet	= 1,855 16 km
				= 1/4 geographical mile	
				= 1/60 of degree of arc	
Area dimensions					
1 mm^2	= 0,001 550 01 square inch (inch^2)		1 sq. inch		= 6,451 578 cm^2
1 cm^2	= 0,155 006 35 square inch (inch^2)		1 sq. foot	= 144 sq. inches	= 929,027 2 cm^2
1 m^2	= 10,763 983 28 square feet (foot^2)				= 0,092 903 m^2
1 m^2	= 1,195 995 96 square yards		1 sq. yard	= 9 sq. feet	= 8 361,244 80 cm^2
1 a	= 100 m^2	= 0,024 711 acres	1 acres	= 160 sq. poles	
1 ha	= 100 a	= 2,471 063 acres		= 4 840 sq. yards	
1 km^2	= 100 ha	= 0,386 100 square miles		= 40,468 4 a	= 4 046,842 5 m^2
1 geographical square mile		= 55,062 91 km^2	1 sq. mile	= 640 acres	= 2,59 km^2
			1 sq. pole		= 25,298 676 m^2
			1 circular inch	= π/4 sq. inches	= 5,067 057 cm^2

Continuation of table, see Page 30.

Continuation of table, Length, area and volume dimensions, from Page 29.

German – English		English – German		
Volume dimensions				
1 cm^3	= 0,061 024 cubic inches ($inch^3$)	1 cub. inch		= 16,386 979 cm^3
1 dm^3	= 0,035 315 cubic feet ($foot^3$)	1 cub. foot	= 1728 cub. inches	= 28,316 700 dm^3
	= 61,024 061 cubic inches	1 cub. yard	= 27 cub. feet	= 0,764 551 m^3
1 m^3	= 1,307 957 cubic yards	1 reg. ton	= 100 cub. feet	= 2,831 670 m^3
	= 35,314 850 cubic feet	1 imperial gallon	= 277,26 cub. inches	= 4,543 454 l
1 m^3	= 0,353 148 register tons			
1 l	= 0,220 097 imperial gallons	1 bushel	= 8 gallons	= 36,347 632 l
1 l	= 0,027 512 bushels	1 imperial quarter	= 8 bushels	= 64 gallons
1 l	= 0,003 439 imperial quarters		= 290,781 056 l	= 2,907 811 hl
1 hl	= 100 l			
	= 0,343 901 imperial quarters			

Units of volume, flow rate

The following table shows the conversion from the fps system into SI for units of volume and flow rates:

	Conversion from								
↓	in^3 to cm^3	ft^3 to l	yd^3 to m^3	pt (UK) to l	liq · pt (US) to l	gal (UK) to l	gal (US) to l	barrel petrol. to l	$ft^3 \cdot min$ cfm to m^3/h
1,0	16,4	28,3	0,765	0,568	0,473	4,55	3,79	159	1,70
1,1	18,0	31,1	0,841	0,625	0,520	5,00	4,16	175	1,87
1,2	19,7	34,0	0,917	0,682	0,568	5,46	4,54	191	2,04
1,3	21,3	36,8	0,994	0,739	0,615	5,91	4,92	207	2,21
1,4	22,9	39,6	1,07	0,796	0,662	6,36	5,30	223	2,38
1,5	24,6	42,5	1,15	0,852	0,710	6,82	5,68	238	2,55
1,6	26,2	45,3	1,22	0,909	0,757	7,27	6,06	254	2,72
1,7	27,9	48,1	1,30	0,966	0,804	7,73	6,44	270	2,89
1,8	29,5	51,0	1,38	1,02	0,852	8,18	6,81	286	3,06
1,9	31,1	53,8	1,45	1,08	0,899	8,64	7,19	302	3,23

Continuation of table, see Page 31.

The table also applies to decimal multiples and submultiples.
Examples: 1 in^3 = 16,4 cm^3; 3 gal (UK) = 13,6 l; 30 gal (UK) = 136 l.
Source: Bosch, Automotive Handbook.

Continuation of table, Units of volume, flow rate, from Page 30.

	Conversion from								
↓	in^3 to cm^3	ft^3 to l	yd^3 to m^3	pt (UK) to l	liq · pt (US) to l	gal (UK) to l	gal (US) to l	barrel petrol. to l	$ft^3 \cdot min$ cfm to m^3/h
2,0	32,8	56,6	1,53	1,14	0,946	9,09	7,57	318	3,40
2,1	34,4	59,5	1,61	1,19	0,994	9,55	7,95	334	3,57
2,2	36,1	62,3	1,68	1,25	1,04	10,0	8,33	350	3,74
2,3	37,7	65,1	1,76	1,31	1,09	10,5	8,71	366	3,91
2,4	39,3	68,0	1,83	1,36	1,14	10,9	9,08	382	4,08
2,5	41,0	70,8	1,91	1,42	1,18	11,4	9,46	397	4,25
2,6	42,6	73,6	1,99	1,48	1,23	11,8	9,84	413	4,42
2,7	44,2	76,5	2,06	1,53	1,28	12,3	10,2	429	4,59
2,8	45,9	79,3	2,14	1,59	1,32	12,7	10,6	445	4,76
2,9	47,5	82,1	2,22	1,65	1,37	13,2	11,0	461	4,93
3,0	49,2	85,0	2,29	1,70	1,42	13,6	11,4	477	5,10
3,2	52,4	90,6	2,45	1,82	1,51	14,5	12,1	509	5,44
3,4	55,7	96,3	2,60	1,93	1,61	15,5	12,9	541	5,78
3,6	59,0	102	2,75	2,05	1,70	16,4	13,6	572	6,12
3,8	62,3	108	2,91	2,16	1,80	17,3	14,4	604	6,46
4,0	65,5	113	3,06	2,27	1,89	18,2	15,1	636	6,80
4,2	68,8	119	3,21	2,39	1,99	19,1	15,9	668	7,14
4,4	72,1	125	3,36	2,50	2,08	20,0	16,7	700	7,48
4,6	75,4	130	3,52	2,61	2,18	20,9	17,4	731	7,82
4,8	78,7	136	3,67	2,73	2,27	21,8	18,2	763	8,16
5,0	81,9	142	3,82	2,84	2,37	22,7	18,9	795	8,50
5,2	85,2	147	3,98	2,95	2,46	23,6	19,7	827	8,83
5,4	88,5	153	4,13	3,07	2,56	24,5	20,4	859	9,17
5,6	91,8	159	4,28	3,18	2,65	25,5	21,2	890	9,51
5,8	95,0	164	4,43	3,30	2,74	26,4	22,0	922	9,85
6,0	98,3	170	4,59	3,41	2,84	27,3	22,7	954	10,2
6,2	102	176	4,74	3,52	2,93	28,2	23,5	986	10,5
6,4	105	181	4,89	3,64	3,03	29,1	24,2	1018	10,9
6,6	108	187	5,05	3,75	3,12	30,0	25,0	1049	11,2
6,8	111	193	5,20	3,86	3,22	30,9	25,7	1081	11,6
7,0	115	198	5,35	3,98	3,31	31,8	26,5	1113	11,9
7,5	123	212	5,73	4,26	3,55	34,1	28,4	1192	12,7
8,0	131	227	6,12	4,55	3,79	36,4	30,3	1272	13,6
8,5	139	241	6,50	4,83	4,02	38,6	32,2	1351	14,4
9,0	147	255	6,88	5,11	4,26	40,9	34,1	1431	15,3
9,5	156	269	7,26	5,40	4,50	43,2	36,0	1510	16,1

The table also applies to decimal multiples and submultiples.
Examples: 1 in^3 = 16,4 cm^3; 3 gal (UK) = 13,6 l; 30 gal (UK) = 136 l.
Source: Bosch, Automotive Handbook.

Units of temperature The following table shows the conversion into various units of temperature:

T_K K Kelvin	t_C °C Degree Celsius	t_F °F Degree Fahrenheit	T_R °R Degree Rankin
$T_K = 273{,}15 + t_C$	$t_C = T_K - 273{,}15$	$t_F = \frac{9}{5} \cdot T_K - 459{,}67$	$T_R = \frac{9}{5} \cdot T_K$
$T_K = 255{,}38 + \frac{5}{9} \cdot t_F$	$t_C = \frac{5}{9}\left(t_F - 32\right)$	$t_F = 32 + \frac{9}{5} \cdot t_C$	$T_R = \frac{9}{5}\left(t_C + 273{,}15\right)$
$T_K = \frac{5}{9} \cdot T_R$	$t_C = \frac{5}{9}\left(T_R - 273{,}15\right)$	$t_F = T_R - 459{,}67$	$T_R = 459{,}67 + t_F$
Conversion of some temperatures			
0,00	–273,15	–459,67	0,00
+255,37	–17,78	0,00	+459,67
+273,15	0,00	+32,00	+491,67
+273,16[1)]	+0,01	+32,02	+491,69
+300,00	+26,85	+80,33	+540,00
+310,94	+37,78	+100	+559,67
+373,15	+100,00	+212	+671,67
+400,00	+126,85	+260,33	+720,00
+500,00	+226,85	+440,85	+900,00

[1)] The triple point of water is +0,01 °C.
This is the temperature point of pure water at which solid ice, liquid water and water vapour occur simultaneously in equilibrium (at 1013,25 hPa).
1 kelvin is the 273,16th part of the thermodynamic temperature of the triple point of water (13th General Conference on Weights and Measures 1967).
Temperature differential: 1 Kelvin = 1 degree Celsius = 1,8 degree Fahrenheit = 1,8 degree Rankin.

Roman numeral system

Definition In the Roman numeral system, a distinction is made between cardinal numbers and ordinal numbers:

Cardinal numbers				Ordinal numbers		
I = 1	X = 10	C = 100	M = 1000	V = 5	L = 50	D = 500

Conversion table The following table shows some conversion examples:

I	1	VII	7	XL	40	XCIX	99	DC	600
II	2	VIII	8	L	50	C	100	DCC	700
III	3	IX	9	LX	60	CC	200	DCCC	800
IV	4	X	10	LXX	70	CCC	300	CM	900
V	5	XX	20	LXXX	80	CD	400	CMXCIX	999
VI	6	XXX	30	XC	90	D	500	M	1000

Rules In the formation of Roman numerals, the following rules apply:

- Notation starts from the left
- Identical cardinal numbers are added consecutively. A maximum of 3 identical cardinal numbers may appear consecutively, while ordinal numbers are only written once:
  Permissible: III = 3; XX = 20
  Not permissible: XXXX = 40; VV = 10
- Smaller numbers are **added** to the **right** of larger numbers: VI = 6.
  Smaller numbers are **subtracted** to the **left** of larger numbers: IV = 4
- Ordinal numbers may be added but **not subtracted**:
  Permissible: LV = 55
  Not permissible: VL = 45
- Cardinal numbers may only be subtracted from the next largest cardinal number or ordinal number:
  Permissible: IV = 4; XL = 40; CD = 400
  Not permissible: IC = 99; XM = 990
- Examples:
  1 673 = MDCLXXIII; 1 891 = MDCCCXCI; 1 981 = MCMLXXXI.

Alphabets

German alphabet: Gothic The German alphabet in Gothic script is shown with the most significant ligatures. This was the most commonly used script in the German-speaking world from the middle of the 16th Century to the beginning of the 20th Century.

𝔄 𝔞 A a	𝔅 𝔟 B b	ℭ 𝔠 C c	𝔇 𝔡 D d	𝔈 𝔢 E e	𝔉 𝔣 F f	𝔊 𝔤 G g	ℌ 𝔥 H h	ℑ 𝔦 I i
𝔍 𝔧 J j	𝔎 𝔨 K k	𝔏 𝔩 L l	𝔐 𝔪 M m	𝔑 𝔫 N n	𝔒 𝔬 O o	𝔓 𝔭 P p	𝔔 𝔮 Q q	ℜ 𝔯 R r
𝔖 ſ S s	𝔗 𝔱 T t	𝔘 𝔲 U u	𝔙 𝔳 V v	𝔚 𝔴 W w	𝔛 𝔵 X x	𝔜 𝔶 Y y	ℨ 𝔷 Z z	
𝔄̈ 𝔞̈ Ä ä	𝔒̈ 𝔬̈ Ö ö	𝔘̈ 𝔲̈ Ü ü	𝔠𝔥 ch	𝔠𝔨 ck	𝔣𝔣 ff	𝔣𝔦 fi	𝔣𝔩 fl	𝔩𝔩 ll
ſ𝔦 si	ſſ ss	ſ𝔱 st	ß ß (sz)	𝔱𝔷 tz				

German alphabet: Handwriting The German alphabet in “Sütterlin” script is shown. This script was taught in German schools from 1915 to approximately 1940. The lower case letters are used today principally to indicate vectors.

A a	B b	C c	D d	E e	F f	G g	H h	I i
J j	K k	L l	M m	N n	O o	P p	Q q	R r
S s	T t	U u	V v	W w	X x	Y y	Z z	
Ä ä	Ö ö	Ü ü	ß					

Greek alphabet The Greek alphabet and some variant shapes of specific letters are shown:

Α α Alpha (a)	Β β Beta (b)	Γ γ Gamma (c)	Δ δ Delta (d)	Ε ϵ Epsilon (e)	Ζ ζ Zeta (z)	Η η Eta (e)	Θ ϑ Theta (th)	Ι ι Iota (i)
Κ κ Kappa (k)	Λ λ Lambda (l)	Μ μ Mu (m)	Ν ν Nu (n)	Ξ ξ Xi (x)	Ο ο Omicron (o)	Π π Pi (P)	Ρ ρ Rho (r)	Σ σ, ς Sigma (s)
Τ τ Tau (t)	Υ υ Upsilon (ü)	Φ φ, ϕ Phi (f)	Χ χ Chi (ch)	Ψ ψ Psi (ps)	Ω ω Omega (o)			

Cyrillic (Russian) alphabet The Cyrillic (Russian) alphabet is shown:

А а A a	Б б B b	В в W w	Г г G g	Д д D d	Е е Je je	Ё ё Jo jo	Ж ж Sch sch	З з S s
И и I i	Й й I i	К к K k	Л л L l	М м M m	Н н N n	О о O o	П п P p	Р р R r
С с Ss ss	Т т T t	У у U u	Ф ф F f	Х х Ch ch	Ц ц Z z	Ч ч Tsch tsch	Ш ш Sch sch	Щ щ Schtsch schtsch
Ъ ъ – – hard	Ы ы Y y muffled	Ь ь – – soft	Э э E e	Ю ю Ju ju	Я я Ja ja			

Phonetic alphabets (for voice traffic)

Phonetic alphabets for phonetic dictation

In order to achieve comprehensible communication of text information, phonetic alphabets are used in which names are assigned to the individual letters. For example, English text information can be communicated using the Telecom B phonetic alphabet. In 1927, the first internationally recognised phonetic alphabet was introduced by the International Telecommunications Union (ITU):

National phonetic alphabet (Telecom B)					
A	Alfred	J	Jack	S	Samuel
B	Benjamin	K	King	T	Tommy
C	Charles	L	London	U	Uncle
D	David	M	Mary	V	Victor
E	Edward	N	Nellie	W	William
F	Frederick	O	Oliver	X	X-ray
G	George	P	Peter	Y	Yellow
H	Harry	Q	Queen	Z	Zebra
I	Isaac	R	Robert		

International ITU phonetic alphabet					
A	Amsterdam	J	Jerusalem	S	Santiago
B	Baltimore	K	Kilogramme	T	Tripoli
C	Casablanca	L	Liverpool	U	Upsala
D	Denmark	M	Madagascar	V	Valencia
E	Edison	N	New York	W	Washington
F	Florida	O	Oslo	X	Xantippe
G	Gallipoli	P	Paris	Y	Yokohama
H	Havana	Q	Québec	Z	Zurich
I	Italia	R	Roma		

International phonetic alphabet for aviation, radio transmission, NATO, maritime

In 1956, the currently used phonetic alphabet was brought into force by the ICAO in international air traffic communication, which was also adopted by NATO, for international radio transmission (under the ITU Radio Regulations RR) and for maritime use (by the IMO). The words are predominantly pronounced in English, with the pronunciation differing in some cases from the written form. The ITU uses twin code words for figures:

Word	Pronunciation[1)]	Word	Pronunciation[1)]	Word	Pronunciation[1)]
International phonetic alphabet of the ICAO, NATO, ITU-RR, IMO					
A Alfa	**AL**-FAH	J Juliett	**JEW**-LEE-ETT	T Tango	**TANG**-GO
B Bravo	**BRAH**-VOH	K Kilo	**KEY**-LOH	U Uniform	**YOU**-NEE-FORM, **OO**-NEE-FORM
C Charlie	**CHAR**-LEE, **SHAR**-LEE	L Lima	**LEE**-MAH	V Victor	**VIK**-TAH
D Delta	**DELL**-TAH	M Mike	MIKE	W Whiskey	**WISS**-KEY
E Echo	**ECK**-OH	N November	NO-**VEM**-BER	X X-ray	**ECKS**-RAY
F Foxtrot	**FOKS**-TROT	O Oscar	**OSS**-CAH	Y Yankee	**YANG**-KEY
G Golf	GOLF	P Papa	PAH-**PAH**	Z Zulu	**ZOO**-LOO
H Hotel	**HOH**-TELL	Q Quebec	**KEH**-BECK		
I India	**IN**-DEE-AH	R Romeo	**ROW**-ME-OH		
		S Sierra	SEE-**AIR**-RAH		
Figure codes of the ICAO, NATO, IMO					
0 Zero	**ZEE**-RO	4 Four	**FOW**-ER	8 Eight	AIT
1 One	WUN	5 Five	FIFE	9 Nine	**NIN**-ER
2 Two	TOO	6 Six	SIX	1000 Thousand	**TOU**-SAND
3 Three	TREE	7 Seven	**SEV**-EN		
Figures codes of the ITU-RR					
0 Nadazero	NAH-DAH-ZAY-ROH	4 Kartefour	KAR-TAY-FOWER	8 Oktoeight	OK-TOH-AIT
1 Unaone	OO-NAH-WUN	5 Pantafive	PAN-TAH-FIFE	9 Novenine	NO-VAY-NINER
2 Bissotwo	BEES-SOH-TOO	6 Soxisix	SOK-SEE-SIX		
3 Terrathree	TAY-RAH-TREE	7 Setteseven	SAY-TAY-SEVEN		

1) Syllables indicated bold are emphasized.

Mathematics

General symbols, numbers, definitions

Mathematical symbols

The following table shows a selection of the most important mathematical symbols.

Symbol	Designation	Symbol	Designation
$+$	Plus	$\sqrt{\ }$	Root of ($\sqrt[n]{\ }$ = nth root of)
$-$	Minus	$n!$	n factorial (example: $3! = 1 \cdot 2 \cdot 3 = 6$)
$\cdot$ or $\times$	Multiplied by	$\lvert x \rvert$	Absolute value of x
/ or :	Divided by	$\rightarrow$	Approaches
$=$	Equals	∞	Infinity
$\neq$	Not equal to	i or j	Imaginary unit, $i^2 = -1$
$<$	Less than	$\perp$	Perpendicular to
$\leqq$	Less than or equal to	$\parallel$	Parallel to
$>$	Greater than	$\sphericalangle$	Angle
$\geqq$	Greater than or equal to	$\triangle$	Triangle
$\approx$	Approximately equal to	lim	Limit
$\ll$	Much less than	Δ	Delta (difference between two values)
$\gg$	Much greater than	d	Total differential
$\triangleq$	Corresponds to	∂	Partial differential
...	And so forth, to	$\int$	Integral
$\sim$	Proportional	log	Logarithm
Σ	Summation of	ln	Logarithm to base e, $e = 1 + 1/1! + 1/2! + 1/3! + \ldots$
Π	Product	lg	Logarithm to base 10

Frequently used numbers

The following table shows a selection of numbers or constants which are frequently used in mathematics.

Symbol	Value	Symbol	Value	Symbol	Value
e	2,718 282	ln 10	2,302 585	π	3,141 593
e^2	7,389 056	1/(ln 10)	0,434 294	$\sqrt{\pi}$	1,772 454
$1/e$	0,367 879	$\sqrt{2}$	1,414 214	$1/\pi$	0,318 310
lg e	0,434 294	$1/\sqrt{2}$	0,707 107	π^2	9,869 604
$\sqrt{e}$	1,648 721	$\sqrt{3}$	1,732 051	$180/\pi$	57,295 780
1/(lg e)	2,302 585			$\pi/180$	0,017 453

Golden section

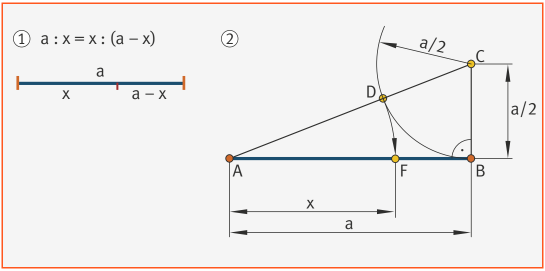
The golden section (division in extreme and mean ratio) is defined as the ratio a/x, where:

Equation 1

$$a : x = x : (a - x) \qquad \text{where} \quad \frac{a}{x} = \frac{\left(\sqrt{5}+1\right)}{2} \approx 1{,}618$$

Figure 1
Golden section

① Division in extreme and mean ratio
② Construction, general formulation



Pythagorean numbers

Pythagorean numbers are integers x, y, z, to which the following equation applies:

Equation 2

$$x^2 + y^2 = z^2$$

Triangles formed from the sides x, y, z in any unit of length are right-angled. If we apply:

Equation 3

$$x = 2\,p\,q \qquad y = p^2 - q^2 \qquad z = p^2 + q^2$$

and where p and q are random integers, this gives the Pythagorean numbers:

p	q	x	y	z	p	q	x	y	z
2	1	4	3	5	4	2	16	12	20
3	1	6	8	10	5	2	20	21	29
4	1	8	15	17	4	3	24	7	25
5	1	10	24	26	5	3	30	16	34
3	2	12	5	13	5	4	40	9	41

The following should apply:

Equation 4

$0 < q < p$	Natural numbers
p, q	Co-prime
$p + q$	Odd

Prime numbers

The following table shows prime numbers and compound numbers that are not divisible by 2, 3 or 5, with their smallest factors. The numbers listed are below 1 000.

7		107		209	11	311		409		511	7	613		713	23	817	19	917	7
11		109		211		313		413	7	517	11	617		719		821		919	
13		113		217	7	317		419		521		619		721	7	823		923	13
17		119	7	221	13	319	11	421		523		623	7	727		827		929	
19		121	11	223		323	17	427	7	527	17	629	17	731	17	829		931	7
23		127		227		329	7	431		529	23	631		733		833	7	937	
29		131		229		331		433		533	13	637	7	737	11	839		941	
31		133	7	233		337		437	19	539	7	641		739		841	29	943	23
37		137		239		341	11	439		541		643		743		847	7	947	
41		139		241		343	7	443		547		647		749	7	851	23	949	13
43		143	11	247	13	347		449		551	19	649	11	751		853		953	
47		149		251		349		451	11	553	7	653		757		857		959	7
49	7	151		253	11	353		457		557		659		761		859		961	31
53		157		257		359		461		559	13	661		763	7	863		967	
59		161	7	259	7	361	19	463		563		667	23	767	13	869	11	971	
61		163		263		367		467		569		671	11	769		871	13	973	7
67		167		269		371	7	469	7	571		673		773		877		977	
71		169	13	271		373		473	11	577		677		779	19	881		979	11
73		173		277		377	13	479		581	7	679	7	781	11	883		983	
77	7	179		281		379		481	13	583	11	683		787		887		989	23
79		181		283		383		487		587		689	13	791	7	889	7	991	
83		187	11	287	7	389		491		589	19	691		793	13	893	19	997	
89		191		289	17	391	17	493	17	593		697	17	797		899	29		
91	7	193		293		397		497	7	599		701		799	17	901	17		
97		197		299	13	401		499		601		703	19	803	11	907			
101		199		301	7	403	13	503		607		707	7	809		911			
103		203	7	307		407	11	509		611	13	709		811		913	11		

Binomial coefficients

The binomial coefficient $\binom{n}{k}$ is defined as:

Equation 5

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \qquad \text{where} \quad n \geqq k$$

n	$\binom{n}{0}$	$\binom{n}{1}$	$\binom{n}{2}$	$\binom{n}{3}$	$\binom{n}{4}$	$\binom{n}{5}$	$\binom{n}{6}$	$\binom{n}{7}$
1	1	1						
2	1	2	1					
3	1	3	3	1				
4	1	4	6	4	1			
5	1	5	10	10	5	1		
6	1	6	15	20	15	6	1	
7	1	7	21	35	35	21	7	1
8	1	8	28	56	70	56	28	8
9	1	9	36	84	126	126	84	36
10	1	10	45	120	210	252	210	120
11	1	11	55	165	330	462	462	330
12	1	12	66	220	495	792	924	792
13	1	13	78	286	715	1 287	1 716	1 716
14	1	14	91	364	1 001	2 002	3 003	3 432
15	1	15	105	455	1 365	3 003	5 005	6 435

n	$\binom{n}{8}$	$\binom{n}{9}$	$\binom{n}{10}$	$\binom{n}{11}$	$\binom{n}{12}$	$\binom{n}{13}$	$\binom{n}{14}$	$\binom{n}{15}$
8	1							
9	9	1						
10	45	10	1					
11	165	55	11	1				
12	495	220	66	12	1			
13	1 287	715	286	78	13	1		
14	3 003	2 002	1 001	364	91	14	1	
15	6 435	5 005	3 003	1 365	455	105	15	1

Arithmetic

Laws and rules

The following laws and rules are defined in arithmetic:

Rules of signs
Equation 6

The following rules of signs apply:

$$a+(-b) = a-b \qquad a-(-b) = a+b \qquad a\cdot(-b) = -a\,b$$

$$(-a)\cdot(-b) = a\,b \qquad (-a)\cdot b = -a\,b \qquad (-a)/b = -\frac{a}{b}$$

$$a/(-b) = -\frac{a}{b} \qquad (-a)/(-b) = \frac{a}{b}$$

Commutative law
Equation 7

The commutative law of addition and multiplication is:

$$a+b = b+a \qquad a\cdot b = b\cdot a$$

Associative law
Equation 8

The associative law of addition and multiplication is:

$$(a+b)+c = a+(b+c) \qquad (a\,b)\cdot c = a\cdot(b\,c) = a\cdot b\cdot c$$

Binomials

The following equations give examples of products of algebraic sums and binomial formulae.

Equation 9

$$(a+b)\cdot(c+d) = a\,c+a\,d+b\,c+b\,d$$

$$(a\pm b)^2 = a^2 \pm 2\,a\,b+b^2$$

$$(a+b)\cdot(a-b) = a^2-b^2$$

$$(a\pm b)^3 = a^3 \pm 3\,a^2\,b+3\,a\,b^2 \pm b^3$$

$$(a\pm b)^n = \sum_{k=0}^{n} (\pm 1)^k \binom{n}{k} a^k\, b^{n-k}$$

$$(a+b+c)^2 = a^2+b^2+c^2+2\,a\,b+2\,a\,c+2\,b\,c$$

Mean values
Equation 10

The arithmetic mean is:

$$\frac{a+b}{2}\;;\quad \frac{a+b+c}{3}\;;\ldots \qquad A_n = \frac{1}{n}\left(a_1+a_2+\ldots+a_n\right)$$

Equation 11

The geometric mean is:

$$\sqrt{a\cdot b}\;;\quad \sqrt[3]{a\cdot b\cdot c}\;;\ldots \qquad G_n = \sqrt[n]{a_1\cdot a_2\cdot\ldots\cdot a_n}$$

Equation 12

The harmonic mean is:

$$\frac{2\,a\,b}{a+b} = 2\bigg/\left(\frac{1}{a}+\frac{1}{b}\right) \qquad H_n = \frac{n}{1/a_1+1/a_2+\ldots+1/a_n}$$

Equation 13

The following applies:

$$H_n \leqq G_n \leqq A_n$$

Powers
Equation 14

The following equations give examples of how powers are formed:

$$a^n \cdot a^m = a^{n+m} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad a^n \cdot b^n = (ab)^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \qquad (a^m)^n = a^{m \cdot n} \qquad a^{-n} = \frac{1}{a^n}$$

$$0^n = 0 \qquad a^0 = 1 \quad (a \neq 0)$$

Roots
Equation 15

The following equations give examples of how roots are formed:

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad \left(\sqrt[n]{a}\right)^n = a \qquad \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \qquad \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m \qquad \sqrt[np]{a^{mp}} = \sqrt[n]{a^m}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a} \qquad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

Algebra

Algebraic equation of the 2nd degree

An algebraic equation of the 2nd degree (referred to as a quadratic equation) is described below.

Quadratic equation
Equation 16

The solutions to a quadratic equation are as follows:

Quadratic equation: $a x^2 + b x + c = 0$

Solutions: $x_1, x_2 = \dfrac{-b \pm \sqrt{b^2 - 4 a c}}{2 a}$

Discriminant
Equation 17

If we define the discriminant Δ for general instances as:

$$\Delta = b^2 - 4 a c$$

we arrive at the following solutions:

Equation 18

$\Delta > 0$	2 different real solutions
$\Delta = 0$	2 identical real solutions
$\Delta < 0$	2 conjugate complex solutions

Normal form
Equation 19

The normal form of the quadratic equation is:

Normal form: $x^2 + p x + q = 0$

Solutions: $x_1, x_2 = -\dfrac{p}{2} \pm \sqrt{(p/2)^2 - q}$

Algebraic equation of the 1st degree
Equation 20

Two equations of the 1st degree with 2 unknown quantities can be calculated with the aid of matrices:

$$a_{11}\,x_1 + a_{12}\,x_2 = k_1$$
$$a_{21}\,x_1 + a_{22}\,x_2 = k_2$$

Determinant

The equations give the corresponding determinant D and the counter determinants D_{x1} and D_{x2}:

Equation 21

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\,a_{22} - a_{21}\,a_{12}$$

$$D_{x1} = \begin{vmatrix} k_1 & a_{12} \\ k_2 & a_{22} \end{vmatrix} = k_1\,a_{22} - k_2\,a_{12}$$

$$D_{x2} = \begin{vmatrix} a_{11} & k_1 \\ a_{21} & k_2 \end{vmatrix} = a_{11}\,k_2 - a_{21}\,k_1$$

Solution
Equation 22

For $D \neq 0$, this gives the clear solution:

$$x_1 = \frac{D_{x1}}{D} \qquad\qquad x_2 = \frac{D_{x2}}{D}$$

Logarithms
Equation 23

The general logarithm is defined as:

$$\log_b a = c \quad \text{and means that} \quad b^c = a \qquad a > 0,\ b > 1$$

In this case, b denotes the base, a denotes the anti-logarithm and c denotes the logarithm.

Logarithmic laws
Equation 24

The following relationships are regarded as logarithmic laws:

$$\log_b (a\,c) = \log_b a + \log_b c$$

$$\log_b \frac{a}{c} = \log_b a - \log_b c$$

$$\log_b \left(a^n\right) = n \log_b a$$

$$\log_b \sqrt[n]{a} = \frac{1}{n} \log_b a$$

in addition to the following special cases:

Equation 25

$$\log_b 0 = -\infty \qquad \log_b 1 = 0 \qquad \log_b b = 1 \qquad \log_b \infty = \infty$$

Natural logarithm

Logarithms to the base e = 2,71828128459... are called natural logarithms.
These are written as ln a instead of $\log_e$ a.

The following relationships apply:

Equation 26

$$\ln (e^{\pm n}) = \pm n$$

$$\ln (a \cdot 10^n) = \ln a + \ln (10^n) \qquad \ln (a/10^n) = \ln a - \ln (10^n)$$

The module M_b to base b is defined as:

Equation 27

$$M_b = \log_b e = 1/\ln b$$

$$\log_b a = M_b \ln a$$

Common (Briggs') logarithm

Logarithms to the base 10 are called common or Briggs' logarithms.
These are written as lg a instead of $\log_{10}$ a.

The following relationships apply:

Equation 28

$$\lg (10^{\pm n}) = \pm n$$

$$\lg (a \cdot 10^n) = \lg a + n \qquad \lg (a/10^n) = \lg a - n$$

The logarithmic laws which apply to the common logarithm are (see also Equation 24):

Equation 29

$$\lg (u \cdot v) = \lg u + \lg v$$

$$\lg \frac{u}{v} = \lg u - \lg v \qquad \lg \frac{v}{u} = -\lg \frac{u}{v}$$

$$\lg u^n = n \cdot \lg u \qquad \lg \sqrt[n]{u} = \frac{1}{n} \cdot \lg u$$

Conversion of logarithms
Equation 30

The following relationships apply between natural and common logarithms:

$$M_{10} = 0{,}434\,294\,4819 = \lg e = 1 / \ln 10 = 1 / 2{,}302\,585\,0930$$

$$\ln x = \ln 10 \; \lg x = 2{,}302\,585\,0930 \; \lg x$$

$$\lg x = \lg e \; \ln x = 0{,}434\,294\,4819 \; \ln x$$

$$\ln 10 \; \lg e = 1$$

Complex numbers
Equation 31

A complex number z consists of a real and an imaginary part:

$$z = x + i\,y$$

The following applies to the imaginary unit i:

Equation 32

$$i = \sqrt{-1} \qquad i^2 = -1 \qquad i^3 = -i \qquad i^4 = 1 \qquad 1/i = -i$$

Conjugation
Equation 33

The complex number conjugated to z is:

$$z^* = x - i\,y$$

Absolute value
Equation 34

The absolute value of z is:

$$r = \sqrt{z \cdot z^*} = \sqrt{(x + i\,y) \cdot (x - i\,y)} = \sqrt{x^2 + y^2}$$

Argument
Equation 35

The argument of z is:

$$\varphi = \arctan\,(y/x)$$

Normal form
Equation 36

The normal form of z is:

$$z = x + i\,y = r\,(\cos\varphi + i\,\sin\varphi)$$

Exponential form
Equation 37

The following is defined for the exponential form of the complex number z:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi \qquad \text{Euler's equation}$$

$$z = r\cdot(\cos\varphi + i\sin\varphi) = r\cdot e^{i\varphi}$$

$$e^{-i\varphi} = \cos\varphi - i\sin\varphi$$

The trigonometric functions and their relationship with the exponential function are presented in the section Circular functions (trigonometric functions) from Page 49 to Page 51.

Power
Equation 38

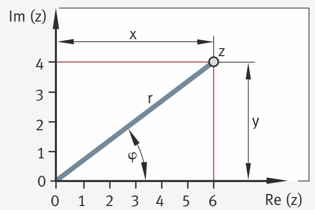
The power of z is:

$$z^n = r^n\cdot(\cos n\varphi + i\sin n\varphi)$$

Representation
Figure 2
Representation of the complex number

Im (z) = imaginary axis
Re (z) = real axis

The complex number z corresponds to a point in a Gaussian plane:



The following is an example of a calculation based on the graph:

Equation 39

$$z = 6 + i4 \quad \text{where} \quad x = 6;\quad y = 4$$

$$r = \sqrt{36+16} = \sqrt{52} \approx 7{,}2$$

$$\varphi = \arctan\left(y/x\right) = \arctan 0{,}667 \approx 33{,}7°$$

$$z = 7{,}2\cdot(\cos 33{,}7° + i\sin 33{,}7°)$$

Sequences and progressions

Arithmetic sequence
Equation 40

In the case of an arithmetic sequence, the difference d between two consecutive terms is constant:

$$a\,,\ a+d\,,\ a+2\,d\,,\ a+3\,d\,,\ \dots,\ a+(n-1)\,d$$

Arithmetic progression
Equation 41

In the case of an arithmetic progression, each term is the arithmetic mean of its two adjacent terms:

$$a+(a+d)+(a+2\,d)+(a+3\,d)+\ \dots\ +[a+(n-1)\,d]$$

This gives the following for the k-th term and the final term (n):

Equation 42

$$a_k = a+(k-1)\,d \qquad a_n = a+(n-1)\,d$$

and for the sum:

Equation 43

$$S = \frac{n}{2}\cdot(a+a_n) = \frac{n}{2}\cdot\left[2\,a+(n-1)\,d\right]$$

Geometric sequence
Equation 44

In the case of a geometric sequence, the quotient q of two consecutive terms is constant:

$$a_1\,,\ a_1\cdot q\,,\ a_1\cdot q^2,\ a_1\cdot q^3,\ \dots,\ a_1\cdot q^{n-1}$$

Geometric progression
Equation 45

In the case of a geometric progression, each term is the geometric mean of its two adjacent terms:

$$a_1+a_1\cdot q+a_1\cdot q^2+a_1\cdot q^3+\ \dots\ +a_1\cdot q^{n-1}$$

This gives the following for the k-th term and the final term (n):

Equation 46

$$a_k = a_1\cdot q^{k-1} \qquad a_n = a_1\cdot q^{n-1}$$

and for the sum:

Equation 47

$$S = \frac{a_1-a_n\,q}{1-q} \qquad \text{where} \qquad q<1$$

$$S = \frac{a_1\left(1-q^n\right)}{1-q} \qquad \text{where} \qquad q<1$$

$$S = \frac{a_n\,q-a_1}{q-1} \qquad \text{where} \qquad q>1$$

$$S = \frac{a_1\left(q^n-1\right)}{q-1} \qquad \text{where} \qquad q>1$$

Analysis

Derivatives and differentials

The differentiation rules specified below apply to the formation of derivatives (differential quotients).

Derivative of sum or difference
Equation 48

The following applies to the derivative of a sum or difference:

$$y = u(x) \pm v(x) \qquad y' = u'(x) \pm v'(x)$$

Derivative of product or quotient
Equation 49

The following applies to the derivative of a product or quotient:

$$y = u(x) \cdot v(x) \qquad y' = v(x) \cdot u'(x) + u(x) \cdot v'(x)$$

$$y = \frac{u(x)}{v(x)} \qquad y' = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{\left[v(x)\right]^2}$$

Chain rule
Equation 50

The following chain rule also applies:

$$y = f(z)$$

$$z = g(w) \qquad \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dw} \cdot \frac{dw}{dx}$$

$$w = h(x)$$

Differential forms of the basic functions

A selection of 1st derivatives of elementary functions can be found below:

Function	1st derivative	Function	1st derivative
$y = a$	$y' = 0$	$y = \sin x$	$y' = \cos x$
$y = x$	$y' = 1$	$y = \sin(ax)$	$y' = a \cdot \cos(a\,x)$
$y = mx + b$	$y' = m$	$y = \cos x$	$y' = -\sin x$
$y = ax^n$	$y' = n \cdot a \cdot x^{n-1}$	$y = \tan x$	$y' = 1/\cos^2 x$
$y = \sqrt{x}$	$y' = 1/\left(2 \cdot \sqrt{x}\right)$	$y = \cot x$	$y' = -1/\sin^2 x$
$y = 1/x$	$y' = -1/x^2$	$y = \ln \sin x$	$y' = \cot x$
$y = a^x$	$y' = a^x \cdot \ln a$	$y = \ln \tan x$	$y' = 2/\sin(2\,x)$
$y = e^x$	$y' = e^x$	$y = \arcsin x$	$y' = 1/\sqrt{1-x^2}$
$y = e^{ax}$	$y' = a \cdot e^{ax}$	$y = \arccos x$	$y' = -1/\sqrt{1-x^2}$
$y = x^x$	$y' = x^x \cdot \left(1 + \ln x\right)$	$y = \arctan x$	$y' = 1/\left(1+x^2\right)$
$y = \log_a x$	$y' = \frac{1}{x} \cdot \log_a e$	$y = \operatorname{arccot} x$	$y' = -1/\left(1+x^2\right)$
$y = \ln x$	$y' = \frac{1}{x}$	$y = \sinh x$	$y' = \cosh x$

Integrals of antiderivatives (basic integrals)

Integration is the reverse of differentiation.

A selection of basic integrals can be found below:

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ for $[n \neq -1]$	$\int \cosh x \, dx = \sinh x + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$
$\int e^x \, dx = e^x + C$	$\int \frac{dx}{\cosh^2 x} = -\tanh x + C$
$\int e^{a\,x} \, dx = \frac{1}{a} e^{a\,x} + C$	$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C = -\arccos x + C$
$\int \ln x \, dx = x \ln x - x + C$	$\int \frac{dx}{\sqrt{x^2+1}} = \operatorname{arcsinh} x + C$ $= \ln\left(x + \sqrt{x^2+1}\right) + C$
$\int a^{b\,x} \, dx = \frac{1}{b \ln a} a^{b\,x} + C$	$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arccosh} x + C$ $= \ln\left(x + \sqrt{x^2-1}\right) + C$
$\int a^x \ln a \, dx = a^x + C$	$\int \frac{dx}{1+x^2} = \arctan x + C = -\operatorname{arccot} x + C$
$\int \sin x \, dx = -\cos x + C$	$\int \frac{dx}{1-x^2} = \operatorname{arctanh} x + C = \frac{1}{2} \ln \frac{1+x}{1-x} + C$ for $[x^2 < 1]$
$\int \cos x \, dx = \sin x + C$	$\int \frac{dx}{1-x^2} = \operatorname{arccoth} x + C = \frac{1}{2} \ln \frac{x+1}{x-1} + C$ for $[x^2 > 1]$
$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{\sqrt{1+x}}{\sqrt{1-x}} \, dx = \arcsin x - \sqrt{1-x^2} + C$
$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{x\sqrt{x^2-1}} = \arccos \frac{1}{x} + C$
$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x\sqrt{1 \pm x^2}} = -\ln \frac{1+\sqrt{1 \pm x^2}}{x} + C$

Geometry

Circular functions (trigonometric functions)

Circular or trigonometric functions are defined as:

Equation 51

$$\sin \alpha = \frac{a}{c} \qquad \cos \alpha = \frac{b}{c} \qquad \tan \alpha = \frac{a}{b} \qquad \cot \alpha = \frac{b}{a}$$

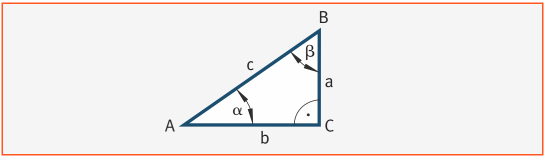


Figure 3
Right-angled triangle

This is represented on the unit circle as follows:

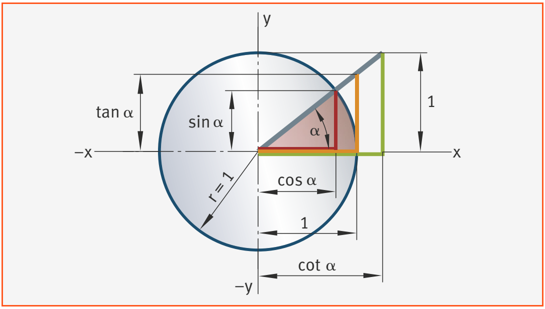


Figure 4
Trigonometric functions on the unit circle

φ =	±α	90 ± α	180 ± α	270 ± α
sin φ =	±sin α	cos α	∓sin α	–cos α
cos φ =	cos α	∓sin α	–cos α	±sin α
tan φ =	±tan α	∓cot α	±tan α	∓cot α
cot φ =	±cot α	∓tan α	±cot α	∓tan α

Conversion from degree size to arc size:

Equation 52

$$\widehat{\alpha} = \operatorname{arc} \alpha = \frac{\pi \cdot \alpha}{180°} \operatorname{rad} = \frac{\alpha}{57{,}3°}$$

$$\widehat{1°} = \operatorname{arc} 1° = \frac{\pi}{180} = 0{,}017\,435$$

$$\operatorname{arc} 57{,}3° = 1$$

Relationships between trigonometric functions
Equation 53

The relationships described below also apply between the trigonometric functions:

$$\cos^2\alpha + \sin^2\alpha = 1 \qquad \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{1}{\cot\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha} \qquad \operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$$

$$\sin 2\alpha = 2\sin\alpha\,\cos\alpha \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\tan 2\alpha = \frac{2}{\cot\alpha - \tan\alpha} \qquad \cot 2\alpha = \frac{\cot\alpha - \tan\alpha}{2}$$

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha \qquad \cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$\sin\frac{\alpha}{2} = \frac{1}{2}\cdot\sqrt{2 - 2\cos\alpha} \qquad \cos\frac{\alpha}{2} = \frac{1}{2}\cdot\sqrt{2 + 2\cos\alpha}$$

Addition theorems for trigonometric functions
Equation 54

The addition theorems for trigonometric functions are:

$$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\cdot\tan\beta}$$

$$\cot(\alpha \pm \beta) = \frac{\cot\alpha\cdot\cot\beta \mp 1}{\cot\beta \pm \cot\alpha}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{\alpha \pm \beta}{2}\cdot\cos\frac{\alpha \mp \beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha + \beta}{2}\cdot\cos\frac{\alpha - \beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha + \beta}{2}\cdot\sin\frac{\alpha - \beta}{2}$$

$$\tan\alpha \pm \tan\beta = \frac{\sin(\alpha \pm \beta)}{\cos\alpha\cdot\cos\beta}$$

$$\cot\alpha \pm \cot\beta = \frac{\sin(\beta \pm \alpha)}{\sin\alpha\cdot\sin\beta}$$

Euler's equation
Equation 55

With the aid of Euler's equation $e^{\pm ix} = \cos x \pm i \sin x$ we arrive at:

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad ; \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \text{where} \quad i = \sqrt{-1}$$

Hyperbolic functions

The following relationships apply to gyperbolic functions (where, for example, "sinh" is read as "hyperbolic sign"):

Equation 56

$$\sinh x = \left(e^{x} - e^{-x}\right)/2 \qquad \cosh x = \left(e^{x} + e^{-x}\right)/2$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad \coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$

$$\sinh x + \cosh x = e^{x} \qquad \sinh 2x = 2 \sinh x \cosh x$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad \cosh 2x = \cosh^2 x + \sinh^2 x$$

Ultimately arriving at (where "ar" is read as "Area"):

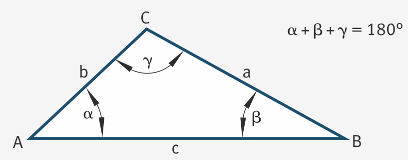
Equation 57

$$y = \sinh x \qquad x = \operatorname{ar} \sinh y$$

Plane triangle

The plane triangle is marked as follows:

Figure 5
Plane triangle



Sine law
Equation 58

This gives the sine law:

$$a : b : c = \sin \alpha : \sin \beta : \sin \gamma$$

Cosine law
Equation 59

and the cosine law:

$$c^2 = a^2 + b^2 - 2ab \cdot \cos \gamma$$

The following applies on the same basis to the remaining angles:

Equation 60

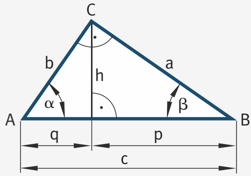
$$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos \beta$$

Right-angled triangle

The right-angled triangle is represented as follows:

Figure 6
Right-angled triangle



The following rules apply to right-angled triangles:

Pythagorean theorem

Equation 61

The Pythagorean theorem:

$$c^2 = a^2 + b^2 \quad ; \qquad c = \sqrt{a^2 + b^2}$$

Altitude theorem

Equation 62

The altitude theorem:

$$h^2 = p \cdot q \quad ; \qquad h = \sqrt{p \cdot q}$$

Cathetus theorem

Equation 63

The cathetus theorem:

$$a^2 = p \cdot c \quad ; \quad a = \sqrt{p \cdot c} \qquad \text{and} \qquad b^2 = q \cdot c \quad ; \quad b = \sqrt{q \cdot c}$$

Trigonometric functions

The following aspect ratios apply to the right-angled triangle:

sin α = a : c	Opposite side : hypotenuse	sin β = b : c
cos α = b : c	Adjacent side : hypotenuse	cos β = a : c
tan α = a : b	Opposite side : adjacent side	tan β = b : a
cot α = b : a	Adjacent side : opposite side	cot β = a : b

This gives the trigonometric functions:

Cathetus	a =	$\sqrt{c^2 - b^2}$	b · tan α	b · cot β	c · sin α	c · cos β
	b =	$\sqrt{c^2 - a^2}$	a · tan β	a · cot α	c · sin β	c · cos α
Hypotenuse	c =	$\sqrt{a^2 + b^2}$	$\frac{a}{\sin\alpha}$	$\frac{a}{\cos\beta}$	$\frac{b}{\sin\beta}$	$\frac{b}{\cos\alpha}$
Angle	α =	90° – β	$\sin\alpha = \frac{a}{c}$	$\tan\alpha = \frac{a}{b}$	$\cos\alpha = \frac{b}{c}$	$\cot\alpha = \frac{b}{a}$
	β =	90° – α	$\sin\beta = \frac{b}{c}$	$\tan\beta = \frac{b}{a}$	$\cos\beta = \frac{a}{c}$	$\cot\beta = \frac{a}{b}$
Area	A =	$\frac{a \cdot b}{2}$	$\frac{a \cdot c \cdot \sin\beta}{2}$	$\frac{a^2 \cdot \tan\beta}{2}$	$\frac{b \cdot c \cdot \cos\beta}{2}$	$\frac{b^2 \cdot \cot\beta}{2}$
		$\frac{c^2 \cdot \sin\alpha \cdot \cos\alpha}{2}$	$\frac{b \cdot c \cdot \sin\alpha}{2}$	$\frac{b^2 \cdot \tan\alpha}{2}$	$\frac{a \cdot c \cdot \cos\alpha}{2}$	$\frac{a^2 \cdot \cot\alpha}{2}$

Trigonometric functions sine and cosine

The following tables show a number of values for the trigonometric functions sine and cosine.

Table 1
Sine to cosine

Degree	sin							
°	0′	10′	20′	30′	40′	50′	60′	
0	0	0,002 91	0,005 82	0,008 73	0,011 64	0,014 54	0,017 45	89
1	0,017 45	0,020 36	0,023 27	0,026 18	0,029 08	0,031 99	0,034 90	88
2	0,034 90	0,037 81	0,040 71	0,043 62	0,046 53	0,049 43	0,052 34	87
3	0,052 34	0,055 24	0,058 14	0,061 05	0,063 95	0,066 85	0,069 76	86
4	0,069 76	0,072 66	0,075 56	0,078 46	0,081 36	0,084 26	0,087 16	85
5	0,087 16	0,090 05	0,092 95	0,095 85	0,098 74	0,101 64	0,104 53	84
6	0,104 53	0,107 42	0,110 31	0,113 20	0,116 09	0,118 98	0,121 87	83
7	0,121 87	0,124 76	0,127 64	0,130 53	0,133 41	0,136 29	0,139 17	82
8	0,139 17	0,142 05	0,144 93	0,147 81	0,150 69	0,153 56	0,156 43	81
9	0,156 43	0,159 31	0,162 18	0,165 05	0,167 92	0,170 78	0,173 65	80
10	0,173 65	0,176 51	0,179 37	0,182 24	0,185 09	0,187 95	0,190 81	79
11	0,190 81	0,193 66	0,196 52	0,199 37	0,202 22	0,205 07	0,207 91	78
12	0,207 91	0,210 76	0,213 60	0,216 44	0,219 28	0,222 12	0,224 95	77
13	0,224 95	0,227 78	0,230 62	0,233 45	0,236 27	0,239 10	0,241 92	76
14	0,241 92	0,244 74	0,247 56	0,250 38	0,253 20	0,256 01	0,258 82	75
15	0,258 82	0,261 63	0,264 43	0,267 24	0,207 04	0,272 84	0,275 64	74
16	0,275 64	0,278 43	0,281 23	0,284 02	0,286 80	0,289 59	0,292 37	73
17	0,292 37	0,295 15	0,297 93	0,300 71	0,303 48	0,306 25	0,309 02	72
18	0,309 02	0,311 78	0,314 54	0,317 30	0,320 06	0,322 82	0,325 57	71
19	0,325 57	0,328 32	0,331 06	0,333 81	0,336 55	0,339 29	0,342 02	70
20	0,342 02	0,344 75	0,347 48	0,350 21	0,352 93	0,355 65	0,358 37	69
21	0,358 37	0,361 08	0,363 79	0,366 50	0,369 21	0,371 91	0,374 61	68
22	0,374 61	0,377 30	0,379 99	0,382 68	0,385 37	0,388 05	0,390 73	67
23	0,390 73	0,393 41	0,396 08	0,398 75	0,401 41	0,404 08	0,406 74	66
24	0,406 74	0,409 39	0,412 04	0,414 69	0,417 34	0,419 98	0,422 62	65
25	0,422 62	0,425 25	0,427 88	0,430 51	0,433 13	0,435 75	0,438 37	64
26	0,438 37	0,440 98	0,443 59	0,446 20	0,448 80	0,451 40	0,453 99	63
27	0,453 99	0,456 58	0,459 17	0,461 75	0,464 33	0,466 90	0,469 47	62
28	0,469 47	0,472 04	0,474 60	0,477 16	0,479 71	0,482 26	0,484 81	61
29	0,484 81	0,487 35	0,489 89	0,492 42	0,494 95	0,497 48	0,5	60
30	0,5	0,502 52	0,505 03	0,507 54	0,510 04	0,512 54	0,515 04	59
	60′	50′	40′	30′	20′	10′	0′	Degree
	cos							°

Continuation of table, see Page 54.

Continuation of table, Table 1 Sine to cosine, from Page 53.

Degree °	sin 0′	10′	20′	30′	40′	50′	60′	
31	0,515 04	0,517 53	0,520 02	0,522 50	0,524 98	0,527 43	0,529 92	58
32	0,529 92	0,532 38	0,534 84	0,537 30	0,539 75	0,542 20	0,544 64	57
33	0,544 64	0,547 08	0,549 51	0,551 94	0,554 36	0,556 78	0,559 19	56
34	0,559 19	0,561 60	0,564 01	0,566 41	0,568 80	0,571 19	0,573 58	55
35	0,573 58	0,575 96	0,578 33	0,580 70	0,583 07	0,585 43	0,587 79	54
36	0,587 79	0,590 14	0,592 48	0,594 82	0,597 16	0,599 49	0,601 82	53
37	0,601 82	0,604 14	0,606 45	0,608 76	0,611 07	0,613 37	0,615 66	52
38	0,615 66	0,617 95	0,620 24	0,622 51	0,624 79	0,627 06	0,629 32	51
39	0,629 32	0,631 58	0,633 83	0,636 08	0,638 32	0,640 56	0,642 79	50
40	0,642 79	0,645 01	0,647 23	0,649 45	0,651 66	0,653 86	0,656 06	49
41	0,656 06	0,658 25	0,660 44	0,662 62	0,664 80	0,666 97	0,669 13	48
42	0,669 13	0,671 29	0,673 44	0,675 59	0,677 73	0,679 87	0,682 00	47
43	0,682 00	0,684 12	0,686 24	0,688 35	0,690 46	0,692 56	0,694 66	46
44	0,694 66	0,696 75	0,698 83	0,700 91	0,702 98	0,705 05	$1/2 \cdot \sqrt{2}$ (≈ 0.707 11)	45
	60′	50′	40′	30′	20′	10′	0′	Degree °
	cos							

Table 2
Cosine to sine

Degree °	cos 0′	10′	20′	30′	40′	50′	60′	
0	1	1,000 00	0,999 98	0,999 96	0,999 93	0,999 89	0,999 85	89
1	0,999 85	0,999 79	0,999 73	0,999 66	0,999 58	0,999 49	0,999 39	88
2	0,999 39	0,999 29	0,999 17	0,999 05	0,998 92	0,998 78	0,998 63	87
3	0,998 63	0,998 47	0,998 31	0,998 13	0,997 95	0,997 76	0,997 56	86
4	0,997 56	0,997 36	0,997 14	0,996 92	0,996 68	0,996 44	0,996 19	85
5	0,996 19	0,995 94	0,995 67	0,995 40	0,995 11	0,994 82	0,994 52	84
6	0,994 52	0,994 21	0,993 90	0,993 57	0,993 24	0,992 90	0,992 55	83
7	0,992 55	0,992 19	0,991 82	0,991 44	0,991 06	0,990 67	0,990 27	82
8	0,990 27	0,989 86	0,989 14	0,989 02	0,988 58	0,988 14	0,987 69	81
9	0,987 69	0,987 23	0,986 76	0,986 29	0,985 80	0,985 31	0,984 81	80
10	0,984 81	0,984 30	0,983 78	0,983 25	0,982 72	0,982 18	0,981 63	79
	60′	50′	40′	30′	20′	10′	0′	Degree °
	sin							

Continuation of table, see Page 55.

Continuation of table, Table 2 Cosine to sine, from Page 54.

Degree °	cos 0′	10′	20′	30′	40′	50′	60′	
11	0,98163	0,98107	0,98050	0,97992	0,97934	0,97875	0,97815	78
12	0,97815	0,97754	0,97692	0,97630	0,97566	0,97502	0,97437	77
13	0,97437	0,97371	0,97304	0,97237	0,97169	0,97100	0,97030	76
14	0,97030	0,96959	0,96887	0,96815	0,96742	0,96667	0,96593	75
15	0,96593	0,96517	0,96440	0,96363	0,96285	0,96206	0,96126	74
16	0,96126	0,96046	0,95964	0,95882	0,95799	0,95715	0,95630	73
17	0,95630	0,95545	0,95459	0,95372	0,95284	0,95195	0,95106	72
18	0,95106	0,95015	0,94924	0,94832	0,94740	0,94646	0,94552	71
19	0,94552	0,94457	0,94361	0,94264	0,94167	0,94068	0,93969	70
20	0,93969	0,93869	0,93769	0,93667	0,93565	0,93462	0,93358	69
21	0,93358	0,93253	0,93148	0,93042	0,92935	0,92827	0,92718	68
22	0,92718	0,92609	0,92499	0,92388	0,92276	0,92164	0,92050	67
23	0,92050	0,91936	0,91822	0,91706	0,91590	0,91472	0,91355	66
24	0,91355	0,91236	0,91116	0,90996	0,90875	0,90753	0,90631	65
25	0,90631	0,90507	0,90383	0,90259	0,80133	0,90007	0,89879	64
26	0,89879	0,89752	0,89623	0,89493	0,89363	0,89232	0,89101	63
27	0,89101	0,88968	0,88835	0,88701	0,88566	0,88431	0,88295	62
28	0,88295	0,88158	0,88020	0,87882	0,87743	0,87603	0,87462	61
29	0,87462	0,87321	0,87178	0,87036	0,86892	0,86748	$1/2 \cdot \sqrt{3}$ (≈ 0,86603)	60
30	$1/2 \cdot \sqrt{3}$ (≈ 0,86603)	0,86457	0,86310	0,86163	0,86015	0,85866	0,85717	59
31	0,85717	0,85567	0,85416	0,85264	0,85112	0,84959	0,84805	58
32	0,84805	0,84650	0,84495	0,84339	0,84182	0,84025	0,83867	57
33	0,83867	0,83708	0,83549	0,83389	0,83228	0,83066	0,82904	56
34	0,82904	0,82741	0,82577	0,82413	0,82248	0,82082	0,81915	55
35	0,81915	0,81748	0,81580	0,81412	0,81242	0,81072	0,80902	54
36	0,80902	0,80730	0,80558	0,80386	0,80212	0,80038	0,79864	53
37	0,79864	0,79688	0,79512	0,79335	0,79158	0,78980	0,78801	52
38	0,78801	0,78622	0,78442	0,78261	0,78079	0,77897	0,77715	51
39	0,77715	0,77531	0,77347	0,77162	0,76977	0,76791	0,76604	50
40	0,76604	0,76417	0,76229	0,76041	0,75851	0,75661	0,75471	49
41	0,75471	0,75280	0,75088	0,74696	0,74703	0,74509	0,74314	48
42	0,74314	0,74120	0,73924	0,73728	0,73531	0,73333	0,73135	47
43	0,73135	0,72937	0,72737	0,72537	0,72337	0,72136	0,71934	46
44	0,71934	0,71732	0,71529	0,71325	0,71121	0,70916	$1/2 \cdot \sqrt{2}$ (≈ 0.70711)	45
	60′	50′	40′	30′	20′	10′	0′	Degree
	sin							°

Trigonometric functions tangent and cotangent

The following tables show a number of values for the trigonometric functions tangent and cotangent.

Table 1
Tangent to cotangent

Degree °	tan 0′	10′	20′	30′	40′	50′	60′	
0	0	0,002 91	0,005 82	0,008 73	0,011 64	0,014 55	0,017 46	89
1	0,017 46	0,020 36	0,023 28	0,026 19	0,029 10	0,032 01	0,034 92	88
2	0,034 92	0,037 83	0,040 75	0,043 66	0,046 58	0,049 49	0,052 41	87
3	0,052 41	0,055 33	0,058 24	0,061 16	0,064 08	0,067 00	0,069 93	86
4	0,069 93	0,072 85	0,075 78	0,078 70	0,081 63	0,084 56	0,087 49	85
5	0,087 49	0,090 42	0,093 35	0,096 29	0,099 23	0,102 16	0,105 10	84
6	0,105 10	0,108 05	0,110 99	0,113 94	0,116 88	0,119 83	0,122 78	83
7	0,122 78	0,125 74	0,128 69	0,131 65	0,134 61	0,137 58	0,140 54	82
8	0,140 54	0,143 51	0,146 48	0,149 45	0,152 43	0,155 40	0,158 38	81
9	0,158 38	0,161 37	0,164 35	0,167 34	0,170 33	0,173 33	0,176 33	80
10	0,176 33	0,179 33	0,182 33	0,185 34	0,188 35	0,191 36	0,194 38	79
11	0,194 38	0,197 40	0,200 42	0,203 45	0,206 48	0,209 52	0,212 56	78
12	0,212 56	0,215 60	0,218 64	0,221 69	0,224 75	0,227 81	0,230 87	77
13	0,230 87	0,233 93	0,237 00	0,240 08	0,243 16	0,246 24	0,249 33	76
14	0,249 33	0,252 42	0,255 52	0,258 62	0,261 72	0,264 83	0,267 95	75
15	0,267 95	0,271 07	0,274 19	0,277 32	0,280 46	0,283 60	0,286 75	74
16	0,286 75	0,289 90	0,293 05	0,296 21	0,299 38	0,302 55	0,305 73	73
17	0,305 73	0,308 91	0,312 10	0,315 30	0,318 50	0,321 71	0,324 92	72
18	0,324 92	0,328 14	0,331 36	0,334 60	0,337 83	0,341 08	0,344 33	71
19	0,344 33	0,347 58	0,350 85	0,354 12	0,357 40	0,360 68	0,363 97	70
20	0,363 97	0,367 27	0,370 57	0,373 88	0,377 20	0,380 53	0,383 86	69
21	0,383 86	0,387 21	0,390 55	0,393 91	0,397 27	0,400 65	0,404 03	68
22	0,404 03	0,407 41	0,410 81	0,414 21	0,417 63	0,421 05	0,424 47	67
23	0,424 47	0,427 91	0,431 36	0,434 81	0,438 28	0,441 75	0,445 23	66
24	0,445 23	0,448 72	0,452 22	0,455 73	0,459 24	0,462 77	0,466 31	65
25	0,466 31	0,469 85	0,473 41	0,476 98	0,480 55	0,484 14	0,487 73	64
26	0,487 73	0,491 34	0,494 95	0,498 58	0,502 22	0,505 87	0,509 53	63
27	0,509 53	0,513 19	0,516 88	0,520 57	0,524 27	0,527 98	0,531 71	62
28	0,531 71	0,535 45	0,539 20	0,542 96	0,546 73	0,550 51	0,554 31	61
29	0,554 31	0,558 12	0,561 94	0,565 77	0,569 62	0,573 48	$1/3 \cdot \sqrt{3}$ (≈ 0,577 35)	60
30	$1/3 \cdot \sqrt{3}$ (≈ 0,577 35)	0,581 24	0,585 13	0,589 05	0,592 97	0,596 91	0,600 86	59
	60′	50′	40′	30′	20′	10′	0′	Degree °
	cot							

Continuation of table, see Page 57.

Continuation of table, Table 1 Tangent to cotangent, from Page 56.

Degree °	tan 0′	10′	20′	30′	40′	50′	60′	
31	0,600 86	0,601 83	0,608 81	0,612 80	0,616 81	0,620 83	0,624 87	58
32	0,624 87	0,628 92	0,632 99	0,637 07	0,641 17	0,645 28	0,649 41	57
33	0,649 41	0,653 55	0,657 71	0,661 89	0,666 08	0,670 28	0,674 51	56
34	0,674 51	0,678 75	0,683 01	0,687 28	0,691 57	0,695 88	0,700 21	55
35	0,700 21	0,704 55	0,708 91	0,713 29	0,717 69	0,722 11	0,726 54	54
36	0,726 54	0,731 00	0,735 47	0,739 96	0,744 47	0,749 00	0,753 55	53
37	0,753 55	0,758 12	0,762 72	0,767 33	0,771 96	0,776 61	0,781 29	52
38	0,781 29	0,785 98	0,790 70	0,795 44	0,800 20	0,804 98	0,809 78	51
39	0,809 78	0,814 61	0,819 46	0,824 34	0,829 23	0,834 15	0,839 10	50
40	0,839 10	0,844 07	0,849 06	0,854 08	0,859 12	0,864 19	0,869 29	49
41	0,869 29	0,874 41	0,879 55	0,884 73	0,889 92	0,895 15	0,900 40	48
42	0,900 40	0,905 69	0,910 99	0,916 33	0,921 70	0,927 09	0,932 52	47
43	0,932 52	0,937 97	0,943 45	0,948 96	0,954 51	0,960 08	0,965 69	46
44	0,965 69	0,971 33	0,977 00	0,982 70	0,988 43	0,994 20	1	45
	60′	50′	40′	30′	20′	10′	0′	Degree °
	cot							

Table 2
Cotangent to tangent

Degree °	cot 0′	10′	20′	30′	40′	50′	60′	
0	∞	343,773 71	171,885 40	114,588 65	85,939 79	68,750 09	57,289 96	89
1	57,289 96	49,103 88	42,964 08	38,188 46	34,367 77	31,241 58	28,636 25	88
2	28,636 25	26,431 60	24,541 76	22,903 77	21,470 40	20,205 55	19,081 14	87
3	19,081 14	18,074 98	17,169 34	16,349 86	15,604 78	14,924 42	14,300 67	86
4	14,300 67	13,726 74	13,196 88	12,706 21	12,250 51	11,826 17	11,430 05	85
5	11,430 05	11,059 43	10,711 91	10,385 40	10,078 03	9,788 17	9,514 36	84
6	9,514 36	9,255 30	9,009 83	8,776 89	8,555 55	8,344 96	8,144 35	83
7	8,144 35	7,953 02	7,770 35	7,595 75	7,428 71	7,268 73	7,115 37	82
8	7,115 37	6,968 23	6,826 94	6,691 16	6,560 55	6,434 84	6,313 75	81
9	6,313 75	6,197 03	6,084 44	5,975 76	5,870 80	5,769 37	5,671 28	80
10	5,671 28	5,576 38	5,484 51	5,395 52	5,309 28	5,225 66	5,144 55	79
	60′	50′	40′	30′	20′	10′	0′	Degree °
	tan							

Continuation of table, see Page 58.

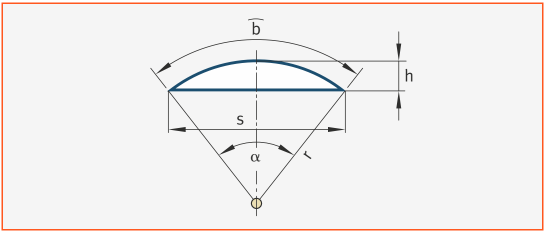
Continuation of table, Table 2 Cotangent to tangent, from Page 57.

Degree	cot							
°	0′	10′	20′	30′	40′	50′	60′	
11	5,144 55	5,065 84	4,989 40	4,915 16	4,843 00	4,772 86	4,704 63	78
12	4,704 63	4,638 25	4,573 63	4,510 71	4,449 42	4,389 69	4,331 48	77
13	4,331 48	4,274 71	4,219 33	4,165 30	4,112 56	4,061 07	4,010 78	76
14	4,010 78	3,961 65	3,913 64	3,866 71	3,820 83	3,775 95	3,732 05	75
15	3,732 05	3,689 09	3,647 05	3,605 88	3,565 57	3,526 09	3,487 41	74
16	3,487 41	3,449 51	3,412 36	3,375 94	3,340 23	3,305 21	3,270 85	73
17	3,270 85	3,237 14	3,204 06	3,171 59	3,139 72	3,108 42	3,077 68	72
18	3,077 68	3,047 49	3,017 83	2,988 69	2,960 04	2,931 89	2,904 21	71
19	2,904 21	2,877 00	2,850 23	2,823 91	2,798 02	2,772 54	2,747 48	70
20	2,747 48	2,722 81	2,698 53	2,674 62	2,651 09	2,627 91	2,605 09	69
21	2,605 09	2,582 61	2,560 46	2,538 65	2,517 15	2,495 97	2,475 09	68
22	2,475 09	2,454 51	2,434 22	2,414 21	2,394 49	2,375 04	2,355 85	67
23	2,355 85	2,336 93	2,318 26	2,299 84	2,281 67	2,263 74	2,246 04	66
24	2,246 04	2,228 57	2,211 32	2,194 30	2,177 49	2,160 90	2,144 51	65
25	2,144 51	2,128 32	2,112 33	2,096 54	2,080 94	2,065 53	2,050 30	64
26	2,050 30	2,035 26	2,020 39	2,005 69	1,991 16	1,976 80	1,962 61	63
27	1,962 61	1,948 58	1,934 70	1,920 98	1,907 41	1,894 00	1,880 73	62
28	1,880 73	1,867 60	1,854 62	1,841 77	1,829 66	1,816 49	1,804 05	61
29	1,804 05	1,791 74	1,779 55	1,767 49	1,755 56	1,743 75	$\sqrt{3}$ (≈ 1,732 05)	60
30	$\sqrt{3}$ (≈ 1,732 05)	1,720 47	1,709 01	1,697 66	1,686 43	1,675 30	1,664 28	59
31	1,664 28	1,653 37	1,642 56	1,631 85	1,621 25	1,610 74	1,600 33	58
32	1,600 33	1,590 02	1,579 81	1,569 69	1,559 66	1,549 72	1,539 87	57
33	1,539 87	1,530 10	1,520 43	1,510 84	1,501 33	1,491 90	1,482 56	56
34	1,482 56	1,473 30	1,464 11	1,455 01	1,445 98	1,437 03	1,428 15	55
35	1,428 15	1,419 34	1,410 61	1,401 95	1,393 36	1,384 84	1,376 38	54
36	1,376 38	1,368 00	1,359 58	1,351 42	1,343 23	1,335 11	1,327 04	53
37	1,327 04	1,319 04	1,311 10	1,303 23	1,295 41	1,287 64	1,279 94	52
38	1,279 94	1,272 30	1,264 71	1,257 17	1,249 69	1,242 27	1,234 90	51
39	1,234 90	1,227 58	1,220 31	1,213 10	1,205 93	1,198 82	1,191 75	50
40	1,191 75	1,184 74	1,177 77	1,170 85	1,163 98	1,157 15	1,150 37	49
41	1,150 37	1,143 63	1,136 94	1,130 29	1,123 69	1,117 13	1,110 61	48
42	1,110 61	1,104 14	1,097 70	1,091 31	1,084 96	1,078 64	1,072 37	47
43	1,072 37	1,066 13	1,059 94	1,053 78	1,047 66	1,041 58	1,035 53	46
44	1,035 53	1,029 52	1,023 55	1,017 61	1,011 70	1,005 83	1	45
	60′	50′	40′	30′	20′	10′	0′	Degree
	tan							°

Arc lengths, arc heights, chord lengths, circular segments

Arc length, arc height, chord length and circular segment are calculated as follows:

Figure 7
Arc of circle



Chord length
Equation 64

Thus, the chord length is:

$$s = 2r\sin\frac{\alpha}{2}$$

Arc height
Equation 65

the arc height is:

$$h = r\left(1 - \cos\frac{\alpha}{2}\right) = \frac{s}{2}\tan\frac{\alpha}{4} = 2r\sin^2\frac{\alpha}{4}$$

Arc length
Equation 66

the arc length is:

$$b = \pi r\frac{\alpha}{180°} = \sqrt{s^2 + \frac{16}{3}h^2} \approx 0{,}017\,453\ r\cdot\alpha$$

Content of the circular segment
Equation 67

the content of the circular segment is:

$$A = \frac{r^2}{2}\left(\frac{\pi}{180°}\alpha - \sin\alpha\right)$$

Content of the circular sector
Equation 68

and the content of the circular sector is:

$$A = \frac{\alpha}{360°}\pi r^2 \approx 0{,}008\,726\,65\ \alpha r^2$$

If $b = r$, then
$\alpha = 57°17'44{,}86'' = 57{,}295\,779\,5° = 206\,264{,}86'' = 1$ rad

Table: Arc lengths, arc heights, chord lengths, circular segments

The following table shows the values relating to arc length, arc height, chord length and circular segment for central angle α and radius r = 1.

Central angle α °	Arc length b	Arc height h	$\frac{b}{h}$	Chord length s	Content of circular segment
1	0,0175	0,0000	458,37	0,0175	0,00000
2	0,0349	0,0002	229,19	0,0349	0,00000
3	0,0524	0,0003	152,80	0,0524	0,00001
4	0,0698	0,0006	114,60	0,0698	0,00003
5	0,0873	0,0010	91,69	0,0872	0,00006
6	0,1047	0,0014	76,41	0,1047	0,00010
7	0,1222	0,0019	65,50	0,1221	0,00015
8	0,1396	0,0024	57,32	0,1395	0,00023
9	0,1571	0,0031	50,96	0,1569	0,00032
10	0,1745	0,0038	45,87	0,1743	0,00044
11	0,1920	0,0046	41,70	0,1917	0,00059
12	0,2094	0,0055	38,23	0,2091	0,00076
13	0,2269	0,0064	35,30	0,2264	0,00097
14	0,2443	0,0075	32,78	0,2437	0,00121
15	0,2618	0,0086	30,60	0,2611	0,00149
16	0,2793	0,0097	28,69	0,2783	0,00181
17	0,2967	0,0110	27,01	0,2956	0,00217
18	0,3142	0,0123	25,52	0,3129	0,00257
19	0,3316	0,0137	24,18	0,3301	0,00302
20	0,3491	0,0152	22,98	0,3473	0,00352
21	0,3665	0,0167	21,89	0,3645	0,00408
22	0,3840	0,0184	20,90	0,3816	0,00468
23	0,4014	0,0201	20,00	0,3987	0,00535
24	0,4189	0,0219	19,17	0,4158	0,00607
25	0,4363	0,0237	18,41	0,4329	0,00686
26	0,4538	0,0256	17,71	0,4499	0,00771
27	0,4712	0,0276	17,06	0,4669	0,00862
28	0,4887	0,0297	16,45	0,4838	0,00961
29	0,5061	0,0319	15,89	0,5008	0,01067
30	0,5236	0,0341	15,37	0,5176	0,01180
31	0,5411	0,0364	14,88	0,5345	0,01301
32	0,5585	0,0387	14,42	0,5513	0,01429
33	0,5760	0,0412	13,99	0,5680	0,01566
34	0,5934	0,0437	13,58	0,5847	0,01711
35	0,6109	0,0463	13,20	0,6014	0,01864
36	0,6283	0,0489	12,84	0,6180	0,02027
37	0,6458	0,0517	12,50	0,6346	0,02198
38	0,6632	0,0545	12,17	0,6511	0,02378
39	0,6807	0,0574	11,87	0,6676	0,02568

Continuation of table, see Page 61.

Continuation of Table: Arc lengths, arc heights, chord lengths, circular segments from Page 60.

Central angle α °	Arc length b	Arc height h	$\frac{b}{h}$	Chord length s	Content of circular segment
40	0,6981	0,0603	11,58	0,6840	0,02767
41	0,7156	0,0633	11,30	0,7004	0,02976
42	0,7330	0,0664	11,04	0,7167	0,03195
43	0,7505	0,0696	10,78	0,7330	0,03425
44	0,7679	0,0728	10,55	0,7492	0,03664
45	0,7854	0,0761	10,32	0,7654	0,03915
46	0,8029	0,0795	10,10	0,7815	0,04176
47	0,8203	0,0829	9,89	0,7975	0,04448
48	0,8378	0,0865	9,69	0,8135	0,04731
49	0,8552	0,0900	9,50	0,8294	0,05025
50	0,8727	0,0937	9,31	0,8452	0,05331
51	0,8901	0,0974	9,14	0,8610	0,05649
52	0,9076	0,1012	8,97	0,8767	0,05978
53	0,9250	0,1051	8,80	0,8924	0,06319
54	0,9425	0,1090	8,65	0,9080	0,06673
55	0,9599	0,1130	8,49	0,9235	0,07039
56	0,9774	0,1171	8,35	0,9389	0,07417
57	0,9948	0,1212	8,21	0,9543	0,07808
58	1,0123	0,1254	8,07	0,9696	0,08212
59	1,0297	0,1296	7,94	0,9848	0,08629
60	1,0472	0,1340	7,81	1,0000	0,09059
61	1,0647	0,1384	7,69	1,0151	0,09502
62	1,0821	0,1428	7,56	1,0301	0,09958
63	1,0996	0,1474	7,46	1,0450	0,10428
64	1,1170	0,1520	7,35	1,0598	0,10911
65	1,1345	0,1566	7,24	1,0746	0,11408
66	1,1519	0,1613	7,14	1,0893	0,11919
67	1,1694	0,1661	7,04	1,1039	0,12443
68	1,1868	0,1710	6,94	1,1184	0,12982
69	1,2043	0,1759	6,85	1,1328	0,13535
70	1,2217	0,1808	6,76	1,1472	0,14102
71	1,2392	0,1859	6,67	1,1614	0,14683
72	1,2566	0,1910	6,58	1,1756	0,15279
73	1,2741	0,1961	6,50	1,1896	0,15889
74	1,2915	0,2014	6,41	1,2036	0,16514

Continuation of table, see Page 62.

Continuation of Table: Arc lengths, arc heights, chord lengths, circular segments from Page 61.

Central angle α °	Arc length b	Arc height h	$\frac{b}{h}$	Chord length s	Content of circular segment
75	1,3090	0,2066	6,34	1,2175	0,17154
76	1,3265	0,2120	6,26	1,2313	0,17808
77	1,3439	0,2174	6,18	1,2450	0,18477
78	1,3614	0,2229	6,11	1,2586	0,19160
79	1,3788	0,2284	6,04	1,2722	0,19859
80	1,3963	0,2340	5,97	1,2856	0,20573
81	1,4137	0,2396	5,90	1,2989	0,21301
82	1,4312	0,2453	5,83	1,3221	0,22045
83	1,4486	0,2510	5,77	1,3252	0,22804
84	1,4661	0,2569	5,71	1,3383	0,23578
85	1,4835	0,2627	5,65	1,3512	0,24367
86	1,5010	0,2686	5,59	1,3640	0,25171
87	1,5184	0,2746	5,53	1,3767	0,25990
88	1,5359	0,2807	5,47	1,3893	0,26825
89	1,5533	0,2867	5,42	1,4018	0,27675
90	1,5708	0,2929	5,36	1,4142	0,28540
91	1,5882	0,2991	5,31	1,4265	0,29420
92	1,6057	0,3053	5,26	1,4387	0,30316
93	1,6232	0,3116	5,21	1,4507	0,31226
94	1,6406	0,3180	5,16	1,4627	0,32152
95	1,6581	0,3244	5,11	1,4746	0,33093
96	1,6755	0,3309	5,06	1,4863	0,34050
97	1,6930	0,3374	5,02	1,4979	0,35021
98	1,7104	0,3439	4,97	1,5094	0,36008
99	1,7279	0,3506	4,93	1,5208	0,37009
100	1,7453	0,3572	4,89	1,5321	0,38026
101	1,7628	0,3639	4,84	1,5432	0,39058
102	1,7802	0,3707	4,80	1,5543	0,40104
103	1,7977	0,3775	4,76	1,5652	0,41166
104	1,8151	0,3843	4,72	1,5760	0,42242
105	1,8326	0,3912	4,68	1,5867	0,43333
106	1,8500	0,3982	4,65	1,5973	0,44439
107	1,8675	0,4052	4,61	1,6077	0,45560
108	1,8850	0,4122	4,57	1,6180	0,46695
109	1,9024	0,4193	4,54	1,6282	0,47845

Continuation of table, see Page 63.

Continuation of Table: Arc lengths, arc heights, chord lengths, circular segments from Page 62.

Central angle α °	Arc length b	Arc height h	$\frac{b}{h}$	Chord length s	Content of circular segment
110	1,9199	0,4264	4,50	1,6383	0,49008
111	1,9373	0,4336	4,47	1,6483	0,50187
112	1,9548	0,4408	4,43	1,6581	0,51379
113	1,9722	0,4481	4,40	1,6678	0,52586
114	1,9897	0,4554	4,37	1,6773	0,53806
115	2,0071	0,4627	4,34	1,6868	0,55041
116	2,0246	0,4701	4,31	1,6961	0,56289
117	2,0420	0,4775	4,28	1,7053	0,57551
118	2,0595	0,4850	4,25	1,7143	0,58827
119	2,0769	0,4925	4,22	1,7233	0,60116
120	2,0944	0,5000	4,19	1,7321	0,61418
121	2,1118	0,5076	4,16	1,7407	0,62734
122	2,1293	0,5152	4,13	1,7492	0,64063
123	2,1468	0,5228	4,11	1,7576	0,65404
124	2,1642	0,5305	4,08	1,7659	0,66759
125	2,1817	0,5383	4,05	1,7740	0,68125
126	2,1991	0,5460	4,03	1,7820	0,69505
127	2,2166	0,5538	4,00	1,7899	0,70897
128	2,2340	0,5616	3,98	1,7976	0,72301
129	2,2515	0,5695	3,95	1,8052	0,73716
130	2,2689	0,5774	3,93	1,8126	0,75144
131	2,2864	0,5853	3,91	1,8199	0,76584
132	2,3038	0,5933	3,88	1,8271	0,78034
133	2,3213	0,6013	3,86	1,8341	0,79497
134	2,3387	0,6093	3,84	1,8410	0,80970
135	2,3562	0,6173	3,82	1,8478	0,82454
136	2,3736	0,6254	3,80	1,8544	0,83949
137	2,3911	0,6335	3,77	1,8608	0,85455
138	2,4086	0,6416	3,75	1,8672	0,86971
139	2,4260	0,6498	3,73	1,8733	0,88497
140	2,4435	0,6580	3,71	1,8794	0,90034
141	2,4609	0,6662	3,69	1,8853	0,91580
142	2,4784	0,6744	3,67	1,8910	0,93135
143	2,4958	0,6827	3,66	1,8966	0,94700
144	2,5133	0,6910	3,64	1,9021	0,96274

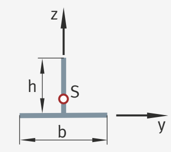
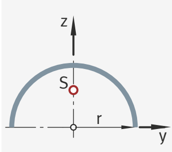
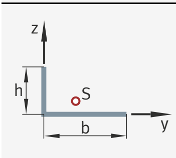
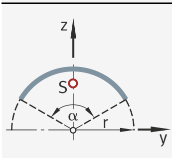
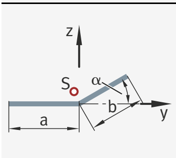
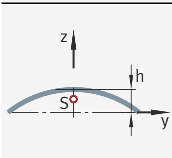
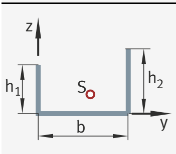
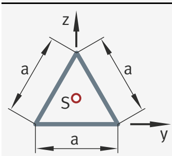
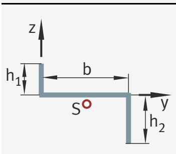
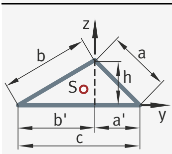
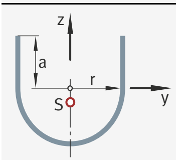
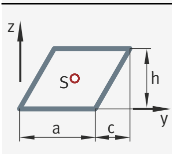
Continuation of table, see Page 64.

Continuation of Table: Arc lengths, arc heights, chord lengths, circular segments from Page 63.

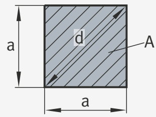
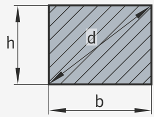
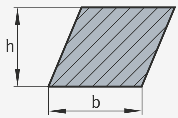
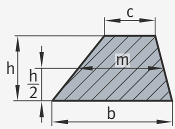
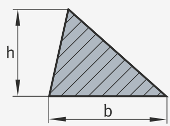
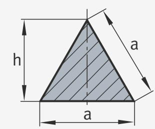
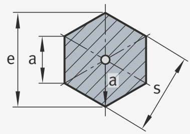
Central angle α °	Arc length b	Arc height h	$\frac{b}{h}$	Chord length s	Content of circular segment
145	2,5307	0,6993	3,62	1,9074	0,97858
146	2,5482	0,7076	3,60	1,9126	0,99449
147	2,5656	0,7160	3,58	1,9176	1,01050
148	2,5831	0,7244	3,57	1,9225	1,02658
149	2,6005	0,7328	3,55	1,9273	1,04275
150	2,6180	0,7412	3,53	1,9319	1,05900
151	2,6354	0,7495	3,52	1,9363	1,07532
152	2,6529	0,7581	3,50	1,9406	1,09171
153	2,6704	0,7666	3,48	1,9447	1,10818
154	2,6878	0,7750	3,47	1,9487	1,12472
155	2,7053	0,7836	3,45	1,9526	1,14132
156	2,7227	0,7921	3,44	1,9563	1,15799
157	2,7402	0,8006	3,42	1,9598	1,17472
158	2,7576	0,8092	3,41	1,9633	1,19151
159	2,7751	0,8178	3,39	1,9665	1,20835
160	2,7925	0,8264	3,38	1,9696	1,22525
161	2,8100	0,8350	3,37	1,9726	1,24221
162	2,8274	0,8436	3,35	1,9754	1,25921
163	2,8449	0,8522	3,34	1,9780	1,27626
164	2,8623	0,8608	3,33	1,9805	1,29335
165	2,8798	0,8695	3,31	1,9829	1,31049
166	2,8972	0,8781	3,30	1,9851	1,32766
167	2,9147	0,8868	3,28	1,9871	1,34487
168	2,9322	0,8955	3,27	1,9890	1,36212
169	2,9496	0,9042	3,26	1,9908	1,37940
170	2,9671	0,9128	3,25	1,9924	1,39671
171	2,9845	0,9215	3,24	1,9938	1,41404
172	3,0020	0,9302	3,23	1,9951	1,43140
173	3,0194	0,9390	3,22	1,9963	1,44878
174	3,0369	0,9477	3,20	1,9973	1,46617
175	3,0543	0,9564	3,19	1,9981	1,48359
176	3,0718	0,9651	3,18	1,9988	1,50101
177	3,0892	0,9738	3,17	1,9993	1,51845
178	3,1067	0,9825	3,16	1,9997	1,53589
179	3,1241	0,9913	3,15	1,9999	1,55334
180	3,1416	1,0000	3,14	2,0000	1,57080

Centre of gravity of lines

The centre of gravity of lines is calculated using:

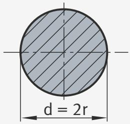
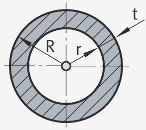
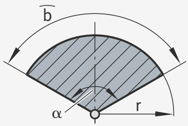
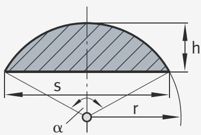
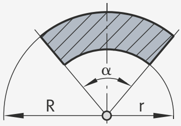
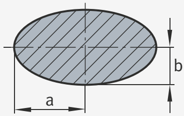
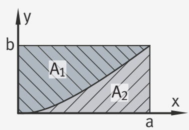
	$z_S = \frac{h^2}{2(b+h)}$		$z_S = \frac{2r}{\pi}$
	$y_S = \frac{b^2}{2(b+h)}$ $z_S = \frac{h^2}{2(b+h)}$		$z_S = \frac{2r\sin\left(\frac{\alpha}{2}\right)}{\alpha}$ α in radians
	$y_S = \frac{a^2+b^2\cos\alpha}{2(a+b)}$ $z_S = \frac{b^2\sin\alpha}{2(a+b)}$		$z_S \approx 2h/3$
	$y_S = \frac{b(b/2+h_2)}{b+h_1+h_2}$ $z_S = \frac{h_1^2+h_2^2}{2(b+h_1+h_2)}$		$z_S = \frac{a}{6}\sqrt{3}$
	$y_S = \frac{b(b/2+h_2)}{b+h_1+h_2}$ $z_S = \frac{h_1^2-h_2^2}{2(b+h_1+h_2)}$		$y_S = \frac{a'(a+a')-b'(b+b')}{2(a+b+c)}$ $z_S = \frac{h(a+b)}{2(a+b+c)}$
	$z_S = \frac{a^2-2r^2}{2a+\pi r}$		$y_S = \frac{a+c}{2}$ $z_S = \frac{h}{2}$

Calculating surfaces Geometric surfaces are calculated using:

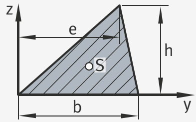
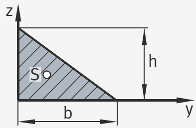
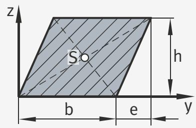
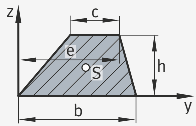
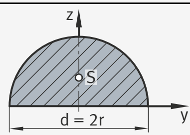
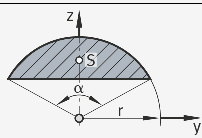
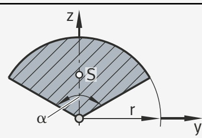
Square		$A = a^2$ $a = \sqrt{A}$ $d = a\sqrt{2}$
Rectangle		$A = b\,h$ $d = \sqrt{b^2 + h^2}$
Parallelogram		$A = b\,h$ $b = \dfrac{A}{h}$
Trapezoid		$m = \dfrac{b+c}{2}$ $A = m\,h$
Triangle		$A = \dfrac{b\,h}{2}$ $b = \dfrac{2\,A}{h}$
Equilateral triangle		$A = \dfrac{a^2}{4}\sqrt{3}$ $h = \dfrac{a}{2}\sqrt{3}$
Regular hexagon		$A = \dfrac{3\,a^2\sqrt{3}}{2}$ $e = 2\,a$ $\quad e \approx 1{,}155\ s$ $s = a\sqrt{3} = e\dfrac{\sqrt{3}}{2}$ $\quad s \approx 0{,}866\ e$

Continuation of table, see Page 67.

Continuation of table, Calculating surfaces, from Page 66.

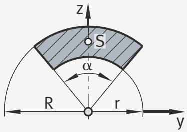
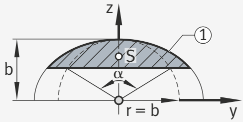
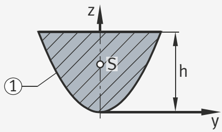
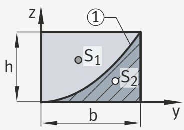
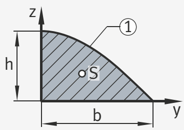
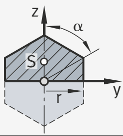
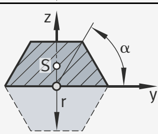
Circle		$A = \frac{d^2\pi}{4} = r^2\pi \approx 0{,}785\,d^2$ $U = 2r\pi = d\pi$
Annulus		$A = \pi\left(R^2 - r^2\right) \approx \left(2r + t\right)\pi t$ $t = R - r$
Circular sector		$A = r^2\pi \frac{\alpha}{360°} = \frac{b\,r}{2}$ $b = r\pi \frac{\alpha}{180°}$
Circular segment		$A = r^2\pi \frac{\alpha}{360°} - \frac{r^2}{2}\sin\alpha$ $\approx \frac{h}{6s}\left(3h^2 + 4s^2\right)$ $t = r - h$ $\qquad r = \frac{h}{2} + \frac{s^2}{8h}$ $s = 2r\sin\frac{\alpha}{2} = \frac{r^2}{t}\sin\alpha$ $h = r\left(1 - \cos\frac{\alpha}{2}\right)$
Annular sector		$A = \frac{\pi\left(R^2 - r^2\right)\alpha}{360°}$
Ellipse		$A = a\,b\,\pi$ $U = (a + b)\,\pi$
Polynomial surface		$y = b\,(x/a)^n$ $A_1 = \frac{n}{n+1}\,a\,b \qquad A_2 = \frac{1}{n+1}\,a\,b$

Centre of gravity of plane surfaces The centre of gravity of plane surfaces is calculated using:

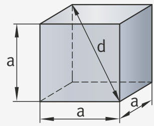
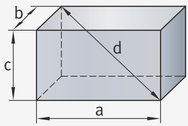
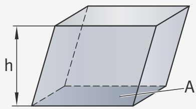
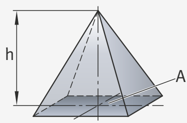
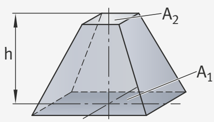
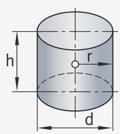
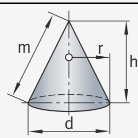
Triangle		$y_S = \frac{b+e}{3}$ $z_S = \frac{h}{3}$
Right-angled triangle		$y_S = \frac{b}{3}$ $z_S = \frac{h}{3}$
Parallelogram		$y_S = \frac{b+e}{2}$ $z_S = \frac{h}{2}$
Trapezoid		$y_S = \frac{b^2 - c^2 + e\left(b + 2\,c\right)}{3\left(b + c\right)}$ $z_S = \frac{h\left(b + 2\,c\right)}{3\left(b + c\right)}$
Semicircle		$z_S = \frac{4\,r}{3\,\pi}$
Circular segment		$z_S = \frac{4\,r\,\sin^3\left(\frac{\alpha}{2}\right)}{3\left(\alpha - \sin\alpha\right)}$ α in radians
Circular sector		$z_S = \frac{4\,r\,\sin\left(\frac{\alpha}{2}\right)}{3\,\alpha}$ α in radians

Continuation of table, see Page 69.

Continuation of table, Centre of gravity of plane surfaces, from Page 68.

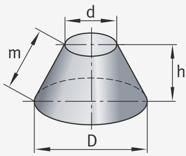
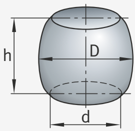
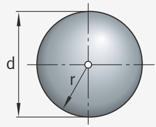
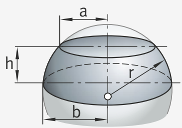
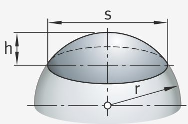
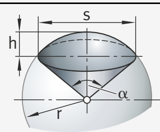
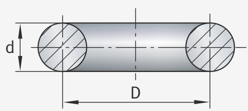
Annular sector		$z_S = \frac{4\left(R^3 - r^3\right)\sin\left(\frac{\alpha}{2}\right)}{3\left(R^2 - r^2\right)\alpha}$ α in radians
Elliptical segment ① Ellipse		$z_S = \frac{4\,r\,\sin^3\left(\frac{\alpha}{2}\right)}{3\left(\alpha - \sin\alpha\right)}$ α in radians
Parabola segment 1 ① Parabola		$z_S = \frac{3}{5}h$
Parabola segment 2 ① Parabola		$y_{S_1} = \frac{3}{8}b$ $\quad y_{S_2} = \frac{3}{4}b$ $z_{S_1} = \frac{3}{5}h$ $\quad z_{S_2} = \frac{3}{10}h$
Cosine segment ① Cosine line		$y_S = \left(1 - \frac{2}{\pi}\right)b$ $z_S = \frac{\pi}{8}h$
Hexagon half 1		$z_S = \frac{4\,r}{3\,\pi} \cdot \frac{\alpha\left(3 + \cos\alpha\right)}{4\sin\alpha}$ α in radians
Hexagon half 2		$z_S = \frac{4\,r}{3\,\pi} \cdot \frac{\alpha}{2\sin\left(\frac{\alpha}{2}\right)}$ α in radians

Calculating solids The volume of solids (volume V, surface area O, lateral area M) is calculated using:

Cube		$V = a^3$ $O = 6\,a^2$ $d = a\sqrt{3}$
Cuboid		$V = a\,b\,c$ $O = 2\left(a\,b + a\,c + b\,c\right)$ $d = \sqrt{a^2 + b^2 + c^2}$
Oblique cuboid		$V = A\,h$ (Cavalieri's principle)
Pyramid		$V = \dfrac{A\,h}{3}$
Truncated pyramid		$V = \dfrac{h}{3}\left(A_1 + A_2 + \sqrt{A_1\,A_2}\right) \approx h\,\dfrac{A_1 + A_2}{2}$
Cylinder		$V = \dfrac{d^2\pi}{4}\,h$ $O = 2\,\pi\,r\,(r + h)$ $M = 2\,\pi\,r\,h$
Cone		$V = \dfrac{r^2\pi\,h}{3}$ $\quad m = \sqrt{h^2 + \left(\dfrac{d}{2}\right)^2}$ $O = \pi\,r\,(r + m)$ $M = \pi\,r\,m$

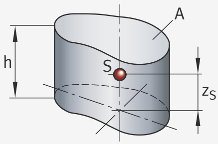
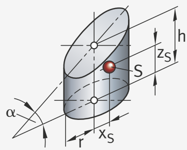
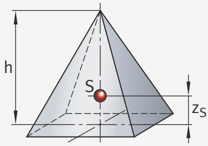
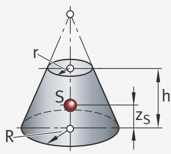
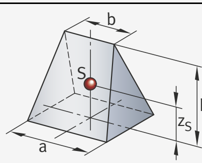
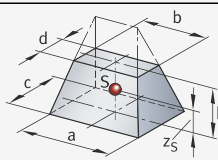
Continuation of table, see Page 71.

Continuation of table, Calculating solids, from Page 70.

Truncated cone		$V = \frac{\pi h}{12}\left(D^2 + D\,d + d^2\right)$ $m = \sqrt{\left(\frac{D-d}{2}\right)^2 + h^2}$ $M = \frac{\pi m}{2}(D+d)$
Barrel		$V = \frac{h\,\pi}{12}\left(2\,D^2 + d^2\right)$ (barrel)
Sphere		$V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3 \approx 4{,}189\,r^3$ $O = 4\,\pi\,r^2 = \pi\,d^2$
Spherical zone		$V = \frac{\pi h}{6}\left(3\,a^2 + 3\,b^2 + h^2\right)$ $M = 2\,\pi\,r\,h$ $a = f(h)$
Spherical segment		$V = \frac{\pi h}{6}\left(\frac{3}{4}s^2 + h^2\right) = \pi\,h^2\left(r - \frac{h}{3}\right)$ $M = 2\,\pi\,r\,h = \frac{\pi}{4}\left(s^2 + 4\,h^2\right)$
Spherical sector		$V = \frac{2}{3}\pi\,r^2 h$ $O = \frac{\pi r}{2}(4\,h + s)$
Circular torus (cylindrical ring)		$V = \frac{D\,\pi^2 d^2}{4}$ $O = D\,d\,\pi^2$

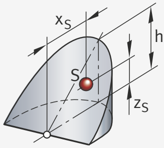
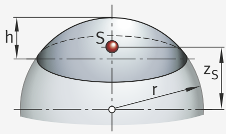
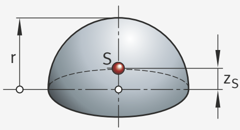
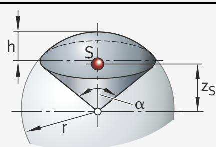
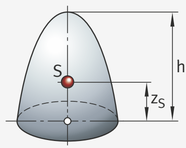
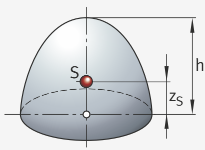
Centre of gravity of homogeneous solids

The centre of gravity of homogeneous solids is calculated using:

Cylinder with random cross section		$z_S = \frac{h}{2}$
Chamfered cylinder		$x_S = \frac{r^2 \tan\alpha}{4h}$ $z_S = \frac{h}{2} + \frac{r^2 \tan^2\alpha}{8h}$
Pyramid, cone		$z_S = \frac{h}{4}$
Truncated cone		$z_S = \frac{h}{4} \cdot \frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}$
Wedge		$z_S = \frac{h}{2} \cdot \frac{a+b}{2a+b}$
Truncated wedge		$z_S = \frac{h}{2} \cdot \frac{ac+ad+bc+3bd}{2ac+ad+bc+2bd}$

Continuation of table, see Page 73.

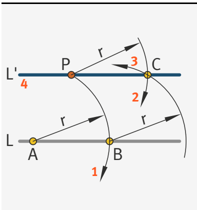
Continuation of table, Centre of gravity of homogeneous solids, from Page 72.

Section of a cylinder		$x_S = \frac{3\pi r}{16}$ $z_S = \frac{3\pi h}{32}$
Spherical segment		$z_S = \frac{3}{4} \cdot \frac{(2r-h)^2}{(3r-h)}$
Hemisphere		$z_S = \frac{3}{8} r$
Spherical sector		$z_S = \frac{3r\left(1+\cos\frac{\alpha}{2}\right)}{8} = \frac{3(2r-h)}{8}$
Paraboloid		$z_S = \frac{h}{3}$
Ellipsoid		$z_S = \frac{3}{8} h$

Basic geometric constructions

The following examples show the basic methods used in geometric construction.

Drawing parallel lines



Given: L and P

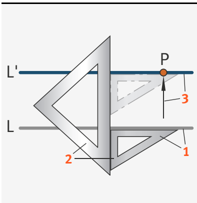
1 Draw an arc about any point A with radius r = $\overline{AP}$ (intersecting point B)

2 Draw an arc with radius r about P

3 Draw an arc with radius r about B (intersecting point C)

4 Draw the required parallel line L' through the two points P and C

Drawing parallel lines: Variant using set square



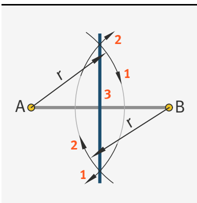
Given: L and P

1 Apply an angle to L

2 Apply a second angle to the first

3 Move the first angle to point P and draw the required parallel line L'

Creating mid-perpendiculars



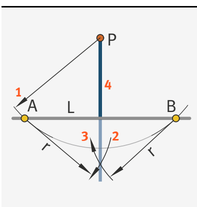
Given: $\overline{AB}$

1 Draw arcs with radius r about A (r must be greater than 1/2 $\overline{AB}$)

2 Draw arcs with radius r about B

3 The connecting line between the intersecting points is the required mid-perpendicular

Drawing perpendicular lines



Given: L and P

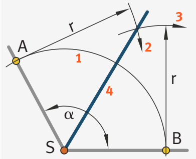
1 Draw any arc about P (intersecting points A and B)

2 Draw an arc with radius r about B

3 Draw an arc with radius r about A (r must be greater than 1/2 $\overline{AB}$)

4 The connecting line of the intersecting point with P is the required perpendicular

Bisecting angles



Given: angle α

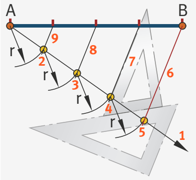
1 Draw any arc about S (intersecting points A and B)

2 Draw an arc with radius r about A (r must be greater than 1/2 $\overline{AB}$)

3 Draw an arc with radius r about B

4 The connecting line of the intersecting point with S is the required angle bisecting line

Dividing lines (equal division)



Given: $\overline{AB}$

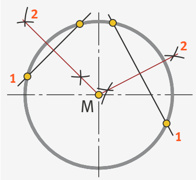
1 From A, draw a half-line at any angle

2 to 5
Mark off divisions of equal length and any size to the required number (in this instance 4) on the half-line of A

6 Connect the last end point (5) with B

7 to 9
Draw parallel lines to $\overline{B5}$: this gives the required subdivisions of the length $\overline{AB}$

Finding the centre of a circle



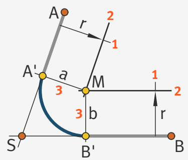
Given: circle without a centre

1 Draw any 2 chords

2 Create the two mid-perpendiculars. The point of intersection M is the required centre

(The chords should be at less than 90° to each other where possible, as this will increase the accuracy of the design)

Rounding an angle



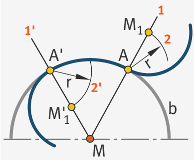
Given: angle ASB and rounding radius r

1 Apply r so that it is perpendicular to the sides

2 Draw the parallel lines to AS and BS.
The point of intersection M is the required rounding centre

3 Draw the perpendiculars a and b from M to the sides.
The intersecting points A' and B' are the transition points

Rounding a circle

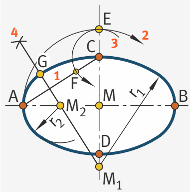


Given: arc b and rounding radius r

1 From M, draw a half-line

2 From intersecting point A, mark off from radius r.
The end point is the required rounding centre M_1,
A is the transition point

Ellipse approximation (false ellipse)



Given: axes $\overline{AB}$ and $\overline{CD}$

1 Draw $\overline{AC}$

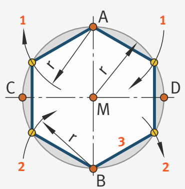
2 Draw an arc with radius $\overline{MA}$ about M to give E

3 Draw an arc with radius $\overline{CE}$ about C to give F

4 Create mid-perpendicular to $\overline{AF}$

5 The intersecting points M_1 and M_2 are the required foci of the ellipse for r_1 and r_2,
A is the transition point

Hexagon and dodecagon



Given: circle

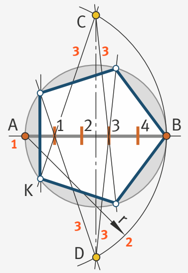
1 Draw arcs with radius r about A

2 Draw arcs with radius r about B

3 Draw hexagonal lines

For the dodecagon, it is necessary to define the intermediate points. Recess in C and D

Regular polygon in the circle, approximation construction



Given: circle

Taking a pentagon as an example:

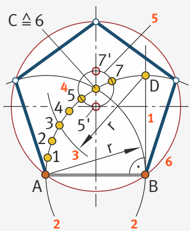
1 Divide $\overline{AB}$ into 5 sections (for example using equal division)

2 Draw an arc with radius $r = \overline{AB}$ about A

3 Join C and D using all odd divisions (1, 3, 5). Joining the intersecting points with the circle gives the required pentagon

For polygons with an even number of corners, C and D must be joined with all of the even divisions (2, 4, 6, ...)

Regular polygon with a given side length, approximation construction



Given: $\overline{AB}$

Taking a pentagon as an example:

1 Create perpendicular at B

2 Draw arcs with radius $r = \overline{AB}$ about A and B to give intersecting points C and D

3 Draw an arc with radius r about D to give point 3

4 Using trial and error and a divider, search for points 1, 2, 4 and 5 (equal distances)

5 Draw an arc with radius $\overline{C5}$ about C to give 5' and 7', 5' is the centre of a pentagon, 7' is the centre of a heptagon

6 Draw a circle around 5' and plot $\overline{AB}$ 5 times. This gives the required pentagon

Calculating interest

Compound interest

If b = the initial amount, p = the interest rate in % and b_n = the final amount after n years, the outcome for the interest factor q and the final amount b_n after n years at compound interest is:

Equation 69

$$q = 1 + \frac{p}{100} \qquad b_n = b \cdot q^n = b \cdot \left(1 + \frac{p}{100}\right)^n$$

Sample calculation

What final amount will 30 000 € increase to in 5 years at an interest rate of 5,5%?
Solution:

Equation 70

$$b_5 = 30\,000\ € \cdot \left(1 + \frac{5{,}5}{100}\right)^5 = 39\,209\ €$$

Growth

The growth of a basic amount b_0 based on compound interest as the result of regular **additional payments** r at the end of any given year can be calculated as a final amount b_n after n years:

Equation 71

$$b_n = b_0 \cdot q^n + \frac{r\left(q^n - 1\right)}{q - 1}$$

Reduction

The reduction of a basic amount b_0 based on compound interest as the result of regular **repayments** r (e.g. pension) at the end of any given year can be calculated as a final amount b_n after n years:

Equation 72

$$b_n = b_0 \cdot q^n - \frac{r\left(q^n - 1\right)}{q - 1}$$

Repayment formula

The repayment formula for $b_n = 0$ is:

Equation 73

$$b_0 \cdot q^n = \frac{r\left(q^n - 1\right)}{q - 1}$$

Compound interest table

The following table shows the growth of an amount of capital, starting from b = 1 €, as the result of interest and compound interest, with interest compounded annually.

Interest rate p	q^n for n years				
%	5	10	15	20	25
2,00	1,104	1,219	1,346	1,486	1,641
2,25	1,118	1,249	1,396	1,561	1,744
2,50	1,131	1,280	1,448	1,639	1,854
2,75	1,145	1,312	1,502	1,720	1,970
3,00	1,159	1,344	1,558	1,806	2,094
3,25	1,173	1,377	1,616	1,896	2,225
3,50	1,188	1,411	1,675	1,990	2,363
3,75	1,202	1,445	1,737	2,088	2,510
4,00	1,217	1,480	1,801	2,191	2,666
4,25	1,231	1,516	1,867	2,299	2,831
4,50	1,246	1,553	1,935	2,412	3,005
5,00	1,286	1,629	2,079	2,653	3,386
5,50	1,307	1,708	2,232	2,918	3,813
6,00	1,338	1,791	2,397	3,207	4,292
7,00	1,403	1,967	2,759	3,870	5,427
8,00	1,469	2,159	3,172	4,661	6,848
9,00	1,539	2,367	3,642	5,604	8,623
10,00	1,611	2,594	4,177	6,727	10,835

Interest rate p	q^n for n years				
%	30	35	40	45	50
2,00	1,811	2,000	2,208	2,438	2,692
2,25	1,949	2,179	2,435	2,722	3,042
2,50	2,098	2,373	2,685	3,038	3,437
2,75	2,257	2,584	2,960	3,390	3,882
3,00	2,427	2,814	3,262	3,782	4,384
3,25	2,610	3,063	3,594	4,217	4,949
3,50	2,807	3,334	3,959	4,702	5,585
3,75	3,017	3,627	4,360	5,242	6,301
4,00	3,243	3,946	4,801	5,841	7,107
4,25	3,486	4,295	5,285	6,508	8,013
4,50	3,745	4,667	5,816	7,248	9,033
5,00	4,322	5,516	7,040	8,985	11,47
5,50	4,984	6,514	8,513	11,13	14,54
6,00	5,743	7,686	10,29	13,76	18,42
7,00	7,612	10,68	14,97	21,00	29,46
8,00	10,06	14,79	21,72	31,92	46,90
9,00	13,27	20,41	31,41	48,33	74,36
10,00	17,45	28,10	45,26	72,89	117,4

Set theory

Set theory symbols

The following table shows a selection of the most important symbols used in set theory.

Symbol	Use	Definition
$\in$	$x \in M$	x is an element of M
$\notin$	$x \notin M$	x is not an element of M
	$\{x_1, \dots, x_n\} \in A$	$x_1, \dots, x_n$ are elements of A
$\{\mid\}$	$\{x \mid \varphi\}$	The set (class) of all x with φ
$\{, \dots,\}$	$\{x_1, \dots, x_n\}$	The set with the elements $x_1, \dots, x_n$
$\subseteq$	$A \subseteq B$	A is the subset of B, A sub B, B is the superset of A (contains A = B)
$\subset$	$A \subset B$	A is properly contained in B (with $A \neq B$)
$\cap$	$A \cap B$	A intersected with B, A intersection B
$\cup$	$A \cup B$	A united with B, A union B
$\complement$	$\bar{A}$ or $\complement A$ or $A^{\complement}$	Complement of A
$\setminus$ or $\complement$	$A \setminus B$ or $\complement_A B$	A minus B, A reduced by B, difference between sets A and B, relative complement of B with respect to A
Δ	$A \Delta B$	Symmetrical difference of A and B
$\emptyset$	$A \cap B = \emptyset$	Empty set, A and B are disjoint
$<\mid,>$	$<x, y>$	Pair of x and y
$\{,\mid\}$	$\{x, y \mid \varphi\}$	Relationship between x, y with φ
x	A x B	Cartesian product of A and B, A cross B
$^{-1}$	R^{-1}	Inverse relation of R, inverse relation to R
$\circ$	$R \circ S$	Relation product of R and S, R linked to S
D	D (f)	Definition range of f
W	W (f)	Value range of f
$\mid$	$f\mid A$	Restriction of f to A
equ	A equ B	A is equivalent to B
card	card A	Cardinal number of A
$\mathbb{N}$ or N		Set of natural numbers
$\mathbb{Z}$ or Z		Set of integers
$\mathbb{Q}$ or Q		Set of rational numbers
$\mathbb{R}$ or R		Set of real numbers
$\mathbb{C}$ or C		Set of complex numbers

Numerics – number systems in data processing

Numeric and alphanumeric data and commands are represented by a combination of binary characters in digital computers. These are generally condensed into words of fixed length, producing standard word lengths of 4, 8, 16, 32, 48 and 64 bits.

Code

Alphanumeric data are usually strung together, character by character, in coded form and in one word. The most frequently used code is composed of 8 bits = 1 byte.

Fixed-point number and floating-point number

Numeric data can be represented in positional (fixed-point number) or floating-point (floating-point number) notation:

Fixed-point representation:

Sign	2^{n-1}	2^{n-2}	...	2^2	2^1	2^0

Floating-point representation:

Sign	Exponent	Mantissa

In the case of fixed-point notation, the highest representable value in terms of its amount is limited by the word length. In the case of a computer in 16-bit format for example, this is $2^{15} - 1 = 32\,767$. Double words can be formed if a larger number range is required.

In the case of a floating-point number, the number of bits of the mantissa defines the relative accuracy of the number and those of the exponent define the magnitude of the number range.

Positional notation systems and representation

The characteristics of a number system are characterised by the digit repertoire and the notation. Here, the value of the digit Z depends on its position within the row of digits (positional notation system).

Integers

Positive integers N_B can be represented on selecting a base B (base number) in the following general form:

Equation 74

$$N_B = \sum_{i=0}^{n-1} Z_i \cdot B^i = Z_{n-1} \cdot B^{n-1} + \ldots + Z_1 \cdot B^1 + Z_0 \cdot B^0$$

with Z_i from {0, 1, 2, ..., (B – 1)} as the digit repertoire for base B.

Decimal system

For the decimal system with digit repertoire $Z_i = 0, 1, 2, ..., 9$ and digit number $n = 3$, the representation is:

Equation 75

$$N_{10} = 257 = 2 \cdot 10^2 + 5 \cdot 10^1 + 7 \cdot 10^0$$

Binary system

For the binary system with digit repertoire $Z_i = 0, 1$ and digit number $n = 5$, the representation is:

Equation 76

$$N_2 = 10\,101 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

Hexadecimal system

As the decimal digits from 0 to 9 do not suffice in the hexadecimal system, the missing digits from 10 to 15 are replaced by the capital letters A to F.

Fractional numbers

The general representation of fractional numbers is:

Equation 77

$$R_B = \sum_{i=0}^{m} Z_i \cdot B^{-i} = Z_1 \cdot B^{-1} + Z_2 \cdot B^{-2} + \ldots + Z_m \cdot B^{-m}$$

Table: Number systems

The following table shows examples of number systems with various bases.

Decimal system	Hexadecimal system	Octal system	Binary system	Tetrad representation	BCD representation	Excess-3 or Stibitz code	Aiken code	1 out of 10 code
0	0	0	0	0 000	0 000	0 011	0 000	0 000 000 001
1	1	1	1	0 001	0 001	0100	0 001	0 000 000 010
2	2	2	10	0 010	0 010	0101	0 010	0 000 000 100
3	3	3	11	0 011	0 011	0110	0 011	0 000 001 000
4	4	4	100	0100	0100	0111	0100	0 000 010 000
5	5	5	101	0101	0101	1000	1011	0 000 100 000
6	6	6	110	0110	0110	1001	1100	0 001 000 000
7	7	7	111	0111	0111	1010	1101	0 010 000 000
8	8	10	1 000	1 000	1 000	1011	1110	0 100 000 000
9	9	11	1 001	1 001	1 001	1100	1111	1 000 000 000
10	A	12	1010	1010	00 010 000	01 000 011	00 010 000	00 000 000 100 000 000 001
11	B	13	1011	1011	00 010 001	01 000 100	00 010 001	00 000 000 100 000 000 010
12	C	14	1100	1100	00 010 010	01 000 101	00 010 010	00 000 000 100 000 000 100
13	D	15	1101	1101	00 010 011	01 000 110	00 010 011	00 000 000 100 000 001 000
14	E	16	1110	1110	00 010 100	01 000 111	00 010 100	00 000 000 100 000 010 000
15	F	17	1111	1111	00 010 101	01 001 000	00 011 011	00 000 000 100 000 100 000

Conversion between number systems

Numbers can be converted from one number system (source system N_Q) to another (target system N_Z).

The following always applies:

Equation 78

$$N_Q = N_Z$$

The following methods are used for converting to a different number system:

Division method

The division method works with numbers from the source notation only. It is based on the division of the number belonging to the source system N_Q by the largest possible powers of the target base while simultaneously splitting the relevant integral quotient that is generated in the division step. The remainder is divided by the next lowest power. This continues until the zero power has been obtained.

This means:

Equation 79

$$N_Q = N_Z$$

$$N_Q = Z_{n-1} \cdot {B_Z}^{n-1} + Z_{n-2} \cdot {B_Z}^{n-2} + \ldots + Z_1 \cdot {B_Z}^1 + Z_0 \cdot {B_Z}^0$$

1st step:

Equation 80

$$N_Q / {B_Z}^{n-1} = Z_{n-1} + \text{Remainder}_1 \qquad Z_{n-1} = \text{1st digit of } N_Z$$

2nd step:

Equation 81

$$\text{Remainder}_1 / {B_Z}^{n-2} = Z_{n-2} + \text{Remainder}_2$$

$$Z_{n-2} = \text{2nd digit of } N_Z$$

etc.

Example

The decimal number $6\,345_{10}$ is to be converted to an octal number:

$6\,345 : 8^4 =$	1	↓	Remainder	2 249
$2\,249 : 8^3 =$	4		Remainder	201
$201 : 8^2 =$	3		Remainder	9
$9 : 8^1 =$	1		Remainder	1
$1 : 8^0 =$	1		Remainder	0

The required octal number is $14\,311_8$ ($6\,345_{10} = 14\,311_8$).

Summand method The summand method is based on a source number with summands of the form $\sum_{i=1}^{n-1} Z_i \cdot B_Z{}^i$, each of which contains B_Z as the factor:

Equation 82

$$N_Q = N_Z$$

$$N_Q = \sum_{i=0}^{n-1} Z_i \cdot B_Z{}^i = \sum_{i=1}^{n-1} Z_1 \cdot B_Z{}^i + Z_0 \cdot B_Z{}^0 \quad \text{where} \quad B_Z{}^0 = 1$$

This gives the following:

Equation 83

$$N_Q = B_Z \cdot \sum_{i=1}^{n-1} Z_1 \cdot B_Z{}^{i-1} + Z_0$$

If we divide N_Q by B_Z, we arrive at the integral share:

Equation 84

$$N_i = \sum_{i=1}^{n-1} Z_i \cdot B_Z{}^{i-1} + \text{Remainder } Z_0 \qquad \text{where } Z_0 = \text{last digit of } N_Z$$

The integral share can now be represented as:

Equation 85

$$N_1 = \sum_{i=2}^{n-1} Z_i \cdot B_Z{}^{i-1} + Z_1 \cdot B_Z{}^0 \qquad \text{where} \quad B_Z{}^0 = 1$$

If we divide again by B_Z, we again arrive at an integral share N_2 and the remainder Z_1 (penultimate digit of the number in the target system) etc.

Example

The decimal number $6\,345_{10}$ is to be converted to an octal number:

6 345 : 8 =	793	Remainder	1
793 : 8 =	99	Remainder	1
99 : 8 =	12	Remainder	3
12 : 8 =	1	Remainder	4
1 : 8 =	0	Remainder	1

The required octal number is $14\,311_8$ ($6\,345_{10} = 14\,311_8$).

This conversion method is particularly suitable for a computing program.

Conversion of binary numbers

For number systems to base 2n, for example number notations based on 2, 8, 16, simpler methods of conversion exist between them. These are based on the fact that the source and target bases are related in a ratio of powers of two. The digit repertoire of the octal system is covered by a three-digit binary number, while that of the hexadecimal system is covered by a four-digit binary number. Conversion of a binary number to an octal or a hexadecimal number is achieved easily by combining groups of three or four of the binary number.

Example

Converting the binary number 110101111_2 to an octal or a hexadecimal number:

$$\overbrace{110}^{6}\;\overbrace{101}^{5}\;\overbrace{111}^{7} \quad \text{Octal number}$$

$$110\;101\;111 \quad \text{Binary number}$$

$$\underbrace{1}_{1}\;\underbrace{1010}_{A}\;\underbrace{1111}_{F} \quad \text{Hexadecimal number}$$

Fundamental arithmetic operations in the binary system

The following arithmetic rules apply to the arithmetic operations of addition, subtraction and multiplication:

Addition	Result	Carry (bit)	Sub-traction	Result	Carry (bit)	Multi-plication	Result	Carry (bit)
0 + 0	0	0	0 – 0	0	0	0 · 0	0	0
0 + 1	1	0	0 – 1	1	–1[1]	0 · 1	0	0
1 + 0	1	0	1 – 0	1	0	1 · 0	0	0
1 + 1	0	+1[1]	1 – 1	0	0	1 · 1	1	0

Source: Koch, G.; Reinhold, U.: Einführung in die Informatik für Ingenieure und Naturwissenschaftler, Teil 1, München: Hanser-Verlag 1977.

[1] If, when applying the operations to multiple-digit numbers, the carry amount (“borrowing”) is included in the calculation, the same rules apply as in the decimal system.

Technical statistics

Functions and areas of application

Functions

The function of technical statistics is to describe sets of elements of the same type having different attribute values by means of statistical parameters. As a result, it is then possible to make objective comparisons and assessments.

In addition, it provides statements on the statistical parameters of larger sets (the population) through the evaluation of a relatively small number of individual data (random samples).

Areas of application

The most important areas of application of technical statistics are:

- statistical quality control
- evaluation of test results
- calculation of errors.

Terms, values and definitions

Population

The population is the set comprising all the units or events that are to be considered in statistical analysis (measurement, observation).

The attribute value that is of interest is described by means of statistical parameters.

Random sample

A random sample is a set taken from the population for the purpose of determining attribute values. Evaluation of this sample facilitates statements about statistical parameters of the population.

Raw data list

The raw data list is the term for the original attribute values (for example measurement values) from a random sample.

Terms and values The following table gives descriptions of some important terms and values in statistics.

Value	Definition	Explanations, relationships
N	Population size	The size of the total population is also designated simply as the population
n	Number of attribute values in the random sample	Attribute values are recorded in the raw data list
x_i	Individual attribute value, such as a measurement value	Ordinal number of the attribute values i = 1, 2, 3, ..., n
$\bar{x}$	Arithmetic mean value of the attribute values in the random sample	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
R	Range of the attribute values	$R = x_{max} - x_{min}$
k	Number of classes into which R is subdivided	Guide value $k = \sqrt{n}$ $k \geqq 10$ for $n \leqq 100$ $k \geqq 20$ for $n \leqq 10^5$
Δx	Class interval	$\Delta x = R/k$
x_j	Values of the class midpoints, arithmetic mean value of the class limits	Ordinal number of the classes j = 1, 2, 3, ..., k
n_j	Population density of the individual classes, absolute frequency	The population density n_j indicates how many values in the raw data list fall into the j-th class: $\sum_{j=1}^{k} n_j = n$
h_j	Relative frequency in the j-th class	$h_j = \frac{n_j}{n}$ $\sum_{j=1}^{k} h_j = 1$
G_j	Cumulative population density	G_j is the population density added up to the j-th class: $G_j = \sum_{i=1}^{j} n_i$
H_j	Cumulative frequency	$H_j = \frac{G_j}{n} = \sum_{i=1}^{j} h_i$
x_0	Reference value of the population	Normally the approximated, rounded mean value or class mark with the greatest frequency: $x_0 \approx \bar{x}$ $d_i = x_i - x_0$ $\bar{x} = x_0 + \frac{1}{n}\sum_{i=1}^{n}(x_i - x_0)$ $\bar{x} = x_0 + \bar{d}$

Continuation of table, see Page 88.

Continuation of table, Terms and values, from Page 87.

Value	Definition	Explanations, relationships
s^2	Variance of the random sample (mean square deviation)	$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - x_0)^2$ $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} d_i^2$ $s^2 = \frac{1}{n-1} \left(\sum_{i=1}^{n} x_i^2 - x_0 \sum_{i=1}^{n} x_i \right)$
s	Standard deviation of the random sample (scatter), square root of variance	$s = \sqrt{\frac{1}{n-1} \left(\sum_{i=1}^{n} x_i^2 - x_0 \sum_{i=1}^{n} x_i \right)}$ s approximates σ for high values of n
μ	Mean value of the population, expected value	The arithmetic mean value $\overline{x}$ of the random sample is an estimated value true to expectancy for the expected value μ of the population
σ	Standard deviation of the population	A measure of the variation of the individual values about the mean value
u	Scatter factor	Certain ranges $\mu \pm u \cdot \sigma$ can be defined in which P% of the measurement values lie
F(x)	Distribution function, cumulative function	The distribution function describes the relationship between the random variables x and the cumulative frequency or probability for values $\leqq$ x, while in empirical distributions it corresponds to the cumulative curve
f(x)	Frequency density function	$f(x) = \frac{dF(x)}{dx}$ As a **continuous** function, the frequency density function corresponds to the representation of the relative frequency density of the random sample by means of a stepped curve (histogram)
R(x)	Reliability function	R(x) = 1 – F(x)

Statistical evaluation (example)

Function A batch of rolling bearing balls (the population) is to be checked against the nominal diameter D of 8 mm and the deviations from this value.

Solution A random sample of 200 balls is taken from the batch. It is assumed that the sample will give a sufficiently accurate representation of the population. The diameter x (the attribute value) is measured to 0,001 mm.

The data obtained from this study of a particular investigation feature are initially present unsorted in the so-called raw data list.

Determining the mean value

Equation 1

Within the measurement accuracy, the mean value determined from the raw data list of 200 measurement values is the required nominal value:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{200}\sum_{i=1}^{200} x_i = 8,000 \text{ mm}$$

Of the 200 measured values, the largest diameter x_{max} = 8,013 mm and the smallest diameter x_{min} = 7,987 mm.

As a result, the range of the attribute values is:

Equation 2

$$R = x_{max} - x_{min} = 0,026 \text{ mm}$$

The attribute values are divided into k = 13 classes with a class interval Δx = 0,002 mm.

Based on the raw data list of 200 measurements, classification can be carried out and presented in tabular form.
The relative frequency density h_j and the cumulative frequency H_j are also determined, as defined in the table Terms and values, Page 87.

Classification and frequency

Based on the raw data list of 200 measurements, the following classification and the values for the relative frequency density h_j and the cumulative frequency H_j are determined.

j	Classification x_{lower} to under mm	x_{upper} mm	x_j mm	n_j	h_j	H_j
1	7,987	7,989	7,988	1	0,005	0,005
2	7,989	7,991	7,990	5	0,025	0,030
3	7,991	7,993	7,992	7	0,035	0,065
4	7,993	7,995	7,994	16	0,080	0,145
5	7,995	7,997	7,996	25	0,125	0,270
6	7,997	7,999	7,998	29	0,145	0,415
7	7,999	8,001	8,000	34	0,170	0,585
8	8,001	8,003	8,002	32	0,160	0,745
9	8,003	8,005	8,004	22	0,110	0,855
10	8,005	8,007	8,006	14	0,070	0,925
11	8,007	8,009	8,008	9	0,045	0,970
12	8,009	8,011	8,010	4	0,020	0,990
13	8,011	8,013	8,012	2	0,010	1,000
				Σ 200	**Σ 1,000**	

Confirmation of the mean value

In order to confirm the mean value $\overline{x}$ where the class interval is identical, the class midpoints x_j are multiplied by their frequencies h_j as weighting factors:

Equation 3

$$\overline{x} = \frac{1}{n} \sum_{j=1}^{k} n_j \cdot x_j = \sum_{j=1}^{k} h_j \cdot x_j$$

$$\overline{x} = 8{,}000 \text{ mm}$$

Representation of relative frequency

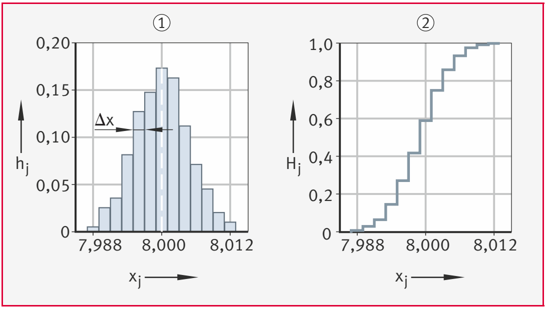
The relative frequency as a function of the class midpoints is represented by means of a bar chart showing the frequency density of the random sample (frequency diagram). This method of representation is also known as a histogram, see Figure 1, ①. It provides a representation of the frequency distribution as an approximation for the distribution function.

In the representation of the cumulative frequency as a function of the class midpoints, the staircase curve shows the so-called cumulative curve or empirical distribution function, see Figure 1, ②.

In comparison with the frequency diagram, the cumulative curve has the advantage that it is possible to easily read off the percentage of the measurements that lie within any interval.

Figure 1
Histogram and cumulative curve

① Histogram/frequency diagram
② Cumulative curve



Variance and standard deviation

The variance s^2 of the random sample is determined as follows:

Equation 4

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{n-1} \sum_{j=1}^{k} n_j \cdot (x_j - \bar{x})^2$$

This can be used to determine the standard deviation s of the random sample:

Equation 5

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1} \sum_{j=1}^{k} n_j \cdot (x_j - \bar{x})^2}$$

The calculation scheme for the standard deviation can be used to determine the variance:

j	x_j	n_i	$x_j - \bar{x}$	$n_j (x_j - \bar{x})^2$
1	7,988	1	−0,012	0,000144
2	7,990	5	−0,010	0,000500
3	7,992	7	−0,008	0,000448
4	7,994	16	−0,006	0,000576
5	7,996	25	−0,004	0,004000
6	7,998	29	−0,002	0,000116
7	8,000	34	0,000	0,0
8	8,002	32	+0,002	0,000128
9	8,004	22	+0,004	0,000352
10	8,006	14	+0,006	0,000504
11	8,008	9	+0,008	0,000576
12	8,010	4	+0,010	0,000400
13	8,012	2	+0,012	0,000288
		Σ 200		**Σ 0,004432**

This gives the variance s^2 as follows:

Equation 6

$$s^2 = \frac{1}{n-1} \sum_{j=1}^{k} n_j (x_j - \bar{x})^2$$

$$s^2 = \frac{1}{199} \cdot 0{,}00\,4432 = 22{,}27 \cdot 10^{-6}$$

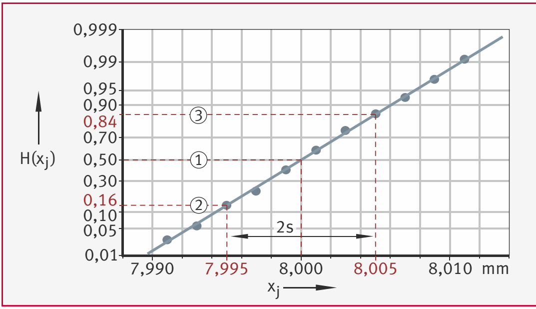
The variance ultimately gives the standard deviation s = 0,0047 mm.

Representation in a probability plot

The values of the cumulative frequency H_j are entered in the probability plot as ordinates against the upper class limits. If a normal distribution is present, the ordinates of the cumulative frequency in the probability plot are distorted such that the S-shaped cumulative curve becomes a straight line, see Figure 2.

Figure 2
Normal distribution in a probability plot

① 50% of cumulative frequency
② 16% of cumulative frequency
③ 84% of cumulative frequency



The following values can be derived from this representation:

mean value $\overline{x}$ at 50% of the cumulative frequency:

Equation 7

$$\overline{x} = 8{,}000 \text{ mm}$$

and the standard deviation s, determined as double the standard deviation from the abscissa values at 16% and 84% of the cumulative frequency:

Equation 8

$$2s = x_{(H = 0{,}84)} - x_{(H = 0{,}16)}$$

$$2s = 8{,}005 \text{ mm} - 7{,}995 \text{ mm} = 0{,}010 \text{ mm}$$

$$s = 0{,}005 \text{ mm}$$

Within the scope of the read-off accuracy, this value shows good agreement with the calculated value. The variation coefficient indicates the standard deviation related to the mean value:

Equation 9

$$V_x = \frac{s}{\overline{x}}$$

$$V_x = \frac{0{,}005 \text{ mm}}{8{,}000 \text{ mm}} = 0{,}000\,63$$

Note on the evaluation of measurement series (confidence interval)

If a large number of random samples each comprising n values is taken from one and the same population with the mean value μ and the standard deviation σ, the mean values $\overline{x}_1$; $\overline{x}_2$; ... of the random samples will show scatter about the true value of μ:

Equation 10

$$\overline{x} = \mu \pm u \cdot \frac{\sigma}{\sqrt{n}}$$

The values for the factor u are listed in the table Value frequency, Page 95.

If only the values $\overline{x}$ and s of a random sample are known and a statement is to be made about the true mean value μ of the population, a so-called confidence interval can be indicated.

The mean value μ should lie within the confidence interval with P% probability:

Equation 11

$$\mu = \overline{x} \pm t \cdot \frac{s}{\sqrt{n}}$$

The values for the factor t are given in the following table.

n		2	3	5	10	20	50	...
t values for P =	90%	6,31	2,92	2,13	1,83	1,73	1,68	1,65
	95%	12,7	4,30	2,78	2,26	2,09	2,01	1,96
	99%	63,7	9,92	4,60	3,25	2,86	2,68	2,58

The true mean value μ of the population, at a probability of 90%, is:

$$\mu = 8{,}000\,\text{mm} \pm 1{,}65\,\frac{0{,}0047\,\text{mm}}{\sqrt{200}}$$

$$\mu = 8{,}000\,\text{mm} \pm 0{,}0005\,\text{mm}$$

Gaussian normal distribution

A Gaussian normal distribution generally occurs when a large number of random influences that are independent of each other act on a single attribute value of a population (collective) while none of the influences plays a dominant role.

If a normal distribution is present, this results in a straight line for the cumulative frequency in the probability plot, see Figure 2, Page 92.

Frequency density function and cumulative function

The staircase curves shown in Figure 1, Page 90, and Figure 2, Page 92, for the relative frequency density and cumulative frequency are incorporated in this case in the constant trends of the frequency density function f(x) and the cumulative function F(x) with a mean value μ and standard distribution σ.

The frequency density function f(x) is described as follows:

Equation 12

$$f(x;\,\mu,\,\sigma) = \frac{1}{\sqrt{2\pi}\cdot\sigma}\cdot e^{-\frac{(x-\mu)^2}{2\cdot\sigma^2}} \qquad t = \frac{x-\mu}{\sigma}$$

using the generalised coordinate t.

The cumulative function F(x) is determined as follows:

Equation 13

$$F(x;\,\mu,\,\sigma) = \int_{-\infty}^{x} f(x;\,\mu,\,\sigma)\,dx$$

Representation of the Gaussian normal distribution

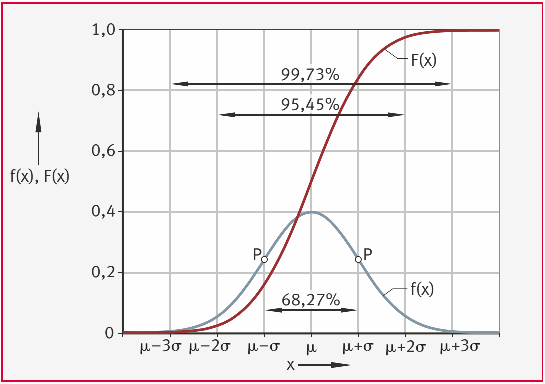
The normal distribution is symmetrical to the mean value μ of the population and shows an inflection point in each case at $x = \mu \pm \sigma$. The greater the value σ, the greater the distance between these two points. This starts at $x = -\infty$ and ends at $x = +\infty$. The total area under this “bell curve” corresponds to 1 = 100%.

Multiple values for the standard deviation can be used to define intervals $x = \mu \pm u \cdot \sigma$ in which P% of the x values lie. It can be seen from the table of value frequency that 99,73% of all values lie in the interval $\pm 3 \cdot \sigma$.

The frequency density function f(x) and the cumulative function F(x) are clearly defined by the mean value μ and the standard deviation σ of the distribution.

Figure 3
Gaussian normal distribution

f(x) = frequency density function
F(x) = cumulative function
P = inflection point



Value frequency

A value frequency within $\pm u \cdot \sigma$ gives the following values:

u	1,00	1,28	1,64	1,96	2,00	2,33	2,58	3,00	3,29
P%	68,27	80	90	95	95,45	98	99	99,73	99,9

Standard normal distribution

The integral of the cumulative function cannot be evaluated on an elementary basis. It is therefore necessary to present the function F(x; μ, σ) in tabular form.

The standard normal distribution is considered on the basis of the mean value $\mu = 0$ and the standard deviation $\sigma = 1$:

Equation 14

$$F(x; 0, 1) = F(x)$$

In many evaluations, it is only the deviations from a stated or known mean value that are of interest, so it is appropriate in these cases to calculate the function $\Phi^*(x)$:

Equation 15

$$\Phi^*(x) = F(x) - \frac{1}{2}$$

For the function Φ^* (x), the following values are derived:

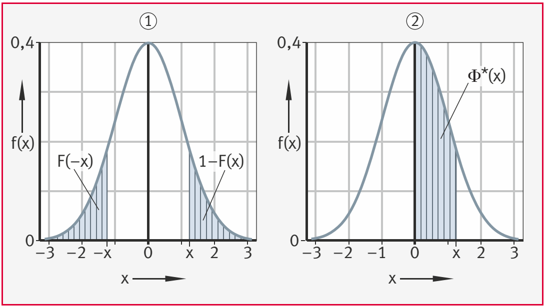
$x \rightarrow \Phi^*(x)$	
x	$\Phi^*(x)$
0,0	0,000
0,1	0,040
0,2	0,079
0,3	0,118
0,4	0,155
0,5	0,191
0,6	0,226
0,7	0,258
0,8	0,288
0,9	0,316
1,0	0,341
1,1	0,364
1,2	0,385
1,3	0,403
1,4	0,419
1,5	0,433
1,6	0,445
1,7	0,455
1,8	0,464
1,9	0,471
2,0	0,477
2,1	0,482
2,2	0,486
2,3	0,489
2,4	0,492
2,5	0,494
2,6	0,495
2,7	0,496
2,8	0,474
2,9	0,498
3,0	0,499

$\Phi^*(x) \rightarrow x$			
$\Phi^*(x)$	**x**	$\Phi^*(x)$	**x**
0,00	0,00	0,25	0,67
0,01	0,03	0,26	0,71
0,02	0,05	0,27	0,74
0,03	0,08	0,28	0,77
0,04	0,10	0,29	0,81
0,05	0,13	0,30	0,84
0,06	0,15	0,31	0,88
0,07	0,18	0,32	0,92
0,08	0,20	0,33	0,95
0,09	0,23	0,34	0,99
0,10	0,25	0,34	1,04
0,11	0,28	0,36	1,08
0,12	0,31	0,37	1,13
0,13	0,33	0,38	1,18
0,14	0,36	0,39	1,23
0,15	0,39	0,40	1,28
0,16	0,41	0,41	1,34
0,17	0,44	0,42	1,41
0,18	0,47	0,43	1,48
0,19	0,50	0,44	1,55
0,20	0,52	0,45	1,64
0,21	0,55	0,46	1,75
0,22	0,58	0,47	1,88
0,23	0,61	0,48	2,05
0,24	0,64	0,49	2,33
–	–	–	–
–	–	–	–
–	–	–	–
–	–	–	–
–	–	–	–
–	–	–	–

The functions F(x) and $\Phi^*(x)$ appear in graphic form as a bell curve:

Figure 4
Standard normal distribution, area under the bell curve

① Function F(x)
② Function $\Phi^*(x)$



The complete area under the bell curve is defined as:

Equation 16

$$F\left(x = +\infty\right) = 1$$

The standard normal distribution is symmetrical to the mean value $\mu = 0$. It is then sufficient to present the function $\Phi^*(x)$ in tabular form for positive values of x only.

The cumulative frequency between the values $\pm x$ is then:

Equation 17

$$\int_{-x}^{+x} F\left(x;\, 0,\, 1\right) dx = 2 \cdot \Phi^*\left(x\right)$$

First approximation

If the random sample n taken is very large in relation to the population N, the following simplification is permitted:

- estimated values for mean value $\overline{x}$ of the random sample = actual parameter for mean value μ
- standard deviation s of the random sample = parameter for standard deviation σ of the population.

Tolerances of a process

If the standard deviation σ is known, this can also be used to determine the natural tolerances of a process, in other words to define an interval that contains (almost) the complete distribution. In practice, the value frequently selected in this case is $\mu \pm 3\sigma$ (99,73%).

Weibull distribution

The Weibull distribution has proved effective in practice for evaluation of the rating life of engineering products. It has been applied as a standard procedure in rolling bearing technology.

Weibull cumulative function

Equation 18

The Weibull cumulative function is as follows:

$$F(t) = 1 - e^{-(t/\eta)^{\beta}}$$

The associated reliability function R(t) is also known as the survival probability and is determined as follows:

Equation 19

$$R(t) = 1 - F(t)$$

F(t) = cumulative function
Probability that a specimen from a random sample or a collective will fail by the time t

R(t) = survival probability;
reliability function

t = attribute value, failure time

β = measure of the scatter of the failure times, failure gradient

η = characteristic rating life;
the time by which 63,2% of the specimens in a test procedure have failed

The characteristic rating life η is determined by using $t = \eta$ in the Weibull cumulative function:

Equation 20

$$F(\eta) = 1 - e^{-1^{\beta}} = 1 - \frac{1}{e}$$

$$F(\eta) = 0{,}632 \triangleq 63{,}2\%$$

Evaluation of a rating life test

For evaluation of tests, a linear representation of the cumulative function is not suitable.

Double logarithmisation gives the following:

Equation 21

$$\left(\frac{t}{\eta}\right)^{\beta} = \ln\frac{1}{1-F(t)}$$

$$\beta\left(\lg t - \lg \eta\right) = \lg \ln\frac{1}{1-F(t)}$$

This relationship appears in the Weibull paper with an abscissa graduation lg t and an ordinate graduation lg ln 1/(1 – F(t)) as a straight line for F(t).

Cumulative failure frequency

For the evaluation of a rating life test comprising n tests, the cumulative frequency H_i is plotted against the rating life values t arranged according to magnitude in accordance with the median ranking method:

Equation 22

$$H_i = \frac{i-0,3}{n+0,4}$$

i = ordinal number of the failure times of the specimens

In order to achieve both statistical authoritativeness of test results and an acceptable test duration, it is necessary to carry out a rating life test with a large random sample n up to a cumulative failure frequency of at least $H_i = 0,5$.

η and β are random values, on a similar basis to $\overline{x}$ and s in the Gaussian normal distribution.

For random samples $n \geqq 50$, the following relationship gives confidence intervals for the values to be expected of the population:

Equation 23

$$\eta \pm \left(\frac{u}{\sqrt{n}}\right) \cdot 1,052 \cdot \left(\frac{\eta}{\beta}\right)$$

$$\beta \pm \left(\frac{u}{\sqrt{n}}\right) \cdot 0,78 \cdot \beta$$

The values for u are given in the table Value frequency, Page 95.

Basic rating life

In rolling bearing technology, the definition states that 10% of the bearings in a large collective are permitted to have failed by the time the basic rating life is reached. A relationship is obtained between η and L_{10} if the values $t = L_{10}$ and F(t) = 0,10 are used in the Weibull cumulative function:

Equation 24

$$L_{10} = \eta \cdot \ln\left(\frac{1}{1-0,1}\right)^{\frac{1}{\beta}} = \eta \cdot 0,10536^{\frac{1}{\beta}}$$

Regression and correlation

The function of regression

The function of regression is to determine, from the value pairs (x_i, y_i) where i = 1, 2, ..., n of a random sample of size n, a relationship between an independent variable x and a dependent random variable y.

The precondition is that the value pairs were each determined on the same i-th element of the analysed elements and the random variable y follows a normal distribution.

Theoretical regression function

For the theoretical regression function, a polynomial of a k-th degree is generally selected whose coefficients α_j with j = 0,1, ..., k are to be determined:

Equation 25

$$f(x) = \alpha_k \cdot x^k + \alpha_{k-1} \cdot x^{k-1} + \ldots + \alpha_j \cdot x^j + \ldots + \alpha_1 \cdot x^1 + \alpha_0 \cdot x^0$$

If there is a linear relationship between x and f(x), the best-fit line gives a good approximation.

Determining the coefficients

The coefficients α_j are determined according to the Gaussian method of least squares:

Equation 26

$$\sum_{i=1}^{n} \left(y_i - f(x_i)\right)^2 = \sum_{j=1}^{n} \left(y_i - \sum_{j=0}^{k} \alpha_j \cdot x_i{}^j\right)^2 = g$$

with $(\alpha_0, \alpha_1, \ldots, \alpha_n)$ = minimum.

Linear approach

The partial derivations $\partial g / \partial \alpha_j = 0$ give (k + 1) linear equations that can be solved using the methods for linear equation systems.

For the linear case:

Equation 27

$$y = \alpha_0 + \alpha_1 \cdot x$$

and with the mean values:

Equation 28

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

the relationship is as follows:

Equation 29

$$\alpha_0 = \bar{y} - \alpha_1 \cdot \bar{x}$$

If the following is used:

Equation 30

$$y - \bar{y} = \alpha_1 (x - \bar{x})$$

the relationship is as follows:

Equation 31

$$\alpha_1 = \frac{\left(\sum x_i\, y_i - n\bar{x}\,\bar{y}\right)}{\left(\sum x_i{}^2 - n\bar{x}^2\right)}$$

Variances

Equation 32

The variances s^2 of the random sample are determined as follows:

$$s_x^2 = \frac{1}{n-1}\left[\sum x_i{}^2 - \left(\left(\sum x_i\right)^2 \cdot \frac{1}{n}\right)\right]$$

$$s_y^2 = \frac{1}{n-1}\left[\sum y_i{}^2 - \left(\left(\sum y_i\right)^2 \cdot \frac{1}{n}\right)\right]$$

The covariance s of the random sample is determined as follows:

Equation 33

$$s_{xy} = \frac{1}{n-1}\sum (x_i - \bar{x})(y_i - \bar{y}) = \frac{1}{n-1}\sum \left(x_i\, y_i - n\bar{x}\,\bar{y}\right)$$

Coefficient

Equation 34

The coefficient α_1 is determined as:

$$\alpha_1 = \frac{s_{xy}}{s_x^2}$$

If all the measurement points lie on the straight line, the variances s^2 are as follows:

Equation 35

$$s_{xy}^2 = s_x^2 \cdot s_y^2$$

The function of correlation

If there are no recognisable reasons for a functional dependence of the random variables y on the independently applied variables x, the correlation calculation (correlation interdependence) is used to check the quality of a subordinate relationship.

A linear dependence can be stated using the correlation coefficient r_{xy}. This is in the following relationship with the values determined in the section Theoretical regression function, Page 100:

Equation 36

$$r_{xy} = \frac{s_{xy}}{\sqrt{s_x^2 \cdot s_y^2}}$$

$$-1 \leqq r_{xy} \leqq 1$$

If $r_{xy} < 0$ this is known as a negative correlation: large values for x are associated with small values for y and vice versa.

The value $B = r_{xy}^2$ is known as the coefficient of determination.

Chemistry

Elements and values

The periodic system of elements

In the periodic system, chemical elements are arranged according to their atomic weight. The atomic number, symbol, name and relative atomic mass (or rather the atomic mass of the most stable isotope in [ ]) is displayed in each case.

Period	Group								
	Main groups		Subgroups						
	1	2	3	4	5	6	7	8	9
1	1 **H** Hydrogen 1,0079								
2	3 **Li** Lithium 6,941	4 **Be** Beryllium 9,0122							
3	11 **Na** Sodium 22,99	12 **Mg** Mag-nesium 24,305							
4	19 **K** Potassium 39,098	20 **Ca** Calcium 40,078	21 **Sc** Scandium 44,956	22 **Ti** Titanium 47,867	23 **V** Vanadium 50,942	24 **Cr** Chromium 51,996	25 **Mn** Manga-nese 54,938	26 **Fe** Iron 55,845	27 **Co** Cobalt 58,933
5	37 **Rb** Rubidium 85,468	38 **Sr** Strontium 87,62	39 **Y** Yttrium 88,906	40 **Zr** Zirconium 91,224	41 **Nb** Niobium 92,906	42 **Mo** Molyb-denum 95,94	43 **Tc** Tech-netium [97,907]	44 **Ru** Ru-thenium 101,07	45 **Rh** Rhodium 102,906
6	55 **Cs** Caesium 132,905	56 **Ba** Barium 137,327	57 – 71 Lantha-nides, see Page 104	72 **Hf** Hafnium 178,49	73 **Ta** Tantalum 180,948	74 **W** Tungsten 183,84	75 **Re** Rhenium 186,207	76 **Os** Osmium 190,23	77 **Ir** Iridium 192,217
7	87 **Fr** Francium [223,02]	88 **Ra** Radium [226,03]	89 – 103 Actinides, see Page 104	104 **Rf** Ruther-fordium [261,11]	105 **Db** Dubnium [262,11]	106 **Sg** Sea-borgium [266,12]	107 **Bh** Bohrium [264,12]	108 **Hs** Hassium [277]	109 **Mt** Meitner-ium [268,14]

			Main groups					
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
								2 **He** Helium 4,0026
			5 **B** Boron 10,811	6 **C** Carbon 12,011	7 **N** Nitrogen 14,007	8 **O** Oxygen 15,999	9 **F** Fluorine 18,998	10 **Ne** Neon 20,18
			13 **Al** Aluminium 26,982	14 **Si** Silicon 28,086	15 **P** Phos- phorus 30,974	16 **S** Sulphur 32,065	17 **Cl** Chlorine 35,453	18 **Ar** Argon 39,948
28 **Ni** Nickel 58,693	29 **Cu** Copper 63,546	30 **Zn** Zinc 65,38	31 **Ga** Gallium 69,723	32 **Ge** Ger- manium 72,64	33 **As** Arsenic 74,922	34 **Se** Selenium 78,96	35 **Br** Bromine 79,904	36 **Kr** Krypton 83,798
46 **Pd** Palladium 106,42	47 **Ag** Silver 107,868	48 **Cd** Cadmium 112,411	49 **In** Indium 114,818	50 **Sn** Tin 118,71	51 **Sb** Antimony 121,76	52 **Te** Tellurium 127,6	53 **I** Iodine 126,9	54 **Xe** Xenon 131,293
78 **Pt** Platinum 195,078	79 **Au** Gold 196,967	80 **Hg** Mercury 200,59	81 **Tl** Thallium 204,383	82 **Pb** Lead 207,2	83 **Bi** Bismuth 208,98	84 **Po** Polonium [208,98]	85 **At** Astatine [209,99]	86 **Rn** Radon [222,02]
110 **Ds** Darm- stadtium [281]	111 **Rg** Roentge- nium [280]	112 **Cn** Coper- nicium [285]		114 **Fl** Flerovium [289]		116 **Lv** Liver- morium [293]		

Lanthanides Addition of lanthanides to table, The periodic system of elements, from Page 102.

57 **La** Lantha-num 138,905	58 **Ce** Cerium 140,116	59 **Pr** Praseo-dymium 140,908	60 **Nd** Neodymium 144,242	61 **Pm** Promethium [144,91]	62 **Sm** Samarium 150,36	63 **Eu** Europium 151,964	64 **Gd** Gadolinium 157,25
	65 **Tb** Terbium 158,925	66 **Dy** Dysprosium 162,5	67 **Ho** Holmium 164,93	68 **Er** Erbium 167,259	69 **Tm** Thulium 168,934	70 **Yb** Ytterbium 173,04	71 **Lu** Lutetium 174,967

Actinides Addition of actinides to table, The periodic system of elements, from Page 102.

89 **Ac** Actinium [227,03]	90 **Th** Thorium [232,04]	91 **Pa** Protactinium [231,04]	92 **U** Uranium [238,03]	93 **Np** Neptunium [237,05]	94 **Pu** Plutonium [244,06]	95 **Am** Americium [243,06]	96 **Cm** Curium [247,07]
	97 **Bk** Berkelium [247,07]	98 **Cf** Californium [251,08]	99 **Es** Einsteinium [252,08]	100 **Fm** Fermium [257,1]	101 **Md** Mendelevium [258,1]	102 **No** Nobelium [259,1]	103 **Lr** Lawrencium [262,11]

Physical properties: Chemical elements The physical properties of the chemical elements are listed in the following table.

Element	Symbol	Atomic number	Relative atomic mass	Density ρ kg/dm³ 1)	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Actinium	Ac	89	(227)	–	1 050	3 200	–	0,12
Aluminium	Al	13	26,98	2,70	660	2 450	238	0,88
Americium	Am	95	(243)	11,7	>850	2 600	–	0,14
Antimony	Sb	51	121,75	6,68	631	1 380	19	0,21
Argon	Ar	18	39,95	1,40 1)	–189	–186	0,02	0,52
Arsenic	As	33	74,92	5,72	817 2)	613	–	0,33
Astatine	At	85	(209,99)	–	302	335	–	0,14
Barium	Ba	56	137,34	3,50	714	1 640	–	0,29
Berkelium	Bk	97	(247)	–	–	–	–	–
Beryllium	Be	4	9,01	1,85	1 280	2 480	168	1,02
Bismuth	Bi	83	208,98	9,8	271	1 560	8,1	0,12
Boron	B	5	10,81	2,34	(2 030)	3 900	–	1,04
Bromine	Br	35	79,90	3,12	–7	58	–	0,45
Cadmium	Cd	48	112,40	8,65	321	765	96	0,23
Caesium	Cs	55	132,91	1,87	29	690	–	0,22

Continuation of table, see Page 105.
Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

1) Gas: density in kg/m³ (at +25 °C and 1013 hPa).
2) In sealed tube at 27,5 bar.

Continuation of table, Physical properties: Chemical elements, from Page 104.

Element	Symbol	Atomic number	Relative atomic mass	Density ρ kg/dm^{3} 1)	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Calcium	Ca	20	40,08	1,55	838	1 490	130	0,66
Californium	Cf	98	(251)	–	–	–	–	–
Carbon	C	6	12,01	2,26	3 730	4 830	168	0,65
Cerium	Ce	58	140,12	6,78	795	3 470	10,9	0,18
Chlorine	Cl	17	35,45	1,56[1)]	–101	–35	0,008	0,47
Chromium	Cr	24	52,00	7,19	1 900	2 642	69	0,44
Cobalt	Co	27	58,93	8,90	1 490	2 900	96	0,43
Copper	Cu	29	63,55	8,96	1 083	2 600	398	0,38
Curium	Cm	96	(247)	7	–	–	–	–
Dysprosium	Dy	66	162,50	8,54	1 410	2 600	10	0,17
Einsteinium	Es	99	(254)	–	–	–	–	–
Erbium	Er	68	167,26	9,05	1 500	2 900	9,6	0,17
Europium	Eu	63	151,96	5,26	826	1 440	–	0,17
Fermium	Fm	100	(257)	–	–	–	–	–
Fluorine	F	9	19,00	1,51[1)]	–220	–188	0,02	0,83
Francium	Fr	87	(223,02)	–	(27)	(680)	–	0,14
Gadolinium	Gd	64	157,25	7,89	1 310	3 000	8,8	0,23
Gallium	Ga	31	69,72	5,91	30	2 400	40	0,37
Germanium	Ge	32	72,59	5,32	937	2 830	62	0,31
Gold	Au	79	196,97	19,3	1 063	2 970	314	0,13
Hafnium	Hf	72	178,49	13,1	2 000	5 400	93	0,14
Helium	He	2	4,003	0,15[1)]	–270	–269	0,16	5,23
Holmium	Ho	67	164,93	8,80	1 460	2 600	–	0,16
Hydrogen	H	1	1,008	0,07[1)]	–259	–253	0,17	14,14
Indium	In	49	114,82	7,31	156	2 000	24	0,23
Iodine	I	53	126,90	4,94	114	183	0,43	0,22
Iridium	Ir	77	192,22	22,5	2 450	4 500	58	0,13
Iron	Fe	26	55,85	7,86	1 540	3 000	72	0,44
Krypton	Kr	36	83,80	2,16[1)]	–157	–152	0,01	0,25
Lanthanum	La	57	138,91	6,17	920	3 470	13,8	0,20

Continuation of table, see Page 106.

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

1) Gas: density in kg/m^3 (at +25 °C and 1013 hPa).

Continuation of table, Physical properties: Chemical elements, from Page 105.

Element	Symbol	Atomic number	Relative atomic mass	Density ρ kg/dm^3 [1]	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Lawrencium	Lr	103	(256)	–	–	–	–	–
Lead	Pb	82	207,2	11,4	327	1740	35	0,13
Lithium	Li	3	6,94	0,53	180	1330	71	3,6
Lutetium	Lu	71	174,97	9,84	1 650	3 330	–	–
Magnesium	Mg	12	24,31	1,74	650	1110	171	1,01
Manganese	Mn	25	54,94	7,43	1 250	2100	30	0,47
Mendelevium	Md	101	(258)	–	–	–	–	–
Mercury	Hg	80	200,59	13,53	−39	357	8,1	0,14
Molybdenum	Mo	42	95,94	10,2	2 610	5 560	142	0,24
Neodymium	Nd	60	144,24	7,00	1020	3 030	16	0,19
Neon	Ne	10	20,18	1,20[1]	−249	−246	0,05	1,03
Neptunium	Np	93	237,05	20,4	640	–	57	–
Nickel	Ni	28	58,71	8,90	1450	2 730	61	0,43
Niobium	Nb	41	92,91	8,55	2 420	4 900	52	0,27
Nitrogen	N	7	14,01	0,81[1]	−210	−196	0,02	1,04
Nobelium	No	102	(256)	–	–	–	–	–
Osmium	Os	76	190,2	22,4	3 000	5 500	87	0,13
Oxygen	O	8	16,00	1,15[1]	−219	−183	0,03	0,92
Palladium	Pd	46	106,4	12,0	1550	3125	69	0,25
Phosphorus	P	15	30,97	1,82	44	280	–	0,67
Platinum	Pt	78	195,09	21,4	1770	3 825	71	0,13
Plutonium	Pu	94	(244)	19,8	640	3 230	9	–
Polonium	Po	84	(208,98)	9,4	254	962	–	0,13
Potassium	K	19	39,10	0,86	64	760	97	0,76
Praseodymium	Pr	59	140,91	6,77	935	3130	12	0,19
Promethium	Pm	61	(145)	–	(1030)	(2 730)	–	0,19
Protactinium	Pa	91	231,04	15,4	(1 230)	–	–	0,12
Radium	Ra	88	226,03	5	700	1530	–	0,12
Radon	Rn	86	(222,02)	4,4[1]	−71	−62	–	0,09
Rhenium	Re	75	186,2	21,0	3180	5 630	48	0,14

Continuation of table, see Page 107.
Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

[1] Gas: density in kg/m^3 (at +25 °C and 1013 hPa).

Continuation of table, Physical properties: Chemical elements, from Page 106.

Element	Symbol	Atomic number	Relative atomic mass	Density ρ kg/dm^{3} 1)	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Rhodium	Rh	45	102,91	12,4	1970	3730	88	0,24
Rubidium	Rb	37	85,47	1,53	39	688	58	0,33
Ruthenium	Ru	44	101,07	12,2	2300	3900	106	0,25
Samarium	Sm	62	150,4	7,54	1070	1900	–	0,20
Scandium	Sc	21	44,96	3,0	1540	2730	63	0,56
Selenium	Se	34	78,96	4,80	217	685	0,2	0,33
Silicon	Si	14	28,09	2,33	1410	2680	80	0,68
Silver	Ag	47	107,87	10,5	961	2210	418	0,23
Sodium	Na	11	22,99	0,97	98	892	138	1,22
Strontium	Sr	38	87,62	2,6	770	1380	–	0,29
Sulphur	S	16	32,06	2,07	113	–	0,26	0,68
Tantalum	Ta	73	180,95	16,6	3000	5430	55	0,12
Technetium	Tc	43	97,907	11,5	2140	(4600)	–	0,25
Tellurium	Te	52	127,60	6,24	450	1390	1,2	0,21
Terbium	Tb	65	158,93	8,27	1360	2800	–	0,18
Thallium	Tl	81	204,37	11,85	303	1460	50	0,13
Thorium	Th	90	232,04	11,7	1700	4200	38	0,14
Thulium	Tm	69	168,93	9,33	1550	1730	–	0,16
Tin	Sn	50	118,69	7,30	232	2270	63	0,22
Titanium	Ti	22	47,90	4,50	1670	3260	16	0,24
Tungsten	W	74	183,85	19,3	3410	5930	130	0,14
Uranium	U	92	238,03	18,90	1130	3820	24	0,12
Vanadium	V	23	50,94	5,8	1900	3450	32	0,51
Xenon	Xe	54	131,30	3,5 1)	–112	–108	0,005	0,16
Ytterbium	Yb	70	173,04	6,98	824	1430	–	0,14
Yttrium	Y	39	88,91	4,5	1500	2930	14	0,29
Zinc	Zn	30	65,37	7,14	419	906	113	0,39
Zirconium	Zr	40	91,22	6,49	1850	3580	21	0,28

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

1) Gas: density in kg/m^{3} (at +25 °C and 1013 hPa).

Physical properties: Liquids

The following table shows the physical properties for a selection of (pure and anhydrous) liquids.

Substance	Density ρ		Melting point	Boiling point	Thermal conductivity λ	Thermal capacity c_p
	kg/dm^3	at °C	°C	°C	W/(m · K)	kJ/(kg · K)
Acetone	0,791	20	−95,35	56,35	0,16	1,21
Acetic acid	1,05	20	16,7	118	–	2,03
Anthracene oil	1,05	15	−20	270 ... 400	0,47	1,33
Benzene	0,83	15	5,4	80	0,14	1,7
Carbon tetrachloride	1,598	18	−22,8	46,3	–	0,845
Diesel fuel	0,83	15	−30	210 ... 380	0,15	2,05
Diethyl ether	0,72	20	−116	35	0,14	2,3
Ethyl acetate	0,975	20	−83,6	77,1	–	2,0
Ethyl alcohol, ≈ 98%	0,80	15	−114	78,5	0,17 ... 23	2,33
Ethyl chloride	0,92	15	−139	12,5	0,16	1,79
Ethylene glycol	1,114	20	−17,4	197,2	0,25	2,4
Glycerine	1,26	20	19	290	0,29	2,43
Heating oil, extra light	<0,86	20	−10	>175	0,14	2,07
Hydrochloric acid, 10%	1,05	15	−14	102	0,50	3,14
Linseed oil	0,93	20	−15	316	0,17	1,88
Machine oil	0,91	15	−5	380 ... 400	0,125	1,80
Mercury	13,55	15	−38,9	357,25	10	0,14
Methyl alcohol	0,80	15	−98	65	0,211	2,55
Methylene chloride	1,335	20	−97	40,1	–	–
Nitric acid	1,51	15	−41,3	86	0,26	1,72
Paraffin oil	0,81	15	−70	150 ... 300	0,13	2,1
Petrol	0,72 ... 0,73	15	−20 ... −50	40 ... 200	0,13	2,1
Petroleum ether	0,66	20	−160	40 ... 70	0,138	1,76
2-propanol (iso...)	0,79	20	−88	83	0,26	2,49
Rapeseed oil	0,91	20	0	300	0,17	1,97
Rectified spirit, 95 perc. by vol.[1)]	0,811	20	−90	78	0,16	2,43
Resin oil	0,96	20	−20	150 ... 300	0,15	–
Silicone oil	0,94	20	–	–	0,22	1,09
Sulphuric acid	1,84	15	10,5	338	0,47	1,42
Table salt solution, 20%	1,15	15	−18	108,8	0,59	3,43
Tar	1,2	20	−15	300	0,19	1,58
Toluene	0,87	15	−97	110	0,14	1,48
Transformer oil	0,87	15	−5	170	0,13	1,88
Trichloroethylene	1,47	18	−83	86,8	–	0,95
Turpentine oil	0,87	15	−10	160	0,10	1,80
Water, distilled	1,0	4	0	100	0,60	4,19

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

1) Ethyl alcohol, denatured.

Physical properties: Solids The following table shows the physical properties for a selection of solids.

Substance	Density ρ kg/dm^3	Melting point °C	Boiling point °C	Thermal conductivity λ W/(m · K)	Thermal capacity c_p kJ/(kg · K)
Agate	2,5 … 2,8	≈ 1 600	≈ 2 590	10,68	0,79
Asphalt	1,1 … 1,5	80 … 100	≈ 300	0,69	0,92
Barium chloride ($BaCl_2$)	3,10	956	1 830	–	0,37
Basalt	2,9	–	–	1,67	0,86
Boiler scale	≈ 2,5	≈ 1 200	–	0,12 … 2,3	0,79
Borax, anhydrous	1,72	741	–	–	0,99
Brass (63 Cu, 37 Zn)	8,5	900	–	116	0,38
Bronze (94 Cu, 6 Sn)	8,73	910	2 300	64	0,37
Charcoal	0,3 … 0,5	–	≈ 3 540	0,08	1,0
Chromium(III) oxide (Cr_2O_3)	5,22	2 330	–	0,4 (powder)	0,75
Coke	1,6 … 1,9	–	–	0,183	0,84
Concrete	1,8 … 2,45	–	–	0,8 … 1,4	0,87
Corundum (Al_2O_3)	3,9 … 4,0	2 050	2 700	12 … 23	0,96
Diamond	3,51	–	–	–	0,52
Flake graphite cast iron	7,25	1 150 … 1 250	2 500	≈ 52	≈ 0,5
Glass fibre mats	0,03 … 0,2	≈ 700	–	0,04	0,84
Glass (window)	2,4 … 2,7	≈ 700	–	0,58 … 1,0	0,84
Granite	2,6 … 2,8	–	–	3,5	0,82
Graphite, pure	2,26	≈ 3 830	≈ 4 200	168	0,71
Greases	0,92 … 0,94	30 … 175	≈ 300	0,2	0,62 … 0,79
Gypsum ($CaSO_4$)	2,3	1 200	–	0,34 … 0,46	1,1
Hard metal K20	14,8	≈ 2 000	≈ 4 000	81,4	0,80
Heat conducting alloy (80 Ni, 20 Cr)	8,3	1 400	2 300	14,6	0,50
Hydrated ferric oxide (rust)	5,1	1 565	–	0,58 (powder)	0,67
Ice	0,92	–	100	2,3	2,1
Leather, dry	0,85 … 1,02	–	–	≈ 0,17	≈ 1,5
Limestone ($CaCO_2$)	2,6 … 2,8	decomposes into CaO & CO_2		2,2	0,91
Litharge (lead monoxide)	9,53	888	1 580	–	0,21
Magnesium alloys	≈ 1,8	≈ 630	1 500	46 … 140	–
Marble ($CaCO_3$)	2,6 … 1,8	1 290	decomposes	2,1 … 3,5	0,88
Mica	2,6 … 3,2	≈ 1 300	≈ 700	0,34	0,87
Monel metal	8,8	1 240 … 1 330	–	19,7	0,43
Porcelain	2,3 … 2,5	≈ 1 600	–	0,8 … 1,0	0,80
Quartz	2,5 … 2,8	≈ 1 400	2 230	9,9	0,80
Red bronze ($CuSn_5$ ZnPb)	8,8	950	2 300	38	0,67
Sand, dry	1,2 … 1,6	1 480	2 230	0,6	0,80
Sandstone	2,2 … 2,5	≈ 1 500	–	2,3	0,71
Soot	1,7 … 1,8	–	–	0,07	0,84
Steel (18 Cr, 8 Ni)	7,9	1 450	–	14	0,51
Steel (18 W)	8,7	1 450	–	26	0,42
Steel, low alloy	7,8 … 7,86	1 450 … 1 530	2 500	46 … 58	0,49
Table salt (NaCl)	2,15	802	1 440	–	0,92
Wood	0,5 … 0,8	–	–	0,17 … 0,34	2,1 … 2,9

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

Physical properties: Gases and vapours The following table shows the physical properties for a selection of gases and vapours.

Substance	Density ρ kg/m^3	Relative density η_{rel} (air = 1)	Boiling point °C	Thermal conductivity λ $W/(m \cdot K)$	Thermal capacity c_p $kJ/(kg \cdot K)$	$\kappa = c_p/c_x$
Acetylene (ethine)	1,17	0,91	−81	0,019	1,68	1,26
Air	1,29	1	−192	0,026	1,00	1,40
Ammonia	0,77	0,60	−33,4	0,024	2,22	1,32
Blast furnace gas	1,28	0,99	−170	0,023	1,05	1,40
Carbon dioxide	1,98	1,52	−78,5	0,015 3	0,88	1,30
Carbon disulphide	3,41	2,64	46	0,007 2	0,67	1,19
Carbon monoxide	1,25	0,97	−191	0,024	1,05	1,40
Cyanogen $(CN)_2$	2,33	1,80	−21,2	–	1,72	1,27
Ethane	1,356	1,049	−88	0,021	–	1,13
Ethyl alcohol vapour	2,07	1,60	78,5	0,032	–	1,13
Ethylene, ethene	1,26	0,98	−102	0,037	1,55	1,25
Freon 12 (Cl_2F_2)	5,08	3,93	−30	–	–	1,14
Generator gas	1,22	0,94	−170	0,023	1,05	1,40
Hydrogen chloride	1,939	1,27	−85	0,014	0,79	1,41
Hydrogen fluoride	0,893	0,713	19,5	–	–	–
Hydrogen sulphide	1,539	1,191	−60,2	–	1,34	–
Isobutane	2,67	2,06	−10	–	–	1,11
Methyl choride	1,545	1,2	−24,0	–	0,74	1,20
Natural gas (methane)	0,718	0,64	−162	–	–	–
n-butane	2,703	2,09	1	–	–	–
Ozone	2,14	1,65	−112	–	–	1,29
Propane	2,019	1,562	−45	–	–	1,14
Propylene (propene)	1,915	1,481	−47	–	–	–
Sulphur dioxide	2,93	2,26	−10	0,010	0,63	1,40
Town gas (illuminating gas)	0,56 ... 0,61	0,47	−210	0,064	2,13	1,40
Water vapour at 100 °C	0,598	0,62	100	0,0191	2,00	1,32

Source: Christiani Datenbank, Dr.-Ing. P. Christiani GmbH.

Melting point of salts

The following table lists the melting points of a number of salts for salt baths.

Salt	Melting point °C	Salt	Melting point °C
Aluminium chloride	192	Potassium chloride	770
Ferric chloride	304	Calcium chloride	772
Potassium nitrate	308	Sodium chloride (table salt)	800
Sodium nitrate	310	Lithium fluoride	848
Zinc chloride	313	Sodium carbonate (soda)	852
Cuprous chloride	432	Potassium fluoride	857
Lithium carbonate	461	Potassium carbonate	897
Lead chloride	498	Barium chloride	955
Lithium chloride	614	Sodium fluoride	992
Cupric chloride	630	Calcium fluoride	1392

Metal salts in water (lethal doses)

The minimum quantities of metal salts in water, which are lethal to living beings, are stated.

Type of metal	Quantity in 1 litre of water	Type of metal	Quantity in 1 litre of water
Copper	0,003 3 g	Manganese	0,3 g
Zinc	0,008 4 g	Magnesium	1,5 g
Iron ((II) oxide)	0,014 0 g	Calcium	2,4 g
Cadmium	0,017 0 g	Potassium	0,1 g
Nickel	0,125 0 g	Sodium	24,17 g
Mercury	0,000 29 g		

Source: SKF-Taschenbuch.

Series

Electrolytic series

If two metals come into contact in the presence of water, acids, bases or salts, electrolytic decomposition of the less noble metal will occur. The less noble metal (has a lower position in the electrolytic series) corrodes and the more noble metal is protected.

Voltage values are stated against a hydrogen electrode.

Substance	Voltage V	Substance	Voltage V
Gold	+1,50	Indium, thallium	−0,34
Chlorine	+1,36	Cadmium	−0,40
Bromine	+1,09	Iron	−0,40
Platinum	+0,87	Chromium	−0,56
Mercury	+0,86	Zinc	−0,76
Silver	+0,80	Aluminium, oxidised	−0,70 ... 1,3
Iodine	+0,58	Manganese	−1,1
Copper	+0,51	Aluminium, bright	−1,45
Arsenic	+0,30	Magnesium	−1,55
Bismuth	+0,23	Beryllium	−1,96
Antimony	+0,20	Calcium	−2,50
Hydrogen	0,00	Sodium	−2,72
Lead	−0,13	Barium	−2,80
Tin	−0,15	Potassium	−2,95
Nickel	−0,22	Lithium	−3,02
Cobalt	−0,29	Fluorine	−4,0

Thermoelectric series

Voltage values for a temperature difference of +100 °C are stated against copper as a reference material (0 °C).

Substance	Voltage mV	Substance	Voltage mV
Chrome nickel	+1,44	Manganin	−0,04
Iron	+1,04	Aluminium	−0,36
Tungsten	+0,05	Platinum	−0,76
Copper	0,00	Nickel	−2,26
Silver	−0,04	Constantan	−4,16

Technically important chemical substances

Commercial names and formulae

The following table shows a selection of technically important, chemical substances with the corresponding commercial names and formulae.

Commercial name	Chemical designation	Formula
Acetone	Acetone (propanone)	$(CH_3)_2 \cdot CO$
Acetylene	Acetylene	C_2H_2
Alum	Potassium aluminium sulphate	$KAl(SO_4)_2 \cdot 12H_2O$
Alumina	Aluminium oxide	Al_2O_3
Ammonia	Ammonia	NH_3
Ammonia solution	Ammonia in aqueous solution	NH_3 in H_2O
Aqua fortis	(see Nitric acid)	
Bauxite	Aluminium hydroxide	$Al_2O_3 \cdot 2H_2O$
Benzene	Benzene	C_6H_6
Blue vitriol	Copper(II) sulphate	$CuSO_4 \cdot 5H_2O$
Borax	Sodium tetraborate	$Na_2B_4O_7 \cdot 10H_2O$
Boric acid	Boric acid	H_3BO_3
Brownstone	Manganese dioxide	MnO_2
Calcined soda	Sodium carbonate, anhydrous	Na_2CO_3
Calcium carbide	Calcium carbide	CaC_2
Calcium chloride	Calcium chloride	$CaCl_2$
Carbon dioxide	Carbon dioxide	CO_2
Carbon monoxide	Carbon monoxide	CO
Carborundum	Silicon carbide	SiC
Caustic lime	Calcium hydroxide	$Ca(OH)_2$
Caustic potash	Potassium hydroxide	KOH
Caustic potash solution	Potassium hydroxide in aqueous solution	KOH
Caustic soda	Sodium hydroxide	$NaOH$
Chalk	Calcium carbonate	$CaCO_3$
Chile saltpetre	Sodium nitrate	$NaNO_3$

Continuation of table, see Page 114.

Source: Mortimer, C. E., Müller, U., Chemie, Stuttgart, Thieme 12. Auflage 2015.
Hollemann, A. F., Wiberg, N., Lehrbuch der Anorganischen Chemie, Berlin, Walter de Gruyter 102. Auflage 2007.

Continuation of table, Commercial names and formulae, from Page 113.

Commercial name	Chemical designation	Formula
China clay	Kaolin	$Al_2O_3 \cdot 2SiO_2 \cdot 2H_2O$
Chlorinated lime	Chlorinated lime	$CaCl(OCl)$
Cinnabar	Mercury sulphide	HgS
Corundum (emery)	Aluminium oxide	Al_2O_3
Dolomite	Calcium magnesium carbonate	$CaMg(CO_3)_2$
Emery	(see Corundum)	
English red	Iron oxide	Fe_2O_3
Epsom salt	Magnesium sulphate	$MgSO_4 \cdot 7H_2O$
Ether	Ethyl ether (diethyl ether)	$(C_2H_3)_2O$
Fixing salt	Sodium thiosulphate	$Na_2S_2O_3 \cdot 5H_2O$
Glauber's salt	Sodium sulphate	Na_2SO_4
Glycerine	Propanetriol	$C_3H_8O_3$
Green vitriol	Iron(II) sulphate	$FeSO_4$
Gypsum	Calcium sulphate	$CaSO_4 \cdot 2H_2O$
Hydrochloric acid	Hydrochloric acid	HCl
Hydrogen sulphide	Hydrogen sulphide	H_2S
Iron oxide	Iron oxide	Fe_2O_3
Lime, burnt	Calcium oxide	CaO
Lime, slaked	(see Caustic lime)	
Lime, with phosphoric acid	Calcium phosphate	$Ca_3(PO_4)_2$
Limestone	Calcium carbonate	$CaCO_3$
Litharge	Lead monoxide	PbO
Lithopone	Mixture of zinc sulphide and barium sulphate	ZnS and $BaSO_4$
Lye	(see Caustic soda)	
Magnesia	Magnesium oxide	MgO
Marble	(see Limestone)	
Mine gas	Methane	CH_4
Minium	(see Red lead)	
Nitric acid	Nitric acid	HNO_3
Oil of vitriol	Concentrated sulphuric acid	H_2SO_4

Continuation of table, see Page 115.

Source: Mortimer, C. E., Müller, U., Chemie, Stuttgart, Thieme 12. Auflage 2015.
Hollemann, A. F., Wiberg, N., Lehrbuch der Anorganischen Chemie, Berlin, Walter de Gruyter 102. Auflage 2007.

Continuation of table, Commercial names and formulae, from Page 114.

Commercial name	Chemical designation	Formula
Petrol	Petrol	(C_nH_{2n+2})
Phosphoric lime	Calcium phosphate	$Ca_3(PO_4)_2$
Potash	Potassium carbonate	K_2CO_3
Prussiate of potash, red	Potassium ferricyanide	$K_3Fe(CN)_6$
Prussiate of potash, yellow	Potassium ferrocyanide	$K_4Fe(CN)_6$
Red lead	Red lead	Pb_3O_4
Rust	Hydrated ferric oxide	$Fe_2O_3 \cdot xH_2O$
Sal ammoniac	Ammonium chloride	NH_4Cl
Silver bromide	Silver bromide	$AgBr$
Soda, crystalline	Sodium carbonate, anhydrous	Na_2CO_3
Soda lye	Sodium hydroxide in aqueous solution	$NaOH$
Sodium bicarbonate	Sodium hydrogen carbonate	$NaHCO_3$
Soldering flux	Aqueous solution of zinc chloride	$ZnCl_2$
Sulphuric acid	Sulphuric acid	H_2SO_4
Sulphurous acid	Dihydrogen sulphite	H_2SO_3
Table salt	Sodium chloride	$NaCl$
Tetra	Tetrachloromethane	CCl_4
Tin chloride	Tin(IV) chloride	$SnCl_4$
Trilene	Trichloroethylene	C_2HCl_3
Vinegar	Acetic acid	$C_2H_4O_2$
Water glass	Sodium silicate or potassium silicate in aqueous solution	Na_4SiO_4 or Na_2SiO_2 K_4SiO_4 or K_2SiO_3
White lead	Basic lead carbonate	$Pb(OH)_2 \cdot 2PbCO_3$
Zinc, with hydrochloric acid	Zinc chloride	$ZnCl_2$

Source: Mortimer, C. E., Müller, U., Chemie, Stuttgart, Thieme 12. Auflage 2015.
Hollemann, A. F., Wiberg, N., Lehrbuch der Anorganischen Chemie, Berlin, Walter de Gruyter 102. Auflage 2007.

Physics

Definitions, values and constants

Atomic building blocks A number of important atomic building blocks of matter are described as follows:

Name	Explanation
Atom	Smallest, chemically uniform particle of an element, consisting of a nucleus and an electron shell; order of magnitude of the diameter 10^{-10} m; the atomic nuclei are smaller by a factor of 10^4 to 10^5; the principal mass of the atom is located in the nucleus (density approximately 10^{14} g/cm^3); all chemical processes (as well as many electrical, magnetic and optical processes) take place in the atomic shell; atoms consist of elementary particles, around 300 are known
Elementary particles	Elementary particles are the smallest known building blocks of matter. The particles included in the standard model of particle physics are: 6 quarks, 6 leptons, the gauge bosons (mediators) and the Higgs boson Photons: Light quantum Leptons: Neutrino, Antineutrino, Electron, Positron Hadrons: – Baryons: – – Nucleons: Proton, Antiproton, Neutron, Antineutron – – Hyperons: Lambda particles, Sigma particles – Mesons: Pion, Kaon
Photons	Quanta of the electromagnetic radiation field
Light quantum	Charge = 0 Mass = 0 Half-life = ∞
Leptons	Particles **extraneous to the nucleus** with **a half-integral spin** (l = 1/2)
Neutrino	Mass, theoretical = 0 (< 0,2 keV) Charge = 0 Half-life = ∞
Electron	Smallest elementary particle with negative charge Charge = –e Rest mass = $9{,}109\,382\,6 \cdot 10^{-31}$ kg Half-life = ∞
Positron	Smallest elementary particle with positive charge Charge = +e Mass = $9{,}109\,382\,6 \cdot 10^{-31}$ kg

Continuation of table, see Page 117.

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Continuation of table, Atomic building blocks, from Page 116.

Name		Explanation
Hadrons		Totality of nuclear-active particles
Baryons		**Nuclear-active** particles with **a half-integral spin** (l = 1/2, 3/2, ...)
	Nucleons	Collective term for protons and neutrons, which are constantly transforming into one another in the atomic nucleus; at the same time, the π-meson field produces the charge transfer
	Proton	Positively charged nuclear building block Charge = +e Rest mass = $1{,}67262171 \cdot 10^{-27}$ kg $\approx$ 1840 electron masses Half-life = ∞
	Neutron	Uncharged nuclear building block Charge = 0 Rest mass = $1{,}674\,927\,28 \cdot 10^{-27}$ kg
	Hyperons	Unstable, superheavy elementary particles Charge = $\pm$ e or 0 Mass = 2 200 to 3 300 electron masses Half-life values around 10^{-10} s
Mesons		**Nuclear-active** particles with **integral spin** (l = 0, 1, 2, ...); example: π- and K-mesons
	Pion, kaon	Unstable, elementary particles, which are either positively or negatively charged, or electrically neutral; Rest masses: π^+, π^-: m = 273,2 electron masses π^0: m = 264,2 electron masses K^+, K^-: m = 966,1 electron masses K^0: m = 974,0 electron masses
	Molecule	Smallest, chemically uniform particle of a compound; composed of atoms; held together by a chemical bond

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Atomic and mass numbers, nuclear and atomic radii

The following table lists the atomic numbers, mass numbers, nuclear radii and atomic radii, complete with corresponding ratios, for a number of selected elements.

Element	Atomic number Z	Mass number M (most common isotope)	Nuclear radius r_K 10^{-15} m	Atomic radius r_A 10^{-10} m	Radii ratio r_A/r_K
Li	3	7	2,3	1,5	65 217
Ne	10	20	3,3	0,5	15 152
Na	11	23	3,4	1,8	52 941
Ar	18	40	4,1	0,9	21 951
K	19	39	4,1	2,2	53 659
Kr	36	84	5,3	1,1	20 755
Rb	37	85	5,3	2,4	45 283
Xe	54	132	6,1	1,3	21 311
Cs	55	133	6,2	2,6	41 935
Rn	86	222	7,3	1,9	26 027

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Values used in nuclear physics and other fields

The following table shows a selection of values used in nuclear physics and other fields.

Name	Unit	Relationship/ formula symbols	Definition
Atomic mass	$u = 1{,}660\,538\,86 \cdot 10^{-27}$ kg Particle number	$u = m_{C12}/M_{C12} = 1/N_A$ $N = \frac{m}{M} N_A$	The relative mass of the nuclide ^{12}C is the unit M = molar mass
Half-life	s, min, d, a	$T_{1/2} = \ln 2/\lambda$ (λ = decay constant)	Time required for half of the original quantity of atoms to decay
Atomic energy	Electron volt $1\ eV = 1{,}602\,176\,53 \cdot 10^{-19}$ J $1\ MeV = 10^6$ eV	W = e U	The energy gained by an electron on travelling through a potential of 1 V is the unit

Continuation of table, see Page 119.

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Continuation of table, Values used in nuclear physics and other fields, from Page 118.

Name	Unit	Relationship/ formula symbols	Definition
Electron mass	1 MeV = 1,782 661 · 10^{-36} kg	$m = \frac{E}{c_0{}^2}$ $m = \frac{m_0}{\sqrt{1-(c/c_0)^2}}$	From the equivalence of energy and mass (according to Einstein)
Absorbed dose	Gray[1)] 1 Gy = 1 J/kg	D = W/m	Energy absorbed per unit mass of irradiated material; 1 rem = 10^{-2} Gy (obsolete)
Activity of a radioactive substance	Becquerel 1 Bq = 1/s	A	A measure of the intensity of radioactive radiation; 1 Ci (Curie) = 3,7 · 10^{10} Bq
Dose equivalent	Sievert[1)] 1 Sv = 1 J/kg	$H = D\ w_R$	A measure of the relative biological effectiveness of the radiation effect; the energy absorbed in the human body as a result of exposure to a specific type of radiation; radiation weighting factor $w_R = 1$ (γ-radiation up to 20, α-radiation, hard neutron radiation)
Energy dose rate	W/kg	$\dot{D}$	–
Ion dose	C/kg	J = Q/m	Charge/mass; 1 R (Roentgen) = 258 · 10^{-6} C/kg (obsolete)
Ion dose rate	A/kg	$\dot{j} = \frac{I}{m} = \frac{Q}{m \cdot t}$	Current/mass or charge/(mass · time)
Effective cross-section	m^2	σ	A measure of the yield of nuclear reactions; imaginary cross-section through the irradiated atoms
Amount of substance	Mole	$n = N/N_A = m/M$	Amount of substance = particle number/Avogadro's constant

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

1) The units Gray (Gy) and Sievert (Sv) are both equivalent to the unit J/kg.
Gy is used to express the pure (physical) absorbed dose of radiation. Sv is used if a factor has been included to take account of the biological effectiveness of the absorbed dose.
Until 1985, the unit rem was used to measure dose equivalent; today, the unit Sv is used.

Physical constants A selection of important physical constants is described below.

Name	Value	Explanation
Gravitational constant	$G = 6{,}674\,28 \cdot 10^{-11}\ m^3/(kg \cdot s^2)$	Force in N which attracts 2 bodies weighing 1 kg each and set 1 m apart
Standard gravitational acceleration	$g_n = 9{,}806\,65\ m/s^2$	Standard value defined by the 3rd General Conference on Measures and Weights in 1901
Molar gas constant	$R = 8{,}314\,472\ J/(mol \cdot K)$	The work that must be done to heat 1 mol of an ideal gas by 1 K under constant pressure; same value for all sufficiently ideal gases
Standard molar volume	$V_m = 22{,}413\,996 \cdot 10^{-3}\ m^3/mol$	Volume occupied by 1 mol of an ideal gas under standard conditions
Avogadro's constant	$N_A = 6{,}022\,1415 \cdot 10^{23}\ mol^{-1}$	Number of atoms or molecules in 1 mol of a substance
Loschmidt's constant	$N_L = 2{,}686\,8 \cdot 10^{25}\ m^{-3}$	Number of atoms or molecules in 1 m^3 of a gas under standard conditions (0 °C and 1013,25 hPa)
Boltzmann's constant	$k = R/N_A$ $= 1{,}380\,650\,5 \cdot 10^{-23}\ J/K$	Average energy increase of a molecule or atom when heated by 1 K
Faraday's constant	$F = N_A \cdot e$ $= 9{,}648\,533\,83 \cdot 10^4\ C/mol$	The charge quantity transported by 1 mol of singly charged ions
Elementary charge	$e = F/N_A$ $= 1{,}602\,176\,53 \cdot 10^{-19}\ C$	The smallest possible charge (charge of an electron)
Permittivity of free space (electric constant)	$\epsilon_0 = \dfrac{1}{\mu_0 \cdot c^2}$ $= 8{,}854\,2 \cdot 10^{-12}\ F/m$	Proportionality factor between the charge density and the electric field strength
Permeability of free space (magnetic constant)	$\mu_0 = 1{,}256\,6 \cdot 10^{-6}\ H/m$ $= 4 \cdot \pi \cdot 10^{-7}\ H/m$	Proportionality factor between the induction and the magnetic field strength
Speed of light in a vacuum	$c_0 = 2{,}997\,9 \cdot 10^8\ m/s$	Propagation rate of electromagnetic waves
Planck's constant (action quantum)	$h = 6{,}626 \cdot 10^{-34}\ J \cdot s$	Combines the energy and frequency of a light quantum as a proportionality factor
Characteristic impedance of a vacuum (impedance of free space)	$Z_0 = 376{,}730\ \Omega$	Propagation resistance for electromagnetic waves in a vacuum
Stefan-Boltzmann's constant	$\sigma = 5{,}670\,40 \cdot 10^{-8}\ W/(m^2 \cdot K^4)$	Combines the radiation energy and temperature of a radiating body

Continuation of table, see Page 121.

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

Continuation of table, Physical constants, from Page 120.

Name	Value	Explanation
Planck's radiation constants	$c_1 = 3{,}741\,771\,38 \cdot 10^{-16}\ \text{W} \cdot \text{m}^2$ $c_2 = 1{,}438\,775\,2 \cdot 10^{-2}\ \text{m} \cdot \text{K}$	Constants of Planck's radiation law in the original wavelength-dependent formulation
Wien's constant	$K = 2{,}897\,8 \cdot 10^{-3}\ \text{m} \cdot \text{K}$	Combines the wavelength of the radiation maximum with the absolute temperature of a radiating body
Rydberg's constant	$R_\infty = 1{,}097\,37 \cdot 10^{7}\ \text{m}^{-1}$	Fundamental, nuclear physical constant occurring in standard formulae for spectral lines
Rest mass of an electron	$m_e = 9{,}109\,382\,6 \cdot 10^{-31}\ \text{kg}$	Mass of a stationary electron
Electron radius	$r_e = 2{,}817\,9403\,25 \cdot 10^{-15}\ \text{m}$	Radius of an electron (spherical formation)
Bohr's radius	$r_1 = 5{,}291\,772\,108 \cdot 10^{-11}\ \text{m}$	Radius of the innermost electron path in Bohr's atom model
Atomic unit of mass	$u = 1{,}660\,538\,86 \cdot 10^{-27}\ \text{kg}$	Unified atomic mass unit (one twelfth of the mass of an atom of the nuclide ^{12}C)
Unit of mass	$1\ \text{UM} = 931{,}494\ \text{MeV}/c^2$ [1)]	Used for energy conversions
Solar constant	$S = 1\,365\ \text{J/m}^2\text{s} = 1\,365\ \text{W/m}^2$	Radiation energy of the Sun which arrives vertically at the upper limits of the Earth's atmosphere (on the ground the value is reduced to 340 W/m^2)

Source: Tipler, Paul Allen, Physik für Wissenschaftler und Ingenieure, Heidelberg Springer Spektrum, 7. Auflage 2015.
Tipler, Paul Allen, Moderne Physik, München, Oldenbourg, 2. Auflage 2010.
Demtröder, W., Experimentalphysik 4 – Kern-, Teilchen- und Astrophysik, Berlin, Springer, 3. Auflage 2010.

[1)] Electron volt (eV) and mega electron volt (MeV) are measures of energy in nuclear physics. 1 eV is the energy gained by an electron when it is accelerated in an electrical field of 1 volt ($1\ \text{eV} = 1{,}602\,176\,53 \cdot 10^{-19}\ \text{J}$).

Electromagnetic radiation

The characteristics of electromagnetic radiation are:

- propagation at the speed of light
- wave nature
- no deflection by electric or magnetic fields
- wavelength $\lambda = c/f = c \cdot T$
  c = speed of light = 299 792 km/s
  f = frequency in Hz
  T = period of oscillation in s.

Other definitions and values used in relation to electromagnetic radiation are described as follows:

Radiation type Wavelength λ	Occurs in the event of energy changes in	Is generated by	Is absorbed by (examples)
Cosmic rays			
0,000 2 ... 0,02 pm	Nucleons (nuclear building blocks)	High-energy nuclear reactions	Around 10 cm of lead
Gamma rays			
0,5 ... 27 pm	Atomic nuclei	Atomic nuclear reactions and radioactive decay	Around 1 cm of lead
X-radiation			
Hard 5,7 ... 80 pm (0,057 ... 0,8 Å)	Internal electron shells	High-vacuum and gas-discharge tubes at high operating voltages	Around 3 ... 0,04 cm of aluminium
Soft 0,08 ... 2 nm (0,8 ... 20 Å)			Around 400 ... 1 μm of aluminium, bone, glass
Ultrasoft 2 ... 37,5 nm (20 ... 375 Å)			Less than 1 μm of aluminium, air
Light rays			
Ultraviolet (short-wave) 0,014 ... 0.18 μm	External electron shells	Spark, arc, glow discharge in vacuum, quartz lamp etc.	Air
Ultraviolet (long-wave) 0,18 ... 0.36 μm			Quartz ($\lambda < 0,15$ μm) Glass ($\lambda < 0,31$ μm)
Violet 0,36 ... 0.42 μm Blue 0,42 ... 0.49 μm Green 0,49 ... 0.53 μm Yellow 0,53 ... 0.65 μm Red 0,65 ... 0,81 μm		Sun, glowing substances etc.	Opaque substances
Infrared (heat rays) 0,81 ... 400 μm		Heated bodies[1)]	Glass

Continuation of table, see Page 123.

1) The following applies to "black body" radiation:
average wavelength λ (in μm) = 2 898/(absolute temperature in K).
Example: at +20 °C (= 293 K) λ = 9,89 μm,
i.e. at +20 °C the maximum heat radiation intensity is at λ = 9,89 μm.
Source: Krist, Handbuch für Techniker und Ingenieure.

Continuation of table, Electromagnetic radiation, from Page 122.

Radiation type Wavelength λ		Occurs in the event of energy changes in	Is generated by	Is absorbed by (examples)
Hertzian waves				
0,01 ... 30 cm		Atoms or molecules	Spark transmitters, velocity-modulated tubes	Metals
Broadcasting waves				
Ultrashort	0,3 ... 10 m	Resonant circuits with capacitance and inductance	Transistor senders	Metals
Short	10 ... 100 m		Propagation of these waves is no longer ray-like and this is why there are no “wave shadows” in valleys and behind mountains; waves are diffracted on the Heaviside layer (ionosphere) and deflected back to Earth; with increasing wavelength, the space wave steps behind the ground wave	
Medium	100 ... 600 m			
Long	600 ... 3000 m			
Telegraphic waves	3 ... 30 km			

Temperature points

General, important temperature points are:

Triple point of water	+0,01 °C
Boiling point of water	+100,00 °C
Boiling point of oxygen	–182,97 °C
Boiling point of nitrogen	–196,00 °C
Boiling point of air	–191,0 °C
Boiling point of sulphur	+444,6 °C
Freezing point of silver	+960,8 °C
Freezing point of gold	+1063,0 °C

Thermal expansion of solids and gases

Almost all solids expand when their temperature is increased and shrink when the temperature decreases. Water does not follow this rule. It exhibits its greatest density at +4 °C and expands irrespective of whether it rises above or falls below this temperature.

Homogeneous solids expand uniformly in all directions (volume expansion). In many cases, we are only interested in expansion in a specific direction (superficial or linear expansion). If a solid’s linear expansion or volume change is impeded in the event of a temperature change, stresses occur within the solid.

Linear thermal expansion coefficient

In the case of solids, the linear thermal expansion coefficient (coefficient of linear expansion) is the relative change in length per degree of temperature increase.

Thus, the change in length Δl of a solid is described in terms of:

Equation 1

$$\Delta l = l_0 \cdot \alpha \cdot \Delta T$$

Δl = length change
l_0 = initial length
α = coefficient of linear thermal expansion
ΔT = temperature increase

A temperature increase ΔT produces the following in the solid:

Equation 2

$$\epsilon_{\Delta T} = \frac{\Delta l}{l_0} = \alpha \cdot \Delta T$$ in the case of unimpeded expansion

$$\sigma_{\Delta T} = E \cdot \epsilon_{\Delta T} = E \cdot \alpha \cdot \Delta T$$ in the case of impeded expansion

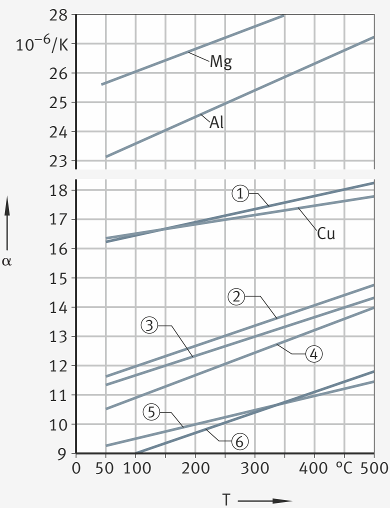
The following diagram shows the influence of temperature on the coefficient of linear expansion α.

Figure 1

Influence of temperature on α in the case of steels and non-ferrous metals

T = temperature
α = coefficient of linear thermal expansion
Mg = magnesium
Al = aluminium
Cu = copper

① 18% Cr + 9% Ni steel
② 0,4% Mo steel
③ Cr-Mo steel
④ Unalloyed steel: (0,2 – 0,6% C)
⑤ 13% Cr steel
⑥ Cast iron



The following table lists a number of exemplary values for coefficients of linear thermal expansion α at +20 °C.

Material	α 10^{-6}/K	Material	α 10^{-6}/K	Material	α 10^{-6}/K
Cast iron	9 ... 10	Copper	16 ... 17	Thermoplastics	70 ... 250
Unalloyed steel	11 ... 12	Aluminium	23 ... 24	Brickwork	5 ... 8
Cr-Mo steel	12 ... 13	Magnesium	25,5	Rubble	3
Cr-Ni steel	16 ... 17	Thermosets	10 ... 80	Glass	8 ... 10

Coefficient of cubical expansion

The coefficient of cubical expansion (volume expansion coefficient) of a solid, liquid or gaseous body is the relative volume change per degree of temperature increase.

Thus, the change in volume ΔV is described in terms of:

Equation 3

$$\Delta V = V_0 \cdot \beta \cdot \Delta T$$

ΔV = volume change
V_0 = initial volume
β = coefficient of thermal volume expansion
ΔT = temperature increase

In the case of homogeneous, solid bodies:

Equation 4

$$\beta = 3 \cdot \alpha$$

In the case of gases, the coefficient of cubical expansion is the same value for all gases and temperatures at constant pressure and relative to the volume V_0 at 0 °C

Equation 5

$$\beta = \frac{1}{V_0} \cdot \frac{\Delta V}{\Delta T} = \frac{1}{273{,}15} \frac{1}{K}$$

Coefficient of superficial thermal expansion

Superficial thermal expansion can be described by the thermal volume expansion coefficient:

Equation 6

$$\Delta A = A_0 \cdot \frac{2}{3} \cdot \beta \cdot \Delta T$$

ΔA = surface change
A_0 = initial surface
β = coefficient of thermal volume expansion
ΔT = temperature increase

Astronomical and terrestrial definitions and values

Astronomical units The following table shows a selection of important astronomical units.

Name	Value	Explanation
Speed of light in a vacuum	$c_0 = 2{,}997\,9 \cdot 10^8$ m/s	Propagation rate of electromagnetic waves
Light-year	$L_y = 9{,}460\,73 \cdot 10^{15}$ m	Distance covered by electromagnetic waves in space in 1 year
Sidereal year (stellar year)	S_y = 365,256 5 mean solar days = 365 d 6 h 9 min 9,54 s	The sidereal year is based on the position of the Sun relative to the fixed stars[1)]
Tropical year (solar year)	T_y = 365,242 2 mean solar days = 365 d 5 h 48 min 46,98 s	The mean vernal equinox is the reference point
Sidereal month (stellar month)	S_m = 27,321 66 d mean solar time = 27 d 7 h 43 min 11,5 s	–
Tropical month (solar month)	T_m = 27,321 58 d mean solar time = 27 d 7 h 43 min 4,7 s	–
Synodic month (lunar month)	S_{ym} = 29,530 59 d mean solar time = 29 d 12 h 44 min 2,9 s	Time between two identical moon phases: new moon to new moon
Orbital period of the Moon around the Earth	t_M = 27,321 66 d = 27,321 58 d	Sidereal year Tropical year
Sidereal day	d_{Sy} = 0,997 269 6 mean solar days = 23 h 56 min 4,091 s	–
Mean solar day	d_{Ty} = 1,002 737 9 sidereal days	–
Day	d = 24 h = 1 440 min = 86 400 s	The day is 3 min 56 s longer than the sidereal day
Astronomical unit	$AU = 1{,}496 \cdot 10^{11}$ m	Mean distance between the Earth and the Sun

[1)] Time interval between two successive passages of the Sun through the same point along the apparent trajectory of the Sun (ecliptic).
The point of the ecliptic is measured in relation to a fixed star.
Average obliquity of the ecliptic is currently ≈ 23° 27′ 15″.

Our Solar System

A number of interesting values relating to our Solar System are:

Solar planets	Equator diameter km	Mass (Earth = 1[1])	Density kg/m^3	Sidereal rotation period	Distance from the Sun 10^6 km	Sidereal orbital period years
Sun	1 392 000	333 000	1 410	25,23 d	–	–
Earth	12 757	1,000	5 517	23,94 h	149,6	1,00
Moon[2]	3 476	0,012	3 340	27 d 7,1 h	0,384 4 (from the Earth)	
Mercury	4 840	0,056	5 620	88 d	58	0,24
Venus	12 228	0,815	5 090	243 d	108	0,62
Mars	6 770	0,108	3 970	24,62 h	228	1,88
Jupiter	140 720	317,8	1 300	9,84 h	778	11,86
Saturn	116 820	95,11	680	10,1 h	1 428	29,46
Uranus	47 100	14,51	1 580	10,8 h	2 872	84,02
Neptune	44 600	17,21	2 220	15,8 h	4 498	164,79
Pluto[3]	2 300	0,002 2	–	6,4 d	5 910	249,17

1) Earth's mass = $5{,}977 \cdot 10^{24}$ kg, volume of the globe = 1 083 319,8 million km^3, average density of the Earth = 5 517 kg/m^3, circumference of the Earth's trajectory = 939 120 000 km.

2) Satellite of the Earth.

3) Reclassified as a dwarf planet in 2006 on the orders of the International Astronomical Union.

The Earth – fundamental figures

A number of important, fundamental figures relating to our Earth are:

Earth's surface	510,1 million km^2
Of which total surface area of land	147,9 million km^2 (29%)
Of which total surface area of water	362,2 million km^2 (71%)
Length of the Equator	40 076,592 km
Equatorial radius a	6 378,388 km
Length of the meridian	40 009,153 km
Polar radius b	6 356,912 km
Length of the tropic	36 778,000 km
Length of a polar circle	15 996,280 km
Flattening (a – b)/a	1 : 297
Length of a degree of longitude (Arc distance between two meridians which are 1° apart)	
Point on the Equator	111,324 km
At 50° latitude	71,669 km
Length of a degree of latitude (Arc distance between two parallel circles which are 1° apart)	
At 89°-90° latitude	111,700 km
At 45°-46° latitude	111,135 km
At 0°-1° latitude	110,575 km

Legend

a m Equatorial radius	b m Polar radius.

Interesting speeds The following table shows a number of interesting speeds (rounded values).

Name	Speed	
	m/s	km/h
Gulf Stream	1,1	4
Wind force 6 (strong breeze)	11–14	39–49
Wind force 12 (hurricane)	>32	>118
Sound in air (at +20 °C)	340	1 200
Point on the Equator	464	1 670
Earthquake waves	3 600	13 000
Orbital speed of a satellite	≈ 7 800	28 400
Speed required to leave the Earth's gravitational field	≈ 11 200	40 300
Speed required to leave the Solar System	15 800	57 000
Earth on its orbital path around the Sun	29 800	107 000
Lightning	50 000 000	180 000 000
Cathode rays (electrons, 50 kV)	100 000 000	360 000 000
Light in a vacuum	299 792 458	1 079 252 849

Dimensionless characteristic values

Dimensionless characteristic values The definitions for similarity characteristics are described in the following table.

Type of similarity	Scale factors (invariant)	Name of characteristic	Definition	Similarity of physical subject matter
Geometric Length	$\varphi_L = \dfrac{L_1}{L_0}$	–	–	All lengths are similar if the scale is the same (pantograph)
Kinematic Length Time	φ_L, φ_t	Mach	$Ma = \dfrac{v}{c_s}$	Ratio of inertia forces to compression forces
Static Length Force	$\varphi_F = \dfrac{\rho_1}{\rho_0} \cdot \varphi_L{}^3$	–	–	Similarity of weights (constant gravitational acceleration)
	φ_L, φ_F	Hooke	$H_0 = \dfrac{F}{E \cdot L^2}$	Sole effect of elastic forces (similarity of expansions)

Continuation of table, see Page 129.

Continuation of table, Dimensionless characteristic values, from Page 128.

Type of similarity	Scale factors (invariant)	Name of characteristic	Definition	Similarity of physical subject matter
Dynamic Length Time Force	$\varphi_L, \varphi_t, \varphi_F$	Newton	$Ne = \frac{F}{\rho \cdot v^2 \cdot L^2}$	Ratio of resistance force to flow force
		Cauchy	$Ca = \frac{v}{\sqrt{E/\rho}}$	Ratio of inertia forces to elastic forces
		Froude	$Fr = \frac{v^2}{g \cdot L}$	Ratio of inertia forces to gravitational forces
		Reynolds	$Re = \frac{v \cdot L}{\nu}$	Ratio of inertia forces to viscous forces
		Weber	$We = \frac{\rho \cdot v^2 \cdot L}{\sigma}$	Ratio of inertia force to surface force
		Euler	$Eu = \frac{\Delta p}{\rho \cdot v^2}$	Ratio of compressive forces to inertia forces
Thermal Length Time Temperature	$\varphi_L, \varphi_t, \varphi_q$	Péclet	$Pe = v \cdot L \left(\frac{\rho \cdot c}{\lambda}\right)$	Ratio of transported to conducted heat quantity
		Prandtl	$Pr = \frac{P_e}{R_e} = \nu \cdot \left(\frac{\rho \cdot c}{\lambda}\right)$	Ratio between kinematic viscosity and thermal diffusivity
		Nußelt	$Nu = \frac{\alpha \cdot L}{\lambda}$	Heat transfer ratio with moving and stagnant layer
		Fourier	$Fo = \left(\frac{\lambda}{\rho \cdot c}\right) \cdot \frac{t}{L^2}$	Ratio of conducted to stored heat

Legend

L	m	g	m/s^2
Length		Gravitational acceleration	
t	s	α	$W/(m^2K)$
Time		Heat transfer coefficient	
F	N	λ	$W/(m \cdot K)$
Force		Thermal conductivity	
ρ	kg/m^3	ν	m^2/s
Density		Kinematic viscosity	
v	m/s	c	$J/(kg \cdot K)$
Velocity		Specific heat capacity.	
E	N/m^2		
Modulus of elasticity			

Mechanics

Definitions

Mechanics Mechanics is a branch of physics which describes the motion events occurring in nature and engineering. It deals with the plane and spatial motion of bodies and the effect of forces.

Dynamics Dynamics deals with the forces (general: interactions) which cause bodies to move. It describes the association of these motions with mass and acting forces.

The branches of dynamics are:

- **Kinematics**
  Kinematics describes the occurrence of motions in space and time, without consideration of forces. Important terms used in dynamics are the motion values displacement, velocity and acceleration
- **Kinetics**
  Kinetics describes the change in the above-mentioned motion values which occurs under the influence of forces in space.

Newton's fundamental law All dynamic motion events are based on Newton's fundamental physical law:

- force = mass · acceleration
  (translatory motion)
- moment = moment of inertia · angular acceleration
  (rotatory motion).

It is the basis for all accelerated motions of bodies.

Values and units

Values and units The following table shows the names, values and units used in mechanics.

Designation	Value	Unit	Explanation
Acceleration	a	m/s^2	$a = dv/dt\,;\ (\ddot{x},\ \ddot{y},\ \ddot{z})$
Angle, angle of rotation	φ	rad[1)]	1 rad = 57°17′45″
Angular acceleration	$\alpha,\ \dot{\omega},\ \ddot{\varphi}$	rad/s^2	$\alpha = \dot{\omega} = d\omega / dt = \ddot{\varphi}$
Angular frequency	ω	1/s	$f = \omega/(2 \cdot \pi)$
Angular impulse	H	Nms	$H = \int M \cdot dt$

Continuation of table, see Page 131.

Continuation of table, Values and units, from Page 130.

Designation	Value	Unit	Explanation
Angular velocity	ω, $\dot{\varphi}$	rad/s	$\omega = d\varphi/dt = \dot{\varphi}$
Area	A	m^2	$1\ m^2 = 10^6\ mm^2$
Cartesian coordinates	x, y, z	m	Right-handed trihedron
Coefficient of friction	μ	1	$\mu = F_f/F_N$ (Coulomb)
Density, mass density	ρ	kg/m^3	$\rho = m/V$
Dynamic viscosity	η	$N \cdot s/m^2$	$\tau = \eta \cdot dv/dh$ (Newton)
Efficiency	η	1	$\eta = P_{eff}/P_{the}$
Force	F	N	$F = m \cdot a$ (Newton)
Frequency	f	1/s	–
Friction energy, energy loss	W_f	Nm	$W_f = \int F_f \cdot ds$
Gravitational acceleration	g	m/s^2	$g = 9{,}806\,65\ m/s^2$
Impulse	I	Ns	$I = \int F \cdot dt$
Kinematic viscosity	ν	m^2/s	$\nu = \eta/\rho$
Kinetic energy	E_k	J = Nm	$E_k = (m/2) \cdot v^2$
Linear momentum	p	$kg \cdot m/s$	$p = \int v \cdot dm$; $p = m \cdot v$
Mass	m	kg	SI base unit
Moment of force, torque	M	Nm	M = force · lever arm
Moment of inertia	J	$kg\ m^2$	Second moment of mass
Moment of momentum, angular momentum	L	$kg \cdot m^2/s$	$L = \int \omega \cdot dJ$; $L = J \cdot \omega$
Path length, curve length	s	m	SI base unit
Potential energy	E_p	J = Nm	$E_p = m \cdot g \cdot h$ (position)
Power	P	W = Nm/s	$P = W/t$
Pressure	p	N/m^2	$p = F/A$
Rotational speed	n	s^{-1}	$n = \omega/(2\pi)$
Time, timespan, duration	t	s	SI base unit
Velocity	v	m/s	$v = ds/dt$; $(\dot{x}, \dot{y}, \dot{z})$
Volume	V	m^3	$1\ m^3 = 10^9\ mm^3$
Weight	F_G	N	$F_G = m \cdot g$
Work	W	Nm = J	$W = \int F \cdot ds$; $W = \int M \cdot d\varphi$

1) The unit can be replaced by the number "1".

Motion equations

Fundamental law of accelerated motion

Newton's fundamental law governing the accelerated motion of a body is: in translation of the centre of gravity "S":

Equation 1

$$\sum \vec{F}_S = m \cdot \vec{a}$$

in rotation about the centre of gravity "S":

Equation 2

$$\sum \vec{M}_S = J_S \cdot \dot{\vec{\omega}}$$

in rotation about the instantaneous centre of rotation "MP":

Equation 3

$$\sum \vec{M}_{MP} = J_{MP} \cdot \dot{\vec{\omega}}$$

Dynamic equilibrium according to d'Alembert

If the inertia forces $(m \cdot a)$ and the moments resulting from the inertia effect $(J \cdot \omega)$ are interpreted as a kinetic reaction (externally imposed forces or moments) during the accelerated motion of a body, this gives:

Equation 4

$$\sum \vec{F}_S + \left(-m\vec{a}\right) = 0 \qquad \sum \vec{F}_S + \vec{F}_K = 0$$

or:

Equation 5

$$\sum \vec{M}_S + \left(J_S \cdot \dot{\vec{\omega}}\right) = 0 \qquad \sum \vec{M}_S + \vec{M}_K = 0$$

Reduction to static equilibrium problem

The kinetic reaction is always opposing the (positively defined) direction of acceleration. Thus, the kinetic problem can be reduced to a static equilibrium problem.

The fundamental equations of statics will then apply:

Equation 6

$$\sum \overrightarrow{F^*} = 0 \qquad \sum \overrightarrow{M^*} = 0$$

The force sums $\sum \overrightarrow{F^*}$ and moment sums $\sum \overrightarrow{M^*}$ contain the respective kinetic reactions.

Solving motion equations The following guidelines apply to the formulation of equilibrium relationships and solving of motion equations.

Defining co-ordinates Define the co-ordinates at the centre of gravity of the body and in the expected direction of motion:

- for the translational motion of the centre of gravity
- for the rotational motion about the centre of gravity.

Plotting kinetic reactions Plot kinetic reactions against the positively defined directions of acceleration:

- inertia forces
- moments resulting from the inertia effect (where present).

Plotting the remaining forces Plot all other impressed forces (external forces, weight forces) and reaction forces (friction forces and support reactions).

Generating a force equilibrium During plane motion, generate a force equilibrium for all forces in the direction of motion, including the kinetic reactions:

Equation 7

$$\sum F_x^* = 0 \qquad \sum F_y^* = 0$$

Generating a moment equilibrium Generate a moment equilibrium about the centre of gravity, provided the body performs a rotational motion about its centre of gravity in addition to the translational motion, taking account of the sign in each case:

Equation 8

$$\sum M_{z(S)} = 0$$

Stating geometric relationships State the geometric relationships between the translational and rotational motions.

Solving equations Solve the equation in accordance with the required acceleration:

- acceleration = const. or f(t):
  capable of elementary integration, taking into account the boundary conditions
- acceleration = f(dist.): can be integrated once

Equation 9

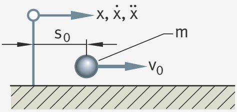
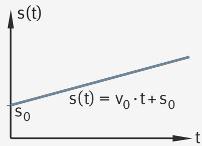
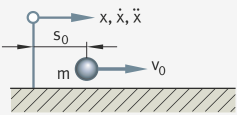
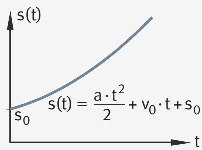
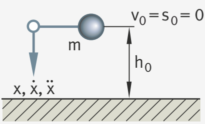
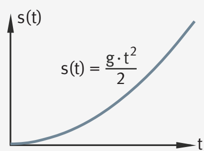
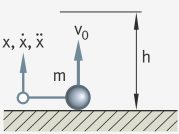
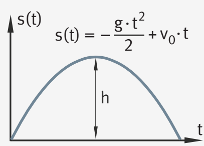
$$\ddot{x} = \frac{\dot{x} \cdot d\dot{x}}{dx} \qquad \ddot{\varphi} = \frac{\dot{\varphi} \cdot d\dot{\varphi}}{d\varphi}$$

- acceleration = f(dist.): an oscillation may be present.

Simple motion events

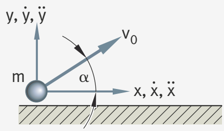
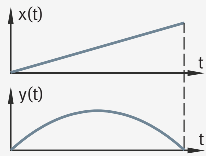
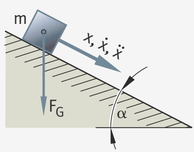
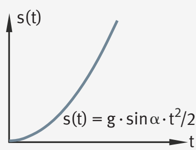
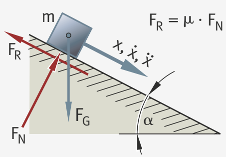
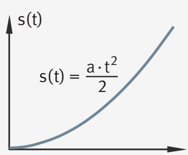
Motions, distance-time diagrams

The following table shows motion events, relationships and the corresponding distance-time diagrams.

Motion event	Relationship	Distance-time diagram
Uniform, rectilinear motion		
Acceleration $\ddot{x} = 0$ Initial conditions s_0, v_0 	$\ddot{x} = 0$ $\dot{x} = v_0 = \text{const.}$ $x = s(t) = v_0 \cdot t + s_0$	
Accelerated, rectilinear motion		
Acceleration $\ddot{x} = a$ Initial conditions s_0, v_0 	$\ddot{x} = a = \text{const.}$ $\dot{x} = v(t) = a \cdot t + v_0$ $x = s(t) = \frac{a \cdot t^2}{2} + v_0 \cdot t + s_0$	
Free fall under acceleration due to gravity		
Acceleration $\ddot{x} = g$ Initial condition, height h_0 	$\ddot{x} = g$ $\dot{x} = v(t) = g \cdot t$ $x = s(t) = \frac{g \cdot t^2}{2}$ $t(h_0) = \sqrt{2 \cdot h_0 / g}$ (time of fall) $v(h_0) = \sqrt{2 \cdot g \cdot h_0}$ (velocity)	
Body thrown vertically upward		
Acceleration $\ddot{x} = -g$ Initial condition v_0, $s_0 = 0$ 	$\ddot{x} = -g$ $\dot{x} = v(t) = -g \cdot t + v_0$ $x = s(t) = -\frac{g \cdot t^2}{2} + v_0 \cdot t$ $h = v_0{}^2 / (2 \cdot g)$ (height reached) $t(h_0) = v_0 / g$ (time of upward travel)	

Continuation of table, see Page 135.

Continuation of table, Motions, distance-time diagrams, from Page 134.

Motion event	Relationship	Distance-time diagram
Body thrown obliquely at angle α		
	$\ddot{x} = 0$ $\dot{x} = v_0 \cdot \cos\alpha$ $x = v_0 \cdot \cos\alpha \cdot t$ $\ddot{y} = -g$ $\dot{y} = -g \cdot t + v_0 \cdot \sin\alpha$ $y = -g \cdot t^2/2 + v_0 \cdot \sin\alpha \cdot t$	
Motion on an inclined plane, without friction (μ = 0)		
	$\ddot{x} = g \cdot \sin\alpha = a$ $\dot{x} = v(t) = g \cdot \sin\alpha \cdot t$ $x = s(t) = g \cdot \sin\alpha \cdot t^2/2$ $t = 0$ $s_0 = 0$ $v_0 = 0$	
Motion on an inclined plane, with friction, friction coefficient μ.		
	$\ddot{x} = g \cdot (\sin\alpha - \mu \cdot \cos\alpha) = a$ $\dot{x} = v(t) = a \cdot t$ $x = s(t) = a \cdot t^2/2$ $t = 0$ $s_0 = 0$ $v_0 = 0$	

Forces in kinetics (selection)

Forces in kinetics The following tables describe a selection of the most important forces in kinetics.

General forces The following general and impressed forces are described in kinetics:

Force	Value	Explanation
Force (general)	F	Muscle power, wind power, driving forces of machines etc.
Imposed forces	Imposed forces are primary forces that act on a body from the outside. They can promote or obstruct motion. As a rule, their magnitude, direction and line of action are known or given.	

Conservative forces In the case of conservative forces, the work (work = force · distance) is independent of the distance curve.

Force	Value	Explanation
Mass attraction force	$F_M = G \cdot \frac{m_1 \cdot m_2}{r^2}$	The mass attraction force between two masses is proportional to the product of the masses and inversely proportional to the square of the distance between their centres of gravity. The proportionality factor is the universal gravitation constant, $G = 6{,}674\,2 \cdot 10^{-11}\ m^3/(kg \cdot s^2)$.
Gravity, weight	$F_G = m \cdot g$	The force of gravity (weight) acts on all bodies in the Earth's proximity. It is based on the mass attraction law and on the centrifugal force (additional force) occurring on the Earth's surface as a result of the Earth's rotation.
Gravity at a greater distance h from the Earth's surface	$F_G = m \cdot g_0 \left(\frac{R_0}{R_0 + h}\right)^2$	At a greater distance from the centre of the Earth ($h > R_0$), gravity decreases with the square of the distance. $g_0 = 9{,}81\ m/s^2$; $R_0 = 6\,370 \cdot 10^3\ m$.
Gravity in the interior of the Earth	$F_G = m \cdot g_0 \cdot \frac{r}{R_0}$	Presuming that the density of the Earth is constant, gravity decreases towards the centre of the Earth, in linear fashion, with the distance from the centre of the Earth.
Spring force	$F_F = c \cdot w$	The spring force results from the product of the spring constant c and spring deflection w. It acts against the positive deflection of the spring.

Dissipative forces In the case of dissipative (non-conservative) forces, the work (work = force · distance) is dependent on the distance curve.

Force		Value	Explanation
Sliding friction force		$F_R = \mu \cdot F_N$	The sliding friction force between two contact surfaces (Coulomb friction) is proportional to the acting normal force. The proportionality factor is the sliding friction coefficient μ. The sliding friction force acts against the relative velocity of the surfaces in contact.
Damping force	according to Stokes	$F_D = b \cdot v$	The resistance according to Stokes is valid for low velocities and is proportional to the velocit (damping constant $b = F \cdot t/s$). The resistance according to Newton is valid for high velocities and is proportional to the square of the velocity (constant factor k). The damping force counteracts the positive velocity direction.
	according to Newton	$F_N = k \cdot v^2$	

Constraining or guiding forces Constraining or guiding forces are caused by the restriction of the freedom of movement of a body or a system of bodies.
The influence of friction-free guides or guiding curves on the body is taken into account by external guiding forces that are perpendicular to the guiding curves.

Force		Value	Explanation
Centripetal force	Curved path	$F_{CP} = m \cdot \frac{v^2}{r}$	When a body moves along a curved path, it is subject to acceleration, known as centripetal acceleration, which is oriented to the centre point of the curvature. The force generating it is referred to as the centripetal force.
	Circular path	$F_{CP} = m \cdot r \cdot \omega^2$	

Kinetic reactions Kinetic reactions represent the action of a moving body as the result of accelerating or decelerating external forces. They always counteract the positive accelerations. If the kinetic reaction of the body is also interpreted as an external force acting on the body (fictitious force), the kinetic problem can be reduced to a static one and can be handled with the aid of the equilibrium conditions (d'Alembert's principle).

Force		Value	Explanation
Tangential to the trajectory curve		$F_{Kt} = -m \cdot \ddot{s}_t$	The kinetic reaction of a body during accelerated motion counteracts the positive direction of the acceleration that is produced by impressed forces and guiding forces. In the case of guided motion on a trajectory curve, it is advisable to split this into components that are tangential and perpendicular to the trajectory curve. Centrifugal force is one of the kinetic reactions that act perpendicular to the trajectory curve.
Perpendicular to the trajectory curve		$F_{Kn} = -m \cdot \ddot{s}_n$	
Centrifugal force	Curved path	$F_{Kn} = -m \cdot \frac{v^2}{r}$	
	Circular path	$F_{Kn} = -m \cdot r \cdot \omega^2$	

Additional forces in the accelerated reference system

If motion events take place in an accelerated reference system, it is advisable to analyse these as relative motions in relation to the system from the point of view of an observer who is also accelerated.

In the accelerated reference system, all bodies are subject to additional inertia forces (apparent forces) which, from the point of view of the observer who is also accelerated, can be interpreted as external forces acting on the body.

Force		Value	Explanation
Inertia force in the translatory accelerated reference system		$F_{Sys} = -m \cdot a_{Sys}$	In the translatory accelerated reference system (system acceleration a_{Sys}), all bodies are subjected to an inertia force. It acts against the positive direction of system acceleration.
Inertia force in the rotating reference system	Centrifugal force	$F_Z = m \cdot r \cdot \Omega^2$	In the rotating reference system (angular velocity Ω), all bodies are subjected to the centrifugal force. To the observer who is also rotating with the body, this appears like a field force corresponding to gravity (weight) that points away from the rotation centre, and which is intrinsic to the system. For the observer, it has the character of a conservative imposed force, which lends a centrifugal acceleration to all free bodies in the same direction.
	Coriolis force	$F_C = 2 \cdot m \cdot v_{rel} \cdot \Omega \cdot \sin\varphi$	The Coriolis force occurs in relative motions in the rotating reference system. It is perpendicular to the plane spanned by the vectors v_{rel} and Ω and points in a right-handed screw direction if the vector v_{rel} is moved over the shortest distance in the direction of Ω. φ is the angle embraced by the two vectors. During guided motion of a body in the rotating reference system, it is subjected to Coriolis acceleration in the direction of the Coriolis force.

Law of conservation of energy

Definition of the energy conservation law

The energy conservation law in mechanics states:

in a self-contained mechanical system, the sum of energies remains constant:

Equation 10

$$E_p + E_k + Q = \text{const.}$$

The energies are composed of potential energy E_p, kinetic energy E_k and heat quantity Q. A mechanical system is referred to as self-contained if no forces are acting on the system from the outside or, if forces are acting on the system from outside, they are negligible.

Frictional energy in the mechanical system

In a mechanical system, only an increase in heat quantity can occur, which is caused by friction losses in the system.

Thus, for the consideration of two points in time 1 and 2 of a motion sequence, the following applies:

Equation 11

$$E_{p1} + E_{k1} + Q_1 = E_{p2} + E_{k2} + Q_2$$

For friction losses occurring along the way between 1 and 2, the following applies:

Equation 12

$$Q_2 - Q_1 = W_{R1,\ 2}$$

This gives the following for the energy conservation law:

Equation 13

$$E_{p1} + E_{k1} = E_{p2} + E_{k2} + W_{R1,\ 2}$$

Conservative systems

For a **conservative system** in which **no friction losses** occur, the energy conservation law is:

Equation 14

$$E_{p1} + E_{k1} = E_{p2} + E_{k2}$$

The energy conservation law can also be formulated as:

Equation 15

$$E_{k2} - E_{k1} = W_{1,\ 2} - W_{R1,\ 2}$$

In other words, the change in the kinetic energy between points 1 and 2 of a motion sequence is equal to the work that is done by the imposed forces along the way from 1 to 2, minus the friction losses occurring along the way from 1 to 2.

Forms of energy in kinetics (selection)

Forms of energy in kinetics

The following table describes a selection of the most important forms of energy in kinetics.

Energy	Value	Explanation
Kinetic energies		
Translational energy of the mass centre of gravity	$E_k = \frac{m}{2} \cdot v_S^2$	The total kinetic energy of a moving mass is composed of the translational energy relative to the centre of gravity velocity and the rotational energy about the centre of gravity.
Rotational energy about the mass centre of gravity	$E_k = \frac{J_S}{2} \cdot \omega^2$	
Rotational energy about the instantaneous centre of revolution or rotation	$E_k = \frac{1}{2} \cdot J_{DP} \cdot \omega^2$ $E_k = \frac{1}{2} \cdot J_{MP} \cdot \omega^2$	In the case of a guided rotational motion of the mass, or whenever the motion's instantaneous centre of rotation can be specified, the total kinetic energy can be specified by the rotational energy about the instantaneous centre of revolution or rotation alone.

Continuation of table, see Page 140.

Continuation of table, Forms of energy in kinetics, from Page 139.

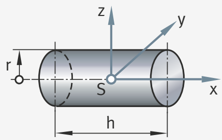
Energy	Value	Explanation
Potential energies		
Energy of the position in the Earth's gravitational field	$E_p = m \cdot g \cdot h$	The energy of the position of a mass in the constant gravitational field (close to the Earth) is the product of the weight and of the altitude of the centre of gravity h above a chosen reference level.
Work in the variable potential field	$W_{1,\,2} = \int_1^2 m \cdot g(h) \cdot dh$	Work in the Earth's gravitational field at greater altitudes: $g(h) = g_0 \cdot R_0^{\,2}/(R_0 + h)^2$
Elastic deformation energy of a spring:		
Translation spring	$E_p = \frac{1}{2} \cdot c \cdot w^2$	The work done on deformation of a spring is stored in the form of elastic energy in the spring. The energy depends on the spring constant ($c = F/s$ or $c' = F \cdot s$) and the deflection w or torsion φ of the spring.
Torsion spring	$E_p = \frac{1}{2} \cdot c' \cdot \varphi^2$	
Elastic energy of supports on deformation as the result of:		
Normal forces	$E_p = \frac{1}{2} \int \frac{F_N^2}{E \cdot A} dx$	If a support or bar is elastically deformed by external forces, displacements are generated by the internal stresses. The work done along the displacements which are generated by these is equal to the elastic energy stored in the bar or support. Generally, this elastic deformation energy can be described by the spring constant and the deflection or torsion at the point of the deformation(see section on springs).
Bending moments	$E_p = \frac{1}{2} \int \frac{M_b^2}{E \cdot I_a} dx$	
Torsion moments	$E_p = \frac{1}{2} \int \frac{M_t^2}{G \cdot I_p} dx$	
Energy of the position in the centrifugal field	$E_p = \frac{1}{2} \cdot m \cdot \Omega^2 \cdot r^2$	In the rotating system, a force field (centrifugal field) exists for the accelerated observer, the strength of which increases from the rotation centre towards the outside. The energy of the position is relative to the rotation axis.
Energy losses		
Due to sliding friction forces	$W_{R1,\,2} = F_N \cdot \mu \cdot s_{1,\,2}$	If sliding friction forces (resistance forces) are present, these do friction work along the acting distance $s_{1,\,2}$ which manifests itself as heat and is lost for the mechanical motion.
In the case of an incomplete elastic impact	$W_{R1,\,2} = \frac{1}{2}\left(1 - e^2\right) \cdot (v_1 - v_2)^2 \cdot \frac{m_1 \cdot m_2}{m_1 + m_2}$	In the case of an incomplete elastic impact of bodies, energy losses occur as the result of internal material friction. e = distance from centre of gravity – fixing points

Mass moments of inertia of homogeneous solids

Second mass moments of inertia, homogeneous solids

The second mass moments of inertia of homogeneous solids can be calculated as follows:

Circular cylinder

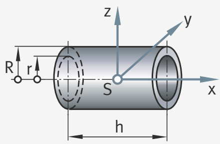


$$m = \rho \pi r^2 h$$

$$J_x = \frac{1}{2} m r^2$$

$$J_y = J_z = \frac{1}{12} m \left(3 r^2 + h^2\right)$$

Hollow cylinder

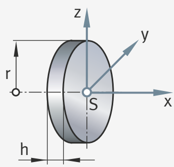


$$m = \rho \pi \left(R^2 - r^2\right) h$$

$$J_x = \frac{1}{2} m \left(R^2 + r^2\right)$$

$$J_y = J_z = \frac{1}{4} m \left(R^2 + r^2 + \frac{h^2}{3}\right)$$

Thin disk

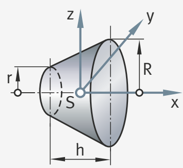


$$m = \rho \pi r^2 h;\ h \ll r$$

$$J_x = \frac{1}{2} m r^2$$

$$J_y = J_z = \frac{1}{4} m r^2$$

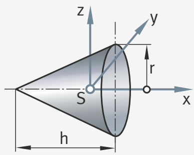
Truncated circular cone



$$m = \frac{1}{3} \rho \pi h \left(R^2 + R r + r^2\right)$$

$$J_x = \frac{3}{10} m \frac{R^5 - r^5}{R^3 - r^3}$$

Circular cone

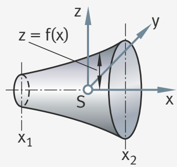


$$m = \frac{1}{3}\rho\,\pi\,r^2\,h$$

$$J_x = \frac{3}{10}\,m\,r^2$$

$$J_y = J_z = \frac{3}{80}\,m\left(4\,r^2 + h^2\right)$$

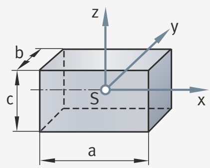
Any body of revolution



$$m = \rho\,\pi\int_{x_1}^{x_2} f^2(x)\,dx$$

$$J_x = \frac{1}{2}\rho\,\pi\int_{x_1}^{x_2} f^4(x)\,dx$$

Cuboid



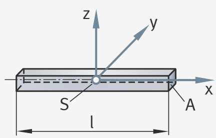
$$m = \rho\,a\,b\,c$$

$$J_x = \frac{1}{12}\,m\left(b^2 + c^2\right)$$

$$J_y = \frac{1}{12}\,m\left(a^2 + c^2\right)$$

$$J_z = \frac{1}{12}\,m\left(a^2 + b^2\right)$$

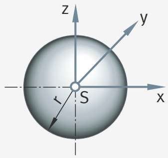
Thin bar



$$m = \rho\,A\,l$$

$$J_y = J_z = \frac{1}{12}\,m\,l^2$$

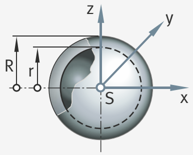
Sphere



$$m = \frac{4}{3}\rho\pi r^3$$

$$J_x = J_y = J_z = \frac{2}{5}mr^2$$

Hollow sphere



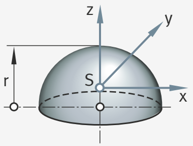
$$m = \frac{4}{3}\rho\pi\left(R^3 - r^3\right)$$

$$J_x = J_y = J_z = \frac{2}{5}m\frac{R^5 - r^5}{R^3 - r^3}$$

for thin-walled hollow spheres:

$$J_x = J_y = J_z = \frac{2}{3}mr^2$$

Hemisphere

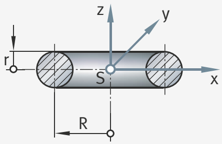


$$m = \frac{2}{3}\rho\pi r^3$$

$$J_x = J_y = \frac{83}{320}mr^2$$

$$J_z = \frac{2}{5}mr^2$$

Circular torus



$$m = 2\rho\pi^2 r^2 R$$

$$J_x = J_y = \frac{1}{8}m\left(4R^2 + 5r^2\right)$$

$$J_z = \frac{1}{4}m\left(4R^2 + 3r^2\right)$$

Principle of linear momentum

Definition of the principle of linear momentum

The fundamental dynamic law for motion of a constant mass m under the influence of an external, resulting, constant force $\vec{F}$ is:

Equation 16

$$m \cdot \vec{a} = m \cdot \frac{d\vec{v}}{dt} = \frac{d(m \cdot \vec{v})}{dt} = \frac{d\vec{p}}{dt} \qquad m = \text{const.}$$

Integration over the duration of the effect gives the principle of linear momentum:

Equation 17

$$\int_{t_0}^{t_1} \vec{F} \cdot dt = \vec{F} \cdot (t_1 - t_0) = m \cdot (\vec{v}_1 - \vec{v}_0) = \vec{p}_1 - \vec{p}_0$$

The value $\vec{p} = m \cdot \vec{v}$ is referred to as the linear momentum or momentum of mass m.

Further statements about the principle of linear momentum

The following statements can be made on the basis of the principle of linear momentum:

- The defined time integral over the external resulting force $\vec{F}$ acting on a mass m is equal to the change in the absolute linear momentum of the mass ($\vec{p} = m \cdot \vec{v}$) in the direction of this force
- If no external resulting force $\vec{F}$ acts on the mass m, its linear momentum $\vec{p}$ remains constant in terms of its magnitude and direction since: $\vec{F} = 0$ and thus $d\vec{p}/dt = 0$, giving $\vec{p} = \text{const.}$
  This statement can be extended to a system of several, individual masses:
  If no external resulting force acts on a system of masses, the system's overall linear momentum remains constant in terms of its magnitude and direction. In other words, the overall centre of gravity of the system either remains at rest or moves uniformly and in a straight line.

Principle of angular momentum

Definition of the principle of angular momentum

On a similar basis to the principle of linear momentum, it follows from the fundamental dynamic law that the motion of a rotating mass with the constant moment of inertia J_0 about its pivotal point "0", under the influence of an external, resulting moment, is:

Equation 18

$$\vec{M}_0 = J_0 \cdot \dot{\vec{\omega}} = J_0 \cdot \frac{d\vec{\omega}}{dt} = \frac{d(J_0 \cdot \vec{\omega})}{dt} = \frac{d\vec{L}}{dt} \qquad J_0 = \text{const.}$$

The following applies:

Equation 19

$$\vec{M}_0 = \vec{r} \times \vec{F}$$

In this instance $\vec{r}$ is the vector from the reference point (pivotal point) "0" to the point of application of the external, resulting force $\vec{F}$.

Integration over the duration of the effect gives the principle of angular momentum:

Equation 20

$$\int_{t_0}^{t_1} \vec{M}_0 \cdot dt = \vec{M}_0 \cdot (t_1 - t_0) = J_0 \cdot (\vec{\omega}_1 - \vec{\omega}_0) = \vec{L}_1 - \vec{L}_0$$

The value $\vec{L} = J_0 \cdot \vec{\omega}$ is referred to as the **angular momentum**, **moment of linear momentum** or **moment of momentum** of the rotating mass J_0 about the reference point "0".

Further statements about the principle of angular momentum

The following statements can be made on the basis of the principle of angular momentum (on a similar basis to the principle of linear momentum):

- The time integral over the resulting moment $\vec{M}_0$ of the external forces $\vec{F}$ acting on a rotating mass J_0 in relation to a reference point "0" is equal to the change in the angular momentum $J_0 \cdot \vec{\omega}$ (moment of momentum $\vec{L}$) of the rotating mass in the direction of the acting moment.
  - If the resulting moment vector points in the direction of the rotation vector $\vec{\omega}$ of the rotating mass, it changes the magnitude of the angular momentum (moment of momentum), i.e. a pure **angular momentum magnitude change**
  - If the resulting moment vector is perpendicular to the direction of the rotation vector $\vec{\omega}$ of the rotating mass, this results in a change to the direction of the angular momentum (moment of momentum), i.e. a pure **angular momentum direction change**.
    In this case we use the statement of the angular momentum principle in the differential form:

Equation 21

$$\vec{M}_a = \frac{d\vec{L}}{dt}$$

  - The change over time of the moment of momentum is equal to the moment of the external forces in relation to any reference point "0"
- If no external resulting moment $\vec{M}_0$ acts on the rotating mass J_0, then its angular momentum (moment of momentum) $\vec{L}$ remains constant in terms of its magnitude and direction in space since: $\vec{M}_a = 0$ and thus $d\vec{L} = 0$, giving $\vec{L} = \text{const}$.
  This statement can be extended to a system of several, individual masses.

Calculations involving the principles of linear momentum and angular momentum

The following analyses are helpful when studying the motion of bodies under the influence of external forces and moments:

- For the purpose of the calculation it is advisable to
  - choose the centre of gravity as the reference point for the angular momenta and to additionally specify the change in linear momentum of the centre of gravity
  - select the instantaneous centre of revolution or rotation as the reference point for the motion. In the case of guided motion, it is sufficient to specify the body's change in angular momentum about the instantaneous centre of revolution
- Both the linear and angular momenta are directional quantities. When studying the motion of bodies, the positive direction of motion (coordinates) must be defined. Their mathematical signs and the signs of the acting, resulting, external forces and moments must be observed.
- The principle of linear momentum and of angular momentum result from the fundamental equation of motion (fundamental dynamic law) through integration over time. Thus, all motion events which can be described using the fundamental equation of motion, can also be studied with the aid of the principles of linear momentum and of angular momentum
- Linear momentum and angular momentum principles are observed in relation to so-called impact phenomena in particular. Impact phenomena cannot be described by the simple fundamental equation of motion, as generally no statement can be made about the magnitude of the acting impact force and the duration of the impact.

Impact laws – central impulse

Definition of impact and impulse

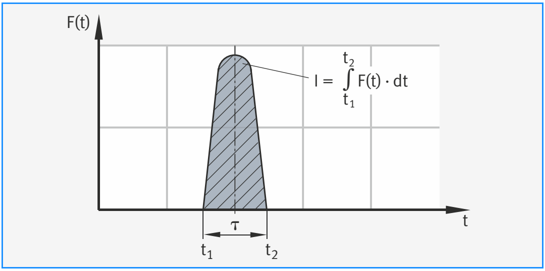
We speak of an impact when a large force F(t) is exerted on a body with the mass m during a very short period of time τ (impact duration) and when this takes place in such a way that the time integral over the acting force assumes a finite value.

The time integral over the force F(t) is referred to as the magnitude of the impulse I:

Equation 22

$$I = \int_{t_1}^{t_2} F(t) \cdot dt = \Delta p \qquad \text{with impact duration} \qquad \tau = t_2 - t_1$$

Figure 1
Magnitude of the impulse



Generally, the time curve for the acting force F(t) cannot be determined separately from the impact duration τ. The impulse I can be described by its effect, which **brings about a finite change in the velocity** of the body. In other words, the linear momentum of the body changes in the direction of the acting impact force.

In accordance with the fundamental dynamic law, the following applies:

Equation 23

$$F(t) = \frac{d(m \cdot v)}{dt}$$

This gives the following for the time integral over the acting impact force:

Equation 24

$$I = \int_{t_1}^{t_2} F(t) \cdot dt = m \cdot v_2 - m \cdot v_1 = m \cdot \Delta v$$

where Δv = change in velocity of the body's centre of gravity as a result of the impact.

Figure 2
Impulse

① Before impact (t_1)
② After impact (t_2)

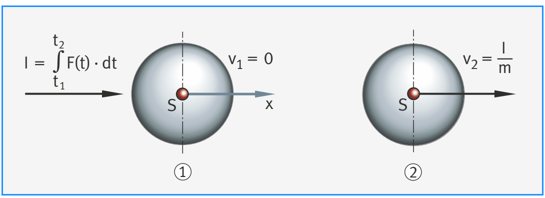
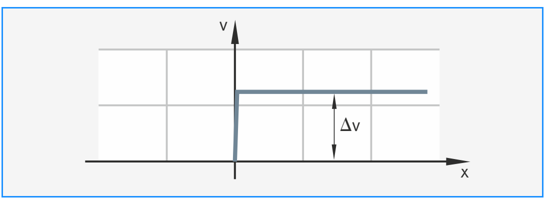


Figure 3
Impulse velocity profile



An impulse causes a “sudden” change in the velocity of the body, which corresponds to a change in linear momentum (see Principle of linear momentum). This change in velocity takes place in such a short space of time that the distance covered by the body during this time is negligible or practically zero.

The influence of other external forces (weights and friction forces etc.) is also negligible during the course of the impact compared with the magnitude of the impact force.

If the impulse is applied “normally” to the body and in the direction of the centre of gravity S and no other external forces are applied (including reaction forces) in the direction of impact, this results in a **sudden change in the translational velocity** in the direction of impact.

Impact laws – angular impulse

Unguided motion

If the impulse is **not** applied in the direction of the centre of gravity S of the body and the line of influence of the impulse is at a distance a from the centre of gravity, the impact not only causes a sudden change in the translational velocity, but also a sudden change in the angular velocity ω.

Angular impulse

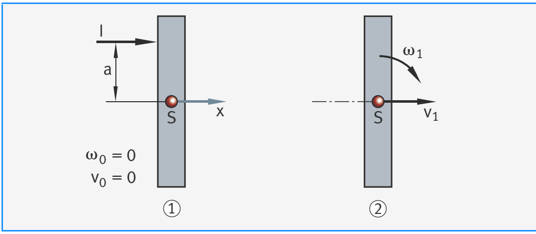
Consequently, there is also a change in the angular momentum (moment of momentum) of the body about the centre of gravity S. This impulse is known as **angular impulse**:

Equation 25

$$H_S = a \cdot I$$

Figure 4
Unguided motions

① Before impact
② After impact



Angular impulse relative to centre of gravity S

If we apply the equation for impulse:

Equation 26

$$I = \int_{t_0}^{t_1} F(t) \cdot dt = m \cdot (v_1 - v_0) = m \cdot v_1$$

this gives the following for the angular impulse relative to the centre of gravity S:

Equation 27

$$H_S = a \cdot \int_{t_0}^{t_1} F(t) \cdot dt = a \cdot I = J_S \cdot (\omega_1 - \omega_0) = J_S \cdot \omega_1$$

Angular impulse relative to instantaneous centre of rotation MP

If the change in angular momentum about the instantaneous centre of rotation MP is described as the angular impulse $H_{MP} = I \cdot b$ (b = distance between the line of action of the impulse I and the instantaneous centre of rotation), this gives:

Equation 28

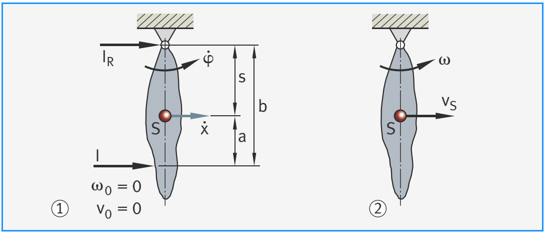
$$H_{MP} = b \cdot \int_{t_0}^{t_1} F(t) \cdot dt = b \cdot I = J_{MP} \cdot (\omega_1 - \omega_0) = J_{MP} \cdot \omega_1$$

Guided motion

In the case of a guided motion, a relationship between the velocity of the centre of gravity and the angular velocity of the body can be described:

Figure 5
Guided motions

① Before impact
② After impact



If several impulses (including reaction impulses) are applied to a body, then the following statements apply:

- The sum of all impulses is equal to the change in the linear momentum
- The sum of all angular impulses about the centre of gravity (total moment of the impacts) is equal to the change in the angular momentum about the centre of gravity
- The total moment of the impacts about the instantaneous centre of rotation (of revolution) is equal to the change in the angular momentum (moment of momentum) about the instantaneous centre of rotation.

For the centre of gravity S for example, this gives the following:

Equation 29

$$I + I_R = m \cdot v_S$$

$$I \cdot a - I_R \cdot (b - a) = J_S \cdot \omega \qquad v_S = (b - a) \cdot \omega = s \cdot \omega$$

and for the instantaneous centre of rotation MP:

Equation 30

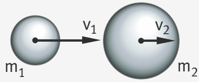
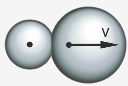
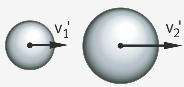
$$I \cdot b = J_{MP} \cdot \omega \qquad v_S = s \cdot \omega$$

Once the positive velocity directions have been defined, the signs of the impulses and angular impulses must be taken into account.

Impact laws for solid bodies

Direct, central impact

In the case of a direct, central impact, the impact normal and the velocity vectors lie on the line connecting the centres of gravity of the two bodies.

Impact period	Principle of linear momentum $I =$	Energy equation $E =$
positive direction		
I. Before impact $v_1 > v_2$ 	$m_1 v_1 + m_2 v_2$	$\frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2$
II. Moment of maximum deformation of the bodies 	$(m_1 + m_2)\, v$	$\frac{(m_1 + m_2)}{2} v^2 + E_{p\ max} + W_{R1,\,2}$
III. After impact $v_1' < v_2'$ 	$m_1 v_1' + m_2 v_2'$	$\frac{m_1}{2} (v_1')^2 + \frac{m_2}{2} (v_2')^2 + W_{R1,\,2}$

Newton's impact hypothesis
Equation 31

In this context, the impact hypothesis is:

$$e\left(v_1 - v_2\right) = v_2' - v_1'$$

Description of the impact periods

Impact period I → II:

- The two bodies are deformed until their centres of gravity are at a minimum distance from one another. At this moment, the velocities of their centres of gravity are equal.

Impact period II → III:

- If the bodies are perfectly elastic ($e = 1$), the energy stored in the bodies in the form of elastic deformation energy is released again and converted into kinetic energy
- If the bodies are perfectly inelastic ($e = 0$, perfectly plastic impact), the deformation is not reversed.
  It occurs in the bodies as heat and is lost for the remainder of the motion sequence. As a result, impact period II → III does not occur:
- The imperfectly elastic impact ($0 < e < 1$) lies between the two limiting cases.

Calculating the velocities

The velocities of the bodies after impact and the occurring losses can be determined using the principle of conservation of linear momentum[1)], the energy conservation law[1)] and Newton's impact hypothesis[1)].

Velocities of the bodies after impact (III)	Potential elastic energy at the moment of maximum deformation (II)	Energy loss during impact I → II and II → III
Perfectly elastic impact e = 1	$v_2' - v_1' = v_1 - v_2$	
$v_1' = \dfrac{(m_1 - m_2)\,v_1 + 2\,m_2\,v_2}{m_1 + m_2}$ $v_2' = v_1' = \dfrac{(m_2 - m_1)\,v_2 + 2\,m_1\,v_1}{m_1 + m_2}$	$E_{p\ max} = \dfrac{1}{2}(v_1 - v_2)^2 \cdot \dfrac{m_1\,m_2}{m_1 + m_2}$	$W_{R1,\,2} = 0$
Imperfectly elastic impact 0 < e < 1	$v_2' - v_1' = e \cdot (v_1 - v_2)$	
$v_1' = \dfrac{m_1\,v_1 + m_2\,v_2 - m_2(v_1 - v_2)\,e}{m_1 + m_2}$ $v_2' = \dfrac{m_1\,v_1 + m_2\,v_2 + m_1(v_1 - v_2)\,e}{m_1 + m_2}$	$E_{p\ max} = \dfrac{1}{2}e^2(v_1 - v_2)^2 \cdot \dfrac{m_1\,m_2}{m_1 + m_2}$	$W_{R1,\,2} = \dfrac{1}{2}\left(1 - e^2\right)(v_1 - v_2)^2 \cdot \dfrac{m_1\,m_2}{m_1 + m_2}$
Perfectly plastic impact e = 0	$v_1' = v_2' = v$	
$v_1' = v_2' = v = \dfrac{m_1\,v_1 + m_2\,v_2}{m_1 + m_2}$	$E_{p\ max} = 0$	$W_{R1,\,2} = \dfrac{1}{2}(v_1 - v_2)^2\,\dfrac{m_1\,m_2}{m_1 + m_2}$

1) The signs of v and v' must be observed when evaluating the relationships.

Oblique, central impact

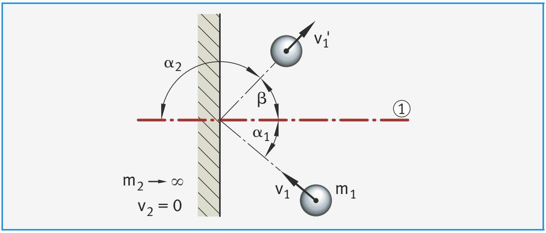
In the case of an oblique, central impact, the impact normal lies on the line connecting the centres of gravity of the two bodies, but the velocity vectors do not.

If no frictional forces occur in the contact area between the two bodies, there will be no change in the bodies' velocity component which is tangential to the contact area. In this instance, the relationships for the straight, central impact can be used by determining the components of the velocities in the direction of the impact normal.

Example system: **oblique central impact of a sphere against a wall**

Figure 6
Oblique, central impact

① Impact normal



The following applies perpendicular to the impact normal:

Equation 32

$$v_1 \cdot \sin\alpha_1 = v_1' \cdot \sin\alpha_2 = v_1' \cdot \sin\beta$$

The following applies in the direction of the impact normal:

Equation 33

$$v_1' \cdot \cos\alpha_2 = -e \cdot v_1 \cdot \cos\alpha_1 = e \cdot v_1 \cdot \cos\beta$$

Therefore:

Equation 34

$$\tan\alpha_2 = -\frac{\tan\alpha_1}{e} = -\tan\beta \qquad v_1' = -e\frac{v_1 \cdot \cos\alpha_1}{\cos\alpha_2}$$

Direct, eccentric impact

In the case of a direct, eccentric impact, the velocity vectors lie in the direction of the impact normal, but the impact normal does not lie on the line connecting the two bodies' centres of gravity.

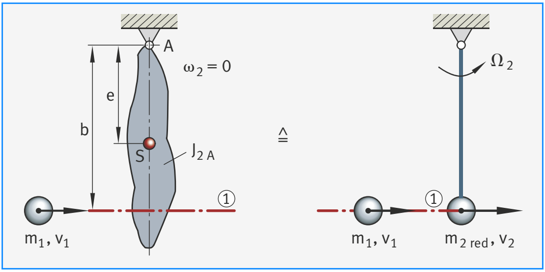
Guided motion

If one of the two bodies (or both) is pivot-mounted, the relationships for a straight, central impact can be used by reducing the rotating mass of the guided bodies to the point of impact.

Example system: **guided motion**

Figure 7
Direct, eccentric impact
Before impact

① Impact normal



Note:

Equation 35

$$m_{2\,red} = \frac{J_{2\,A}}{b^2}$$

With impact factor e, this gives the following:

Equation 36

$$v_1' = \frac{m_1\,v_1 - m_{2\,red} \cdot v_1 \cdot e}{m_1 + m_{2\,red}} \qquad v_2' = \frac{m_1\,v_1 - m_1\,v_1 \cdot e}{m_1 + m_{2\,red}}$$

$$\Omega_2 = \frac{v_2'}{b}$$

Unguided motion

If neither of the two bodies is guided, the equations required to determine the velocities of the bodies (translational velocity and angular velocity) after impact are obtained from the impact laws (for the impulse or angular impulse) and Newton's impact hypothesis for the point of impact.

Example system: **unguided motion**

Figure 8
Positive direction before impact

① Impact normal

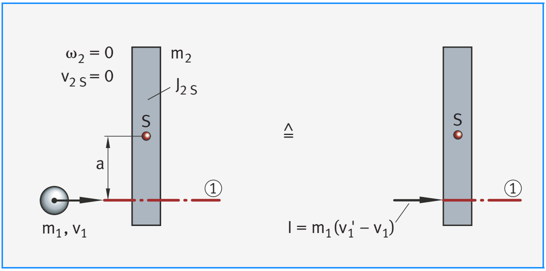
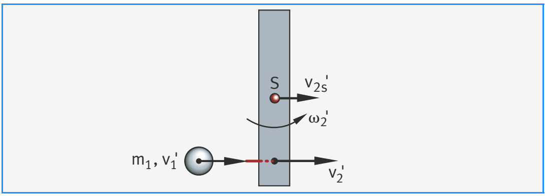


Figure 9
After impact



If we consider the effect of the impulse I occurring between the two bodies, for each body separately, the use of the following equation:

Equation 37

$$I = \int F(t) \cdot dt$$

gives the impulse of body 2 (mass m_2) on body 1 (mass m_1):

Equation 38

$$\int F(t) \cdot dt = m_1 \left(v_1' - v_1\right)$$

For reaction reasons, an impulse of the same magnitude acts on body 2 (mass m_2):

Equation 39

$$\int F(t) \cdot dt = m_2 \left(v_{2\,S}{}' - v_{2\,S}\right) \qquad v_{2\,S} = 0$$

An angular impulse also acts about the centre of gravity of body 2 (mass m_2):

Equation 40

$$a \cdot \int F(t) \cdot dt = J_{2\,S} \left(\omega_2{}' - \omega_2\right) \qquad \omega_2 = 0$$

With Newton's impact hypothesis, this ultimately gives the following for the impact point:

Equation 41

$$e = \frac{v_2{}' - v_1{}'}{v_1 - v_2} = \frac{\left(v_{2\,S}{}' + \omega_2{}' \cdot a\right) - v_1{}'}{v_1} \qquad v_2 = 0$$

These equations can be used to determine the velocities v_1' and $v_{2\,S}'$ and ω_2' after impact.

Oblique, eccentric impact

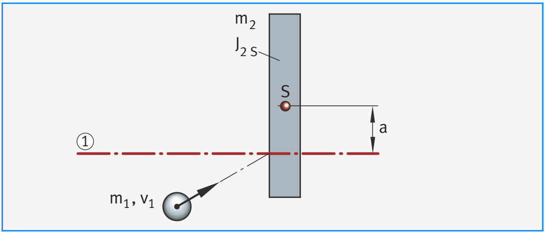
In the case of an oblique, eccentric impact, neither the impact normal nor the line connecting the centres of gravity of the two bodies and the velocity vectors coincide.

If no frictional forces occur in the contact area between the two bodies, there will be no change in the bodies' velocity component which is perpendicular to the impact normal. In this instance, the relationships for the straight, eccentric impact can be used by determining the components of the velocities in the direction of the impact normal.

Example system: **oblique, eccentric impact**

Figure 10
Oblique, eccentric impact

① Impact normal



Mechanical vibrations and acoustics

Mechanical vibrations and acoustics – general definitions

Description of vibration events

In a vibration event, the energy present is transformed from one form of energy to another in defined time periods and periodically transformed, in whole or in part, back into the first form of energy.

Within a mechanical translational or rotational vibration system comprising a mass and a potential energy reservoir, a transformation process occurs between the kinetic and potential energy present.

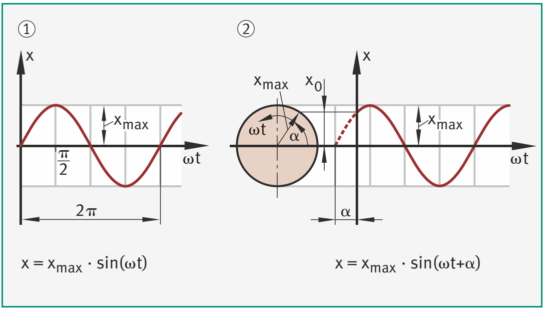
Free, undamped vibration

A free, undamped vibration is present if energy is neither supplied to nor withdrawn from the oscillator during the vibration. As a result, the quantity of energy initially introduced is maintained and a periodic energy transformation takes place. In this case, the system performs stationary natural vibrations whose frequency depends only on the characteristics of the system (mass and potential energy reservoir). Vibrations of this type are also known as **harmonic vibrations**. The vibration pattern over time can be described by means of the constant **vibration amplitude** and a harmonic mathematical function (sin, cos) whose argument contains the **natural frequency** of the system.

Figure 1
Free, undamped vibration

x_{max} = amplitude
2π = period

① Without phase displacement
② With phase displacement α



Damped vibration If the oscillator loses a proportion of the energy present in the system in each vibration period, a damped vibration is present. If linear damping proportional to velocity (Newtonian friction) is present, the vibration amplitude will decrease in accordance with a geometric progression.

Forced vibration If the oscillator is subjected to excitation by an external, periodically acting force F(t) or a moment M(t), forced vibrations will occur.
Due to the excitation force, energy can be supplied to or withdrawn from the oscillator. After a transient phase, the system no longer vibrates at its natural frequency, but with the frequency of the externally acting excitation force.

Resonance If forced vibration is occurring and the excitation acting externally on the system corresponds to the natural frequency, resonance occurs. In the case of undamped systems, the vibration amplitudes in a case of resonance will assume the value “infinite”. The vibration amplitudes, when considered as a function of the excitation frequency, are subdivided by the resonance point (natural frequency = excitation frequency) into **subcritical** and **supercritical** vibration regions.

Coupled vibrations If two vibration systems are coupled with each other by means of mass or elasticity, a periodic exchange of energy takes place between the systems (multiple mass oscillator).

Mathematical description of vibrations In general, mechanical vibration events can be described, depending on their initial conditions, by means of sine or cosine functions or their superposition. In the analysis of vibration events, a Fourier analysis can be useful since each function, which is monotone and constant, can be expressed as the sum of sinusoidal and cosinusoidal basic and harmonic vibrations.

Values and units A number of important values and units relating to mechanical vibrations are described as follows:

Name	Value	Unit	Explanation
Mass	m	kg	The mass undergoing translational vibration
Mass moment of inertia	J	$kg \cdot m^2$	The mass undergoing rotational vibration with a moment of inertia J
Instantaneous value of the vibration	x φ	m rad[1]	The time-dependent value of the oscillation amplitude at a single moment
Amplitude	x_{max}, $\hat{x}$ φ_{max}, $\hat{\varphi}$	m rad	The maximum instantaneous value (peak value) of a vibration
Vibration velocity (speed)	$\dot{x}$ $\dot{\varphi}$	m/s rad/s	The instantaneous value of the alternating velocity in the direction of vibration
Inertia force	$m \cdot \ddot{x}$	N	The d'Alembert's inertia force or the moment of inertia forces acting against the positive acceleration
Moment of inertia forces	$J \cdot \ddot{\varphi}$	$N \cdot m$	
Spring constant	c	N/m	Linear springs (springs with linear spring characteristic)
Spring force	$c \cdot x$	N	In the case of springs with a linear spring characteristic, the spring reaction is proportional to the deflection
Torsion spring constant	c	$N \cdot m/rad$	Torsion springs
Spring moment	$c \cdot \varphi$	$N \cdot m$	In the case of linear springs, the spring reaction is proportional to the deflection
Damping constant (damping coefficient)	b	$N \cdot s/m$	In the case of Newtonian friction, the damping force is proportional to the velocity and the damping constants (linear damping)
Damping constant for rotary motion	b	$N \cdot s \cdot m/rad$	
Damping factor (decay coefficient)	$\delta = b/(2 \cdot m)$ $\delta = b/(2 \cdot J)$	1/s 1/s	The damping constant relative to twice the mass
Damping factor	$D = \delta/\omega_0$	–	$D < 1$: damped vibration, $D \geqq 1$: aperiodic case
Damping ratio	$\hat{x}_n/\hat{x}_{n+1}$ $\hat{\varphi}_n/\hat{\varphi}_{n+1}$	– –	The ratio between two amplitudes separated by one period
Logarithmic damping decrement	$\Lambda = \frac{2 \cdot \pi \cdot D}{\sqrt{1-D^2}}$	–	$\Lambda = \ln\left(\hat{x}_n/\hat{x}_{n+1}\right)$ $\Lambda = \ln\left(\hat{\varphi}_n/\hat{\varphi}_{n+1}\right)$
Time	t	s	The current time coordinate

Continuation of table, see Page 159.

[1] The unit rad can be replaced by "1" in calculation.

Continuation of table, Values and units, from Page 158.

Name	Value	Unit	Explanation
Phase angle	α	rad	This characterises the vibration phase, in other words the vibration state in which the system is currently present
Phase displacement angle	$\epsilon = \alpha_1 - \alpha_2$	rad	The difference in phase angle between two vibration events with the same circular frequency; if the value is positive this is a leading angle
Period time	$T = 2 \cdot \pi / \omega_0$	s	The time in which one single vibration occurs
Characteristic frequency of the natural vibration	$f_0 = 1/T$	Hz	The reciprocal value of the period time
Characteristic circular frequency of the natural vibration	$\omega_0 = 2 \cdot \pi \cdot f_0$	1/s	The number of vibrations in $2 \cdot \pi$ seconds
Natural circular frequency (natural frequency)	$\omega_0 = \sqrt{c/m}$ $\omega_0 = \sqrt{c/J}$	1/s 1/s	The vibration frequency of the natural vibration of the system (undamped)
Natural circular frequency in the case of damping	$\omega_d = \sqrt{\omega^2{}_0 - \delta^2}$	1/s	In the case of a very small damping factor $D \ll 1$, $\omega_d = \omega_0$
Excitation frequency	Ω	1/s	The circular frequency of the excitation
Circular frequency ratio	$\eta = \Omega/\omega_0$	–	Resonance is present at $\eta = 1$

Free, undamped vibration

Description of the energy approach

Where free, undamped mechanical vibrations take place, a periodic exchange generally occurs between **potential** and **kinetic energy.**

The potential energy is present in this case as:

- the energy of position of the vibrating mass in a gravitational field (the Earth's gravitational field, a centrifugal field etc.)
- the elastic deformation energy (elastic strain energy of a spring, a support or a bar structure etc.)
- or both.

The kinetic energy is present in this case as:

- the energy of motion of the vibrating mass.

Motion equation, formulation and solution

The simplest mechanical vibration system, to which a range of vibrating bodies can be reduced, is **linear spring vibration**. The oscillator comprises a theoretically massless elastic element (in this case a spring with a linear spring characteristic) and a point mass.

For formulation and solution of the motion equation for this free, undamped vibration, consideration is given to the following forces:

- translational forces
- inertia forces
- reset forces.

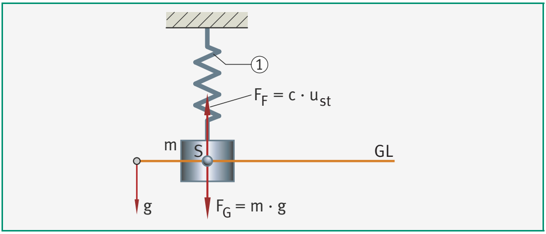
Translational forces

In a resting state, the weight force F_G of the mass m and the spring force F_F are in equilibrium.

Figure 2
Linear spring vibration: translational forces

GL = static equilibrium position

(1) Massless spring



An equilibrium between the two translational forces can be described as follows:

Equation 1

$$F_G = m \cdot g \qquad F_F = c \cdot u_{st}$$

$$\text{Equilibrium:} \qquad F_G = F_F$$

The spring is deflected by an amount u_{st} compared to its untensioned length:

Equation 2

$$u_{st} = \frac{m \cdot g}{c}$$

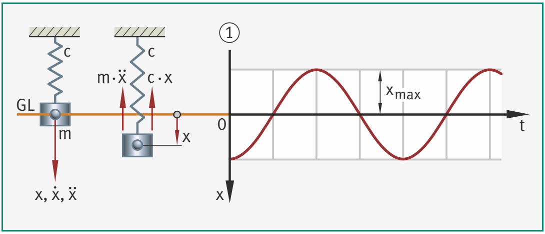
This position is known as the **rest position** or **static equilibrium position**.

If the mass is deflected out of its static equilibrium position in a vertical direction and then released, it performs free, periodic vibrations about the static equilibrium position.

Figure 3
Linear spring vibration: motion pattern

GL = static equilibrium position

① Motion pattern over time



The motion pattern is described by means of the motion equation. This is determined by developing a force approach in accordance with d'Alembert's principle:

- In the direction of motion, plot the coordinates of direction for x, $\dot{x}$ and $\ddot{x}$ **starting from the centre of gravity of the mass in the static equilibrium position**
- Then plot the forces acting at the centre of gravity of the mass in the direction of motion if these are regarded as being **deflected in the positive coordinate of direction** in the vibration.

Inertia forces

The inertia forces are considered using the following approach:

- Force approach according to d'Alembert: plot the kinetic reaction **against the positive direction of acceleration** $\ddot{x}$
- If the vibration event takes place in an **accelerated reference system** (accelerated elevator, accelerated vehicle or rotating system etc.), there will be an additional system force (kinetic reaction, centrifugal force, Coriolis force) that acts on the mass.
  This **additional system force is plotted against the positively defined direction of system acceleration** $\ddot{u}$. In this case, u, $\dot{u}$, $\ddot{u}$ are the motion coordinates of the accelerated reference system.
  This can result, on the one hand, in a change in the static equilibrium position compared with that in the unaccelerated reference system or, on the other hand, a change in the frequency of vibration.

Reset forces

The reset forces are considered using the following approach:

- Plot the **spring force** due to the deflection of the mass out of its static equilibrium position **against the positive direction of deflection**
- **Weight force** and **static spring force**:
  - If only a periodic exchange occurs in a vibration system between potential elastic and kinetic energy under the influence of a constant gravitational field, the weight force F_G (of the mass m) and the spring force F_F (due to the static deflection) will cancel each other out at each moment of the motion if the motion equation is formulated for vibration about the static equilibrium position. **In this case, the weight force and spring force are not used at all in the approach.**
  - If an exchange takes place between potential energy of position and kinetic energy during the vibration, the **weight force** F_G (of the mass m) must be included in the approach.

Motion equation

For an equilibrium of forces in the direction of motion, this gives the following equilibrium relationship in the oscillator presented:

Equation 3

$$\sum F_x = -m \cdot \ddot{x} - c \cdot x = 0$$

The forces occurring are written in the sequence of the derivatives of x, starting with the force having the highest derivative, taking account of their direction.

If this equation is divided by the factor of the highest derivative, this gives the **homogeneous differential equation** as a motion equation for free, undamped vibration of the mass m:

Equation 4

$$\ddot{x} + \frac{c}{m} \cdot x = 0$$

This form of the equation will hereinafter be referred to as the **normal form**. In terms of its mathematical structure, it is typical for all free, undamped and linear vibrations.

This therefore solves the purely mechanical problem of formulating the motion equation. Solving this motion equation is now a mathematical task.

General solution of the motion equation

The **acceleration** of the motion event is a **function of the travel.** As a result, solving this equation by means of double integration over time is not a straightforward matter.

For the existing form of d'Alembert's differential equation with constant coefficients, the general approach to solution is as follows:

Equation 5

$$x = C \cdot e^{s \cdot t}$$

If this approach to solution is applied to the normal form of the differential equation, this gives:

Equation 6

$$C \cdot s^2 \cdot e^{s \cdot t} + \frac{c}{m} \cdot C \cdot e^{s \cdot t} = 0$$

$$s^2 + \frac{c}{m} = 0$$

$$s_{1,2} = \pm\sqrt{-\frac{c}{m}} = \pm i \sqrt{c/m}$$

This gives the general solution of the motion equation:

Equation 7

$$x = C_1 \, e^{+i\sqrt{c/m} \cdot t} + C_2 \, e^{-i\sqrt{c/m} \cdot t}$$

With the aid of Euler's formula $e^{\pm i \varphi} = \cos \varphi \pm i \cdot \sin \varphi$, this can also be expressed as follows:

Equation 8

$$x = C_1 \left(\cos \sqrt{c/m} \cdot t + i \cdot \sin \sqrt{c/m} \cdot t\right)$$
$$+ C_2 \left(\cos \sqrt{c/m} \cdot t - i \cdot \sin \sqrt{c/m} \cdot t\right)$$

$$x = \left(C_1 + C_2\right) \cos \sqrt{c/m} \cdot t + i \cdot \left(C_1 - C_2\right) \sin \sqrt{c/m} \cdot t$$

This relationship only results in a real value as a solution for the motion coordinate x if the constants C_1 and C_2 are conjugated in complex form as follows:

Equation 9

$$C_{1,2} = K_1 \pm i \cdot K_2$$

The general solution for a harmonic vibration is thus expressed as follows:

Equation 10

$$x = 2K_1 \cos\sqrt{c/m}\cdot t - 2K_2 \sin\sqrt{c/m}\cdot t$$

where $\omega_0 = \sqrt{c/m}$ $\quad \omega_0{}^2 = c/m$

ω_0 is known as the **natural circular frequency** of the vibration event. The square of the natural circular frequency is always represented in the normal form of the differential equation by the factor of the linear motion coordinate x.

Based on these considerations, the following **general solution of the differential equation** is always to be expected for the **free, undamped vibration**:

Equation 11

$$x = A \sin\omega_0\, t + B\cos\omega_0\, t \qquad \text{where } \omega_0 = \sqrt{c/m}$$

Consideration of the initial conditions

The two free constants A and B of the general solution are defined by the **initial conditions** of the vibration event, which are normally stipulated.

The different initial conditions will result in the corresponding solutions when the conditions are used in the general solution:

Initial conditions			Solution
$t = 0$	$x = 0$	$\dot{x} = \dot{x}_{max}$	$x = \frac{\dot{x}_{max}}{\omega_0}\cdot \sin\omega_0\, t$
$t = 0$	$x = x_{max}$	$\dot{x} = 0$	$x = x_{max}\cdot \cos\omega_0\, t$
$t = 0$	$\dot{x} = 0$	$\ddot{x} = \lvert\ddot{x}_{max}\rvert$	$x = \frac{\lvert\ddot{x}_{max}\rvert}{\omega_0{}^2}\cdot \cos\omega_0\, t$
$t = 0$	$x = x_0$	$\dot{x} = \dot{x}_0$	$x = \frac{\dot{x}_0}{\omega_0}\cdot \sin\omega_0\, t + x_0\cdot \cos\omega_0\, t$

From a comparison of the maximum vibration deflections x_{max} (amplitudes), two important relationships can be determined between the natural circular frequency of the oscillating body, the vibration amplitude, the maximum velocity and the maximum acceleration:

Equation 12

$$\dot{x}_{max} = \omega_0 \cdot x_{max}$$

$$\lvert\ddot{x}_{max}\rvert = \omega_0{}^2 \cdot \lvert x_{max}\rvert$$

The general solution of the differential equation can therefore be expressed with the aid of the amplitudes in the following form:

Equation 13

$$x = x_{max} \cdot \sin\left(\omega_0\, t + \alpha\right)$$

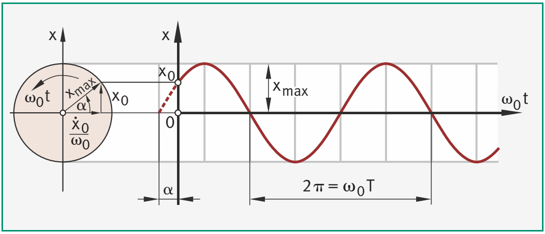
In this case, α is the phase angle (leading angle) with respect to a vibration $x = x_{max} \cdot \sin(\omega_0\, t)$ and x_{max} is the amplitude of the vibration event. The phase angle and amplitude are determined by the initial conditions.

This gives:

Initial condition	Solution
$t = 0 \qquad x = x_0 \qquad \dot{x} = \dot{x}_0$	$x_{max} = \sqrt{{x_0}^2 + \frac{\dot{x}_0^2}{\omega_0^2}}$ $\tan\alpha = \frac{x_0}{\dot{x}_0 / \omega_0}$

Figure 4
Free, undamped vibration: motion pattern

2π = period



Consideration of overlaid vibrations

The preceding method of representation of the motion equation is particularly useful when considering a superposition of several simultaneous vibrations of identical natural circular frequency ω_0 and differing phase angles α.

Two vibrations with the phase angles α_1 and α_2 have a relative phase displacement angle ϵ:

Equation 14

$$\epsilon = \alpha_1 - \alpha_2$$

The following relationships exist between the natural circular frequency, the frequency and the period time of a vibration:

Equation 15

$\omega_0 = 2 \cdot \pi \cdot f_0 = \dfrac{2 \cdot \pi}{T}$	ω_0 = natural circular frequency (vibration in $2\,\pi$ seconds)
	f_0 = frequency (number of vibrations per second)
$T = \dfrac{2 \cdot \pi}{\omega_0} = \dfrac{1}{f_0}$	T = vibration time for one period

In circumferential motion, the relationship between frequency and speed is as follows:

Equation 16

$f = \dfrac{n}{60}$	in Hz (n in min^{-1})
$\omega = \dfrac{2 \cdot \pi \cdot n}{60} = \dfrac{\pi \cdot n}{30}$	in s^{-1} (n in min^{-1})

Reduction to linear spring vibration

A series of free, undamped oscillating bodies in which an exchange takes place between elastic and kinetic energy can be reduced to the basic form of the free, undamped oscillator.

The spring constant c of the elasticity at the position and in the direction of vibration of the mass can be determined on the basis of the condition:

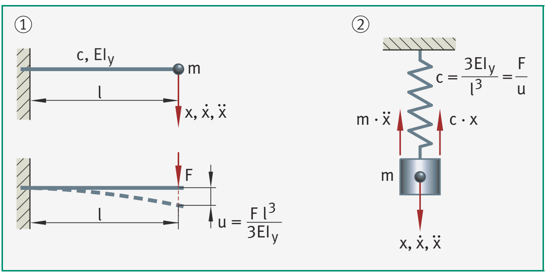
Equation 17

$c = \dfrac{F}{u}$	F = force on the elasticity in the direction of vibration of the mass
	u = deflection at the force application point in the direction of the force F

Figure 5
Free, undamped vibration and transfer to linear spring vibration

c = spring constant
EI_y = bending stiffness

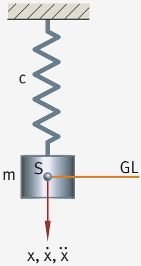
① Oscillator
② Oscillator substitute system

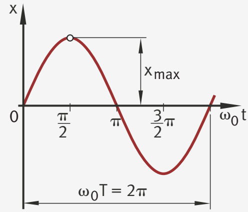


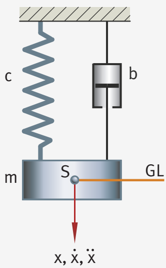
Overview of motion equations and amplification function

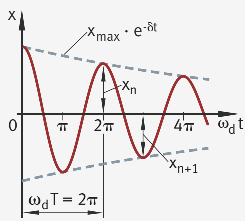
Free, undamped and damped vibration

The following table shows the conditions and relationships for solving the motion equations for free, undamped and damped vibrations:

<table>
<tr><th>Oscillator schematic</th><th>Differential equation and solution</th><th>Time-based vibration pattern</th></tr>
<tr>
<td>

GL = static equilibrium position</td>
<td>Approach:
$-m \cdot \ddot{x} - c \cdot x = 0$
Differential equation:
(normal form of the homogeneous differential equation, 2nd order):
$\ddot{x} + \frac{c}{m} \cdot x = 0$
Initial conditions:
$t = 0; x = 0; \quad \dot{x} = \dot{x}_{max}$
Solution:
$x = x_{max} \cdot \sin \omega_0 t$
With natural circular frequency:
$\omega_0 = \sqrt{\frac{c}{m}}$</td>
<td>

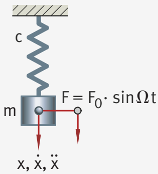
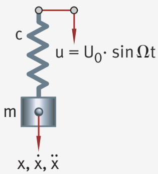
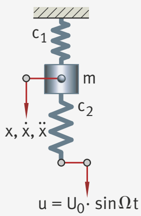
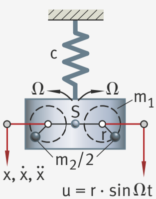
Period time of one vibration:
$T = 2\,\pi/\omega_0$
Vibration frequency:
$f = 1/T = \omega_0/2\,\pi$</td>
</tr>
<tr>
<td>

GL = static equilibrium position
b = damping constant</td>
<td>Approach:
$-m \cdot \ddot{x} - b \cdot \dot{x} - c \cdot x = 0$
Differential equation:
(normal form of the homogeneous differential equation, 2nd order):
$\ddot{x} + \frac{b}{m} \cdot \dot{x} + \frac{c}{m} \cdot x = 0$
Initial conditions:
$t = 0; x = x_0; \quad \dot{x} = 0$
Simplified solution:
$x = e^{-\delta\,t} \cdot x_{max} \cdot \cos \omega_d\,t$
With damping factor:
$\delta = b/(2\,m)$
With natural circular frequency:
$\omega_d = \sqrt{\omega_0{}^2 - \delta^2}$</td>
<td>Damped vibration:
$b < 2\sqrt{c \cdot m}$


Period time of one vibration:
$T = 2\,\pi/\omega_d$
Amplitude ratio for T/2:
$x_n/x_{n+1} = e^{-\delta\,(T/2)}$</td>
</tr>
</table>

Undamped vibration caused by external excitation

The following table shows the conditions and relationships for solving the motion equations for undamped vibrations caused by external excitation (forced vibrations). The solutions apply to the forced state.

Oscillator schematic	Differential equation	Solution and amplitude (non-homogeneous)	Phase angle
Excitation function: $\alpha = \alpha_0 \cdot \sin \Omega t$		**Solution function:** $x = x_{max} \cdot \sin(\Omega t + \alpha)$	
	**Approach:** $-m \cdot \ddot{x} - c \cdot x + F(t) = 0$ **Differential equation:** $m \cdot \ddot{x} + c \cdot x = F_0 \sin \Omega t$	**Solution:** $x = \frac{F_0}{c - m\,\Omega^2} \sin(\Omega t + \alpha)$ **Amplitude:** $x_{max} = \frac{F_0}{c} \cdot \left\lvert \frac{1}{1-\eta^2} \right\rvert = \frac{F_0}{c} \cdot V_{03}$	Subcritical range $\eta = \frac{\Omega}{\omega_0} < 1$ $\alpha = -\epsilon$ $\epsilon = 0°$
	**Approach:** $-m \cdot \ddot{x} - c \cdot x + c \cdot u = 0$ **Differential equation:** $m \cdot \ddot{x} + c \cdot x = c \cdot U_0 \sin \Omega t$	**Solution:** $x = \frac{c \cdot U_0}{c - m\,\Omega^2} \sin(\Omega t + \alpha)$ **Amplitude:** $x_{max} = U_0 \left\lvert \frac{1}{1-\eta^2} \right\rvert = U_0 \cdot V_{03}$	
	**Approach:** $-m \cdot \ddot{x} - c_1 \cdot x - c_2 \cdot x + c_2 \cdot u = 0$ **Differential equation:** $m \cdot \ddot{x} + (c_1 + c_2) \cdot x$ $= c_2 \cdot U_0 \cdot \sin \Omega t$	**Solution:** $x = \frac{c_2 \cdot U_0}{c_1 + c_2 - m\,\Omega^2} \sin(\Omega t + \alpha)$ **Amplitude:** $x_{max} = \frac{c_2 \cdot U_0}{(c_1 + c_2)} \cdot \left\lvert \frac{1}{1-\eta^2} \right\rvert$ $= \frac{c_2 \cdot U_0}{c_1 + c_2} \cdot V_{03}$	Super-critical range $\eta = \frac{\Omega}{\omega_0} > 1$ $\alpha = -\epsilon$ $\epsilon = 180°$
	**Approach:** $-(m_1 + m_2) \cdot \ddot{x} - c \cdot x - m_2 \cdot \ddot{u} = 0$ **Differential equation:** $(m_1 + m_2) \cdot \ddot{x} + c \cdot x$ $= m_2 \cdot r \cdot \Omega^2 \cdot \sin \Omega t$	**Solution:** $x = \frac{m_2 \cdot r \cdot \Omega^2}{c - (m_1 + m_2)\,\Omega^2} \sin(\Omega t + \alpha)$ **Amplitude:** $x_{max} = \frac{m_2 \cdot r}{m_1 + m_2} \cdot \left\lvert \frac{\eta^2}{1-\eta^2} \right\rvert$ $= \frac{m_2 \cdot r}{m_1 + m_2} \cdot V_{01}$	

Amplification function

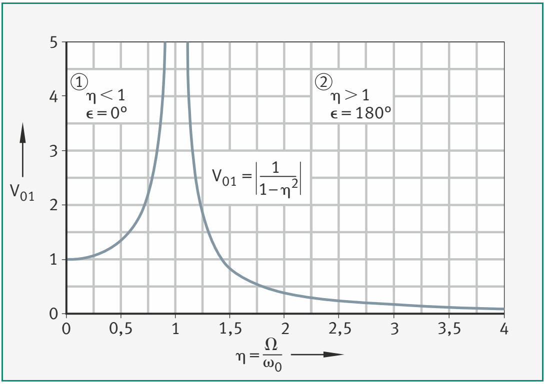
The excitation of undamped vibrations can be achieved in different forms:

- periodic force or spring force excitation

Figure 6
Periodic force or spring force excitation

η = circular frequency ratio
V_{01} = amplification function

① Subcritical range
② Supercritical range

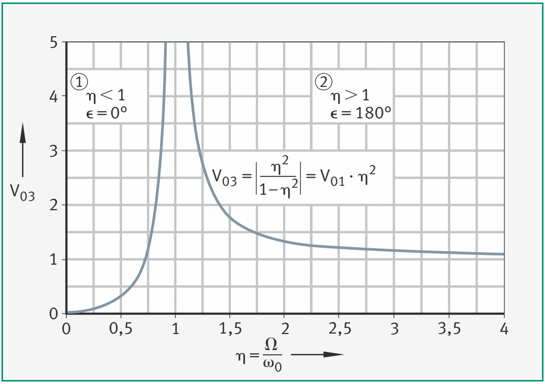


- periodic mass force excitation.

Figure 7
Periodic mass force excitation

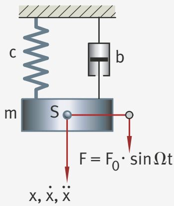
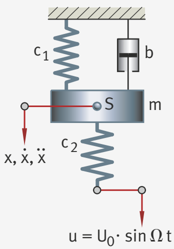
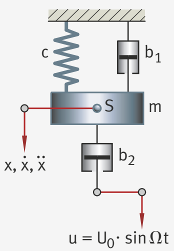
η = circular frequency ratio
V_{03} = amplification function

① Subcritical range
② Supercritical range



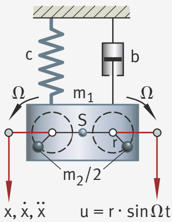
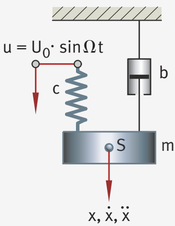
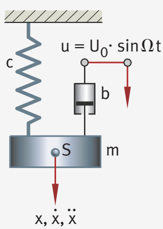
Damped vibration caused by external excitation

The following table shows the conditions and relationships for solving the motion equations for damped vibrations caused by external excitation (forced vibrations). The solutions apply to the turned in state.

Oscillator schematic	Differential equation and amplitude	Phase angle
Excitation function: $\alpha = \alpha_0 \cdot \sin\Omega$	**Solution function:** $x = x_{max} \cdot \sin(\Omega t + \alpha)$	
	**Approach:** $-m\cdot\ddot{x} - b\cdot\dot{x} - c\cdot x + F(t) = 0$ **Differential equation:** $m\cdot\ddot{x} + b\cdot\dot{x} + c\cdot x = F_0\cdot\sin\Omega t$ **Amplitude:** $x_{max} = \frac{F_0}{c}\cdot\frac{1}{\sqrt{\left(1-\eta^2\right)^2 + 4D^2\eta^2}}$ $x_{max} = \frac{F_0}{c}\cdot V_3$	$\alpha = -\epsilon_3$ $\tan\epsilon_3 = \frac{2D\eta}{1-\eta^2}$
	**Approach:** $-m\cdot\ddot{x} - b\cdot\dot{x} - c_1\cdot x - c_2\cdot x + c_2\cdot u = 0$ **Differential equation:** $m\cdot\ddot{x} + b\cdot\dot{x} + (c_1 + c_2)\cdot x = c_2\cdot U_0\cdot\sin\Omega t$ **Amplitude:** $x_{max} = \frac{c_2\cdot U_0}{c_1 + c_2}\cdot\frac{1}{\sqrt{\left(1-\eta^2\right)^2 + 4D^2\eta^2}}$ $x_{max} = \frac{c_2\cdot U_0}{c_1 + c_2}\cdot V_3$	$\alpha = -\epsilon_3$ $\tan\epsilon_3 = \frac{2D\eta}{1-\eta^2}$
	**Approach:** $-m\cdot\ddot{x} - b_1\cdot\dot{x} - b_2\cdot\dot{x} - c\cdot x + b_2\cdot\dot{u} = 0$ **Differential equation:** $m\cdot\ddot{x} + (b_1 + b_2)\cdot\dot{x} + c\cdot x = b_2\cdot U_0\cdot\Omega\cdot\cos\Omega t$ **Amplitude:** $x_{max} = \frac{b_2\cdot U_0}{b_1 + b_2}\cdot\frac{2D\eta}{\sqrt{\left(1-\eta^2\right)^2 + 4D^2\eta^2}}$ $x_{max} = \frac{b_2\cdot U_0}{b_1 + b_2}\cdot V_2$	$\alpha = \gamma_2 = \frac{\pi}{2} - \epsilon_3$ $\tan\gamma_2 = \frac{1-\eta^2}{2D\eta}$

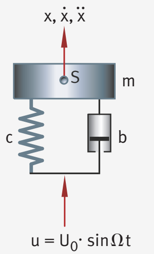
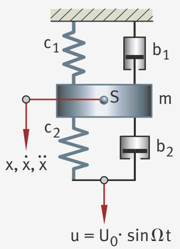
Continuation of table, see Page 171.

Continuation of table, Damped vibration caused by external excitation, from Page 170.

Oscillator schematic	Differential equation and amplitude	Phase angle
Excitation function: $\alpha = \alpha_0 \cdot \sin \Omega t$	**Solution function:** $x = x_{max} \cdot \sin(\Omega t + \alpha)$	
	**Approach:** $-(m_1+m_2)\cdot\ddot{x}-b\cdot\dot{x}-c\cdot x-m_2\cdot\ddot{u}=0$ **Differential equation:** $(m_1+m_2)\cdot\ddot{x}+b\cdot\dot{x}+c\cdot x = m_2\cdot r\cdot\Omega^2\sin\Omega t$ **Amplitude:** $x_{max} = \frac{m_2\cdot r}{m_1+m_2}\cdot\frac{\eta^2}{\sqrt{\left(1-\eta^2\right)^2+4D^2\eta^2}}$ $x_{max} = \frac{m_2\cdot r}{m_1+m_2}\cdot V_1$	$\alpha = -\epsilon_1$ $\tan\epsilon_1 = \frac{2D\eta}{1-\eta^2}$
	**Approach:** $-m\cdot\ddot{x}-b\cdot\dot{x}-c\cdot x+c\cdot u=0$ **Differential equation:** $m\cdot\ddot{x}+b\cdot\dot{x}+c\cdot x = c\cdot U_0\cdot\sin\Omega t$ **Amplitude:** $x_{max} = U_0\cdot\frac{1}{\sqrt{\left(1-\eta^2\right)^2+4D^2\eta^2}}$ $x_{max} = U_0\cdot V_3$	$\alpha = -\epsilon_3$ $\tan\epsilon_3 = \frac{2D\eta}{1-\eta^2}$
	**Approach:** $-m\cdot\ddot{x}-b\cdot\dot{x}-c\cdot x+b\cdot\dot{u}=0$ **Differential equation:** $m\cdot\ddot{x}+b\cdot\dot{x}+c\cdot x = b\cdot U_0\cdot\Omega\cdot\cos\Omega t$ **Amplitude:** $x_{max} = U_0\cdot\frac{2D\eta}{\sqrt{\left(1-\eta^2\right)^2+4D^2\eta^2}}$ $x_{max} = U_0\cdot V_2$	$\alpha = \gamma_2 = \frac{\pi}{2}-\epsilon_3$ $\tan\gamma_2 = \frac{1-\eta^2}{2D\eta}$

Continuation of table, see Page 172.

Continuation of table, Damped vibration caused by external excitation, from Page 171.

Oscillator schematic	Differential equation and amplitude	Phase angle
Excitation function: $\alpha = \alpha_0 \cdot \sin \Omega t$	**Solution function: $x = x_{max} \cdot \sin(\Omega t + \alpha)$**	
	**Approach:** $-m \cdot \ddot{x} - b \cdot \dot{x} - c \cdot x + b \cdot \dot{u} + c \cdot u = 0$ **Differential equation:** $m \cdot \ddot{x} + b \cdot \dot{x} + c \cdot x = b \cdot U_0 \cdot \Omega \cdot \cos \Omega t + c \cdot U_0 \cdot \sin \Omega t$ **Amplitude:** $x_{max} = U_0 \cdot \sqrt{\dfrac{1 + 4 D^2 \eta^2}{\left(1 - \eta^2\right)^2 + 4 D^2 \eta^2}}$ $x_{max} = U_0 \cdot V_{2,3}$	$\alpha = -\epsilon_{2,3}$ $\tan \epsilon_{2,3}$ $= \dfrac{2 D \eta^3}{1 + \eta^2 \left(4 D^2 - 1\right)}$
	**Approach:** $-m \cdot \ddot{x} - (b_1 + b_2) \cdot \dot{x} - (c_1 + c_2) \cdot x + b_2 \cdot \dot{u} + c_2 \cdot u = 0$ **Differential equation:** $m \cdot \ddot{x} + (b_1 + b_2) \cdot \dot{x} + (c_1 + c_2) \cdot x$ $= b_2 \cdot U_0 \cdot \Omega \cdot \cos \Omega t + c_2 \cdot U_0 \cdot \sin \Omega t$ **Amplitude:** $x_{max} = \dfrac{c_2 \cdot U_0}{c_1 + c_2} \cdot \sqrt{\dfrac{1 + 4 D^2 \eta^2}{\left(1 - \eta^2\right)^2 + 4 D^2 \eta^2}}$ $x_{max} = \dfrac{c_2 \cdot U_0}{c_1 + c_2} \cdot V_{2,3}$	$\alpha = -\epsilon_{2,3}$ $\tan \epsilon_{2,3}$ $= \dfrac{2 D \eta^3}{1 + \eta^2 \left(4 D^2 - 1\right)}$

Amplification function

The excitation of damped vibrations can be achieved in different forms:

- direct force, indirect spring force and mass force excitation

Figure 8
Amplification function V_3

η = circular frequency ratio for V_3, ϵ_3

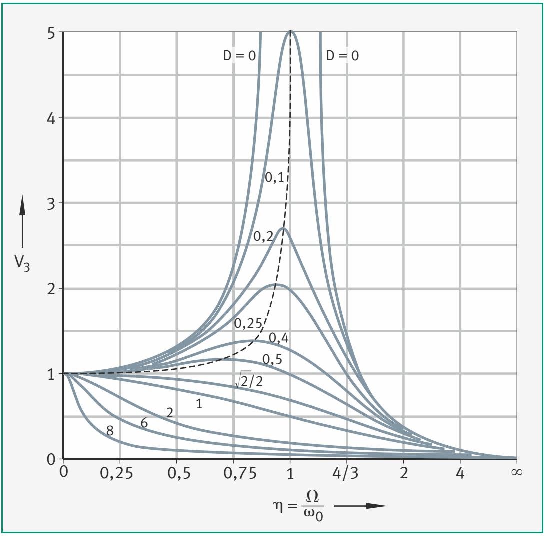
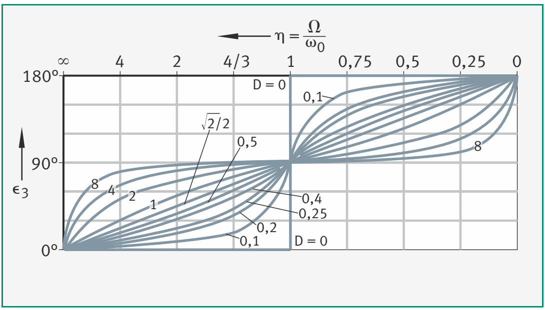


Figure 9
Phase displacement angle ϵ_3

η = circular frequency ratio



- damping force excitation

Figure 10
Amplification function V_2

η = circular frequency ratio

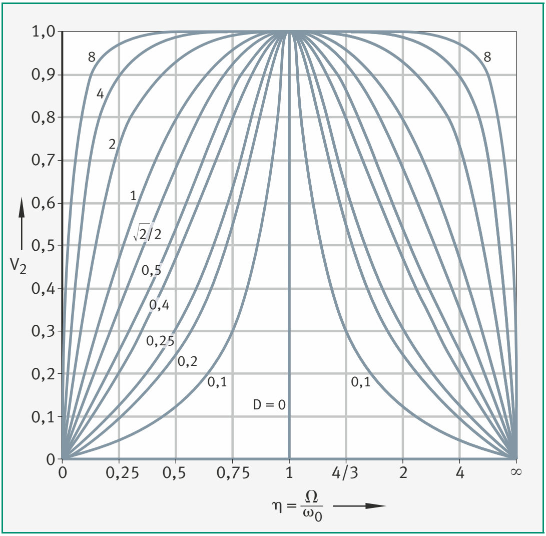
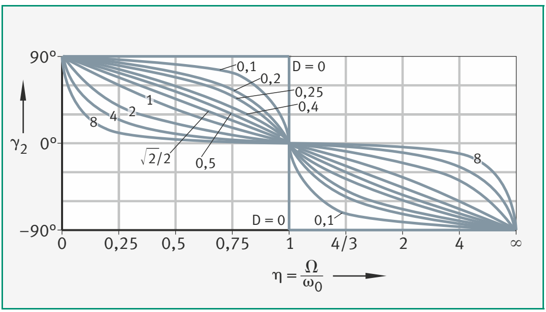


Figure 11
Phase displacement angle γ_2

η = circular frequency ratio



- spring and damping force excitation.

Figure 12
Amplification function
$V_{2,3}$

η = circular frequency ratio

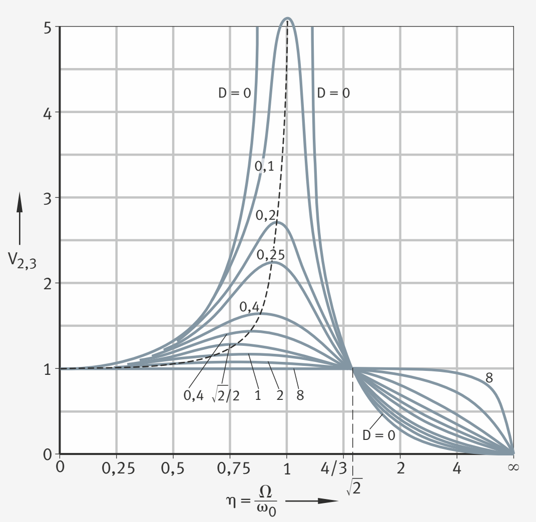
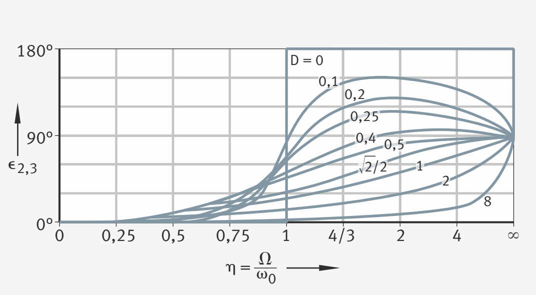


Figure 13
Phase displacement angle
$\epsilon_{2,3}$

η = circular frequency ratio



Acoustics (sound technology)

Sound, sound pressure and sound level

Mechanical vibrations with frequency components within the human hearing range from 20 to 20 000 Hz are described as **audible sound**. Sound at lower or higher frequencies is described as infrasound or ultrasound respectively.

Furthermore, the following terms are used depending on the medium transmitting the sound:

- airborne sound = vibrations in air and gases
- liquid-borne sound = vibrations in liquids
- structure-borne sound = vibrations in solid bodies.

In air and other gases as well as in liquids, sound only propagates in the form of compression waves. The alternating pressure p(t) that is superposed on the static pressure is described as **sound pressure**. The sound pressure is the most important measurement value in these media and is measured by means of microphones or pressure sensors.

For structure-borne sound, the most important measurement value is the vibration velocity v(t) or structure-borne sound speed vertical to the radiating surface of a sound generator.

In general, the acceleration a(t) is measured using piezoelectric quartz sensors and then converted to structure-borne sound speed:

Equation 18

$$a(t) = dv(t)/dt$$

For a frequency band with the centre frequency f, the speed is used as the RMS:

Equation 19

$$\bar{v}(f) = \bar{a}(f)/2 \cdot \pi \cdot f$$

The speed is normally stated in relative terms as a speed level L_v, using the reference value $v_0 = 5 \cdot 10^{-8}$ m/s:

Equation 20

$$L_v = 10 \cdot \lg(\bar{v}/v_0)^2 = 20 \cdot \lg(\bar{v}/v_0) \qquad \text{in dB}$$

The following table shows some values for the acoustic perception of the human ear.

Perception	Volume phon	Sound pressure N/m^2	Sound power W	Sound intensity W/m^2
Auditory threshold[1)]	0 – 10	$2 \cdot 10^{-5}$ [2)]	10^{-12}	10^{-12}
Conversation	50 – 60	0,2	$\approx 10^{-3}$	$\approx 10^{-3}$
Pain threshold	130	20	$\approx 10^{3}$	$\approx 10^{3}$

1) The lowest volume level that is perceptible by the human ear.

2) Reference sound pressure: $p_0 = 2 \cdot 10^{-5}$ N/m^2, internationally defined reference value for the RMS of sound pressure.

Sound level, noise situation and perception

Some example values for the sound level in certain noise situations and the resulting levels of perception are as follows:

Sound level dB(A)[1)]	Noise situation	Perception
0 0 – 10 10 – 20	Complete silence, start of the auditory range Auditory threshold Rustling of leaves	Calm
30 40 50	Whispered speech Quiet radio music Upper limit for mental work requiring concentration	Faint
50 – 70 75	Office work, people talking Start of a disruptive influence on the nervous system	Moderately loud
80 85	Heavy traffic, limit of hearing recovery Start of danger to hearing	Very loud
90 90 – 100	Truck driving noise Car horn	Extremely loud
110 110 – 120	Pneumatic drill Large forging hammer	Unbearable
130 140	Jet aircraft (100 m), pain threshold Rocket launch	Painful

1) At a frequency of 1 000 Hz, the sound pressure level in dB is equal to the volume level in phon.
Harmful limits: 90 phon temporarily, approx. 75 phon for long periods.

Values and units The following table shows a selection of values and names used in sound technology.

Name		Value	Unit	Explanation
Sound velocity	Solid materials	$c_L = \sqrt{\frac{2\,G\,(1-\nu)}{\rho\,(1-2\,\nu)}}$	m/s	Longitudinal waves in large bodies
		$c_T = \sqrt{G/\rho}$		Transversal waves in large bodies
		$c_D = \sqrt{E/\rho}$		Dilatational waves in bars; steel: 5 000 m/s
	Liquids	$c = \sqrt{K/\rho}$		Water: 1485 m/s
	Gases	$c = \sqrt{\kappa \cdot R \cdot T}$		Air: 331 m/s (at 1 bar, 0 °C) Hydrogen: 1 280 m/s (at 1 bar, 0 °C)
Sound speed		$v = a_0 \cdot \omega$ $v = a_0 \cdot 2 \cdot \pi \cdot f$	m/s	Alternating speed of oscillating particles in the direction of vibration
Sound pressure		p	N/m^2 μbar	Alternating pressure caused by sound vibration
Sound power		P	W	Sound energy that passes through a defined surface per unit of time
Sound intensity, sound strength		$I = P/A$ $= p^2/(c \cdot \rho)$	W/m^2	Sound power per unit of area perpendicular to the direction of propagation
Sound level		$L = 10 \cdot \lg (P/P_0)$ $= 10 \cdot \lg (I/I_0)$ $= 20 \cdot \lg (p/p_0)$	Bel B, dB	Logarithmic measure of sound pressure 0 ... 140 dB; $P_0 = 10^{-12}$ W $I_0 = 10^{-12}$ W/m^2 $p_0 = 2 \cdot 10^{-5}$ N/m^2

Continuation of table, see Page 179.

Legend

a_0 Amplitude	m	A Area	m^2
f Frequency	Hz	E Modulus of elasticity	Pa
ρ Density	kg/m^2	G Modulus of rigidity	Pa
κ Isentropic exponent		P Power	W
K Modulus of compression	Pa	R Gas constant	J/(kg · K)
ν Poisson's ratio		T Absolute temperature.	K

Continuation of table, Values and units, from Page 178.

Name	Value	Unit	Explanation
Volume	$\Lambda = 10 \cdot \lg (I/I_0)$ at 1000 Hz	phon	Measure of the subjective perception of sound intensity for the ear
Sound absorption coefficient	$\alpha = (P_a - P_r)/P_r$ $\alpha = (p_a^2 - p_r^2)/p_r^2)$ Index: a = incident r = reflected	1	Measure of the conversion of sound energy into heat as a result of friction for 500 Hz; concrete: 0,01 glass: 0,03 slag wool: 0,36
Sound reduction index	$R = 10 \cdot \lg (I_1/I_2)$ Index: 1 = this side of the wall 2 = the other side of the wall	dB	Logarithmic measure of the sound reduction achieved by a wall; sheet steel, 1 mm: R = 29 dB
Acoustic efficiency	$\eta = P_{acou}/P_{mech}$	1	Ratio of acoustic to mechanical power

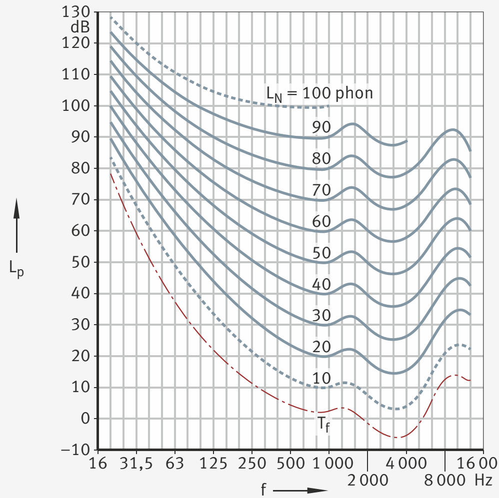
Legend P W Power.

Normal curves for identical volume level

The following diagram shows the normal curves of identical volume level for pure tones in a free sound field in accordance with DIN ISO 226 (April 2006).

Figure 14
Normal curves of identical volume level for pure tones in a free sound field

L_p = sound pressure level
f = frequency
L_N = volume level of a pure tone
T_f = auditory threshold



Hydraulics and pneumatics

Hydraulics

Hydraulic transmissions

Hydraulic transmissions contain pumps, motors and control elements (hydraulic valves) interconnected in a circuit in which power is transmitted by means of circulating hydraulic fluid. The circuit can be of an open or closed design. The controller defines the motion and direction of motion, limits the load in the transmission and, where necessary, adjusts the transmission ratio in accordance with the operating conditions.

Hydraulic pumps

Hydraulic pumps are rotary displacement (rotary piston) or stroke displacement (axial piston) machines with a fixed or variable displacement.

In practice, displacement principles are allocated to specific application areas. The permissible continuous operating pressure is defined by the type of displacement element and the resulting load on the drive mechanism. A further significant feature is the chamber design, which covers the chamber shape and the size of the stroke volume in comparison with the machine size.
In the case of the cell cross-sections of rotary displacement machines, which are normally of a rectangular cross-section, it is more difficult to maintain the required gap tolerances. Since internal leakage losses occur as a function of pressure, the scope of application is restricted to low and medium-pressure systems.
Cylindrical fits can be easily achieved. Axial piston machines are therefore required for use in the high and very high pressure range.

Rotary displacement machines

Rotary displacement machines feed the hydraulic fluid with uniform rotation into cells whose volume is cyclically varied by the design of the limiting walls or the penetration of a tooth. The rotary displacement machine also ensures that the inlet and delivery chambers are sealed from each other. An adjustable stroke volume is only realised in single-stroke vane-cell pumps.

Stroke displacement machines Stroke displacement machines are characterised by the separation of the drive mechanism and the delivery chamber. The cyclical variation of the cell size is carried out by means of a linear piston. Adjustment of the stroke volume is possible by intervention in the drive mechanism or the controller. Due to the internal flow reversal of the fluid, the machines require a slide or valve control mechanism between the displacement chamber and the flow paths.

Hydraulic motors Hydraulic motors convert the fluid energy made available to them into mechanical work. Depending on their output drive motion, a distinction is drawn between torque motors, swivel motors with a limited rotation angle and thrust motors (cylinders). In contrast to hydraulic pumps, hydraulic motors have a constant stroke volume. Adjustable machines are only used in exceptional cases.

Torque motors All of the design principles described for rotary displacement machines and slide-valve controlled axial piston machines are suitable for use as torque motors.
They convert the hydraulic power $P_h = \dot{V} \cdot \Delta p$ (minus the leakage power loss $P_{v,\,v} = \dot{V}_v \cdot \Delta p$, the hydraulic power loss $P_{v,\,h} = \dot{V} \cdot \Delta p_h$ and the mechanical power loss $P_{v,\,r} = M_r \cdot \omega$) into the mechanical motor power $P_m = M \cdot \omega$.

Swivel motors Swivel motors generate swivel motion either directly by swivelling a vane in the subdivided circular cylinder (vane motor with swivel angle of 300°) or by linear movement of a piston by means of a rack and pinion gear.

Thrust motors In the case of thrust motors, a distinction is drawn between single action designs (plunger cylinders) and double action designs (differential cylinders). Differential cylinders can be used for thrust and pull operation by means of alternate piston loading.

Hydraulic pumps

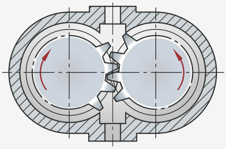
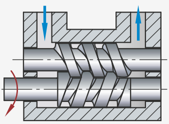
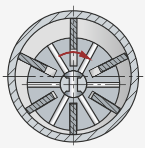
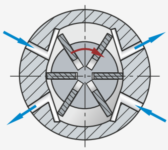
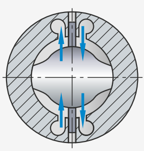
The following section presents the values, units and relationships for hydraulic pumps, as well as common hydraulic pumps and their normal operating values.

Values, units and relationships

The relationships for hydraulic pumps can also be applied analogously to the inverse energy transformation process in hydraulic motors.

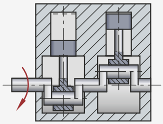
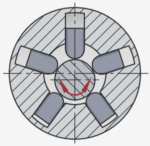
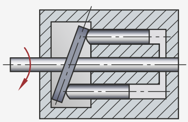
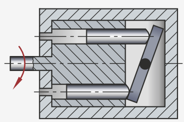
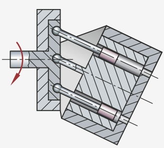
Value	Unit	Name	Relationship, comments
V_H	m^3	Stroke volume = displacement volume (normally stated in cm^3/rev)	The displacement volume is determined from the geometrical data for the pump
$\dot{V}_{th}$	m^3/s	Theoretical delivery rate (based on the assumption of complete filling of the stroke volume during intake)	$\dot{V}_{th} = n \cdot V_H = \omega \cdot V_0$ n = speed $\omega = 2 \cdot \pi \cdot n$ $V_0 = V_H/(2 \cdot \pi)$ base volume
M_{th}	Nm	Theoretical pump moment	$M_{th} = \Delta p \cdot V_H/(2 \cdot \pi)$ $= \Delta p \cdot V_0$
M	Nm	Theoretical drive torque of the pump	Torque provided by the drive unit to the pump shaft
M_r	Nm	Frictional torque within the pump	Friction in the drive mechanism and between the displacement elements
P_m	W	Mechanical drive power of the pump	$P_m = M \cdot \omega$ $P_m = P_{th} + P_{v,r} + P_{v,h}$
$P_{v,r}$	W	Frictional power loss of the pump	$P_{v,r} = M_r \cdot \omega$
P_u	W	Displacement power	$P_u = (M - M_r) \cdot \omega$ The displacement power is transferred to the displacement volume flow and divided up into the displacement power P_{th} in relation to Δp and the hydraulic power loss $P_{v,h}$
P_{th}	W	Displacement power in relation to Δp	$P_{th} = M_{th} \cdot \omega$
$P_{v,h}$	W	Hydraulic power loss	$P_{v,h} = \dot{V}_{th} \cdot \Delta p_h = M_h \cdot \omega$
$\eta_{h,m}$	–	Mechanical-hydraulic efficiency	$\eta_{h,m} = \frac{P_{th}}{P_m} = 1 - \frac{P_{v,r} + P_{v,h}}{P_m}$
$\dot{V}$	m^3/s	Actual delivery rate	$\dot{V} = \dot{V}_{th} - \dot{V}_v$ The pressure differential Δp causes a leakage flow $\dot{V}_v$ through the gap that reduces the displacement volume flow
η_v	–	Volumetric efficiency	$\eta_v = \frac{P_h}{P_{th}} = 1 - \frac{P_{v,v}}{P_{th}} = 1 - \frac{\dot{V}_v}{\dot{V}_{th}}$
η	–	Overall efficiency	$\eta = \frac{P_h}{P_m} = 1 - \frac{\sum P_v}{P_m}$ $\eta = \eta_{h,m} \cdot \eta_V$

Rotary displacement machines The following table shows an overview of types of common rotary displacement machines and their normal operating values.

Displace-ment element	Name		Displacement volume cm^3/rev	Pressure range bar	Speed 1/min	Favourable oil viscosity 10^{-6} m^2/s
Gear	Gear pump 		0,4 ... 1 200	... 200	1 500 ... 3 000	40 ... 80
	Screw pump 		2 ... 800	... 200	1 000 ... 5 000	80 ... 200
Vane	Vane pump	Single stroke 	30 ... 800	... 100	500 ... 1 500	30 ... 50
	Vane pump	Multiple stroke 	3 ... 500	... 160 (200)	500 ... 3 000	30 ... 50
	Rotary piston pump 		8 ... 1 000	... 160	500 ... 1 500	30 ... 50

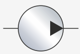
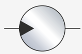
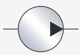
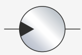
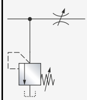
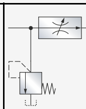
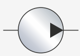
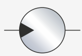
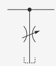
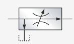
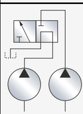
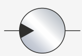
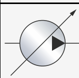
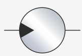
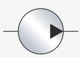
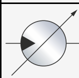
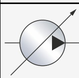
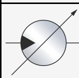
Source: Dubbel (with slight design modifications).

Stroke displacement machines The following table shows an overview of types of common stroke displacement machines and their normal operating values.

Displace-ment element	Name	Displacement volume cm^3/rev	Pressure range bar	Speed 1/min	Favourable oil viscosity $10^{-6}\ m^2/s$
Piston	In-line piston pump 	... 800	... 400	1000 ... 2 000	20 ... 50
	Radial piston pump with internal piston support 	0,4 ... 15 000	... 630	1000 ... 2 000	20 ... 50
	Axial piston pump or swash plate pump 	1,5 ... 3 600	... 400	500 ... 3 000	30 ... 50
	Rear swash plate pump 				
	Bent axis piston pump 				

Source: Dubbel (with slight design modifications).

Hydrostatic stationary transmissions Hydrostatic stationary transmissions can be subdivided according to some of their characteristic features as follows:

Transmission type	Displacement machines (fixed or adjustable stroke volume)		Speed transmission ratio i_G (constant or adjustable, dependent on load or independent of load)	Open loop or closed loop control of speed transmission ratio		Torque trans-mission ratio μ_G
	Pump	Motor		Open loop	Closed loop	
I Figure 1, Page 186			Constant, independent of load	Not possible		Constant
II Primary flow throttle transmission Figure 2, Page 186			i_G adjustable, dependent on load in open loop control, independent of load in closed loop control			–
III Secondary flow throttle transmission Figure 3, Page 186			i_G adjustable, dependent on load in open loop control, independent of load in closed loop control			Constant
IV			Adjustable in steps, independent of load	Connection of a machine	–	–
V Figure 4, Page 186			i_G adjustable, speed transmission ratio is independent of the load of the hydraulic motor	Open loop or closed loop control through adjustment of the stroke volume of one or both (VII) displacement machines		–
VI Figure 4, Page 186						–
VII Figure 4, Page 186						–

For corresponding diagrammatic examples, see Page 186.

The following diagrams show examples of the transmission types I to VII for hydrostatic stationary transmissions.

The following image shows an **open circuit**:
the hydraulic pump and hydraulic motor are not adjustable.

Figure 1
Transmission type I

① Drive unit
② Operating unit
③ Hydraulic pump
④ Hydraulic motor

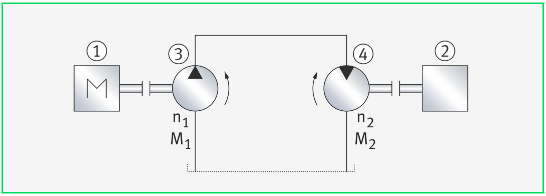


Figure 2
Transmission type II
Primary flow throttle transmission

① Hydraulic pump
② Hydraulic motor

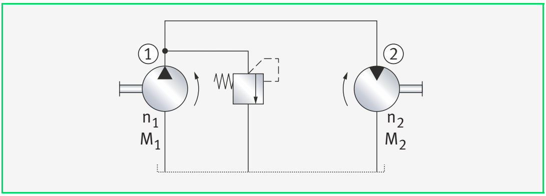
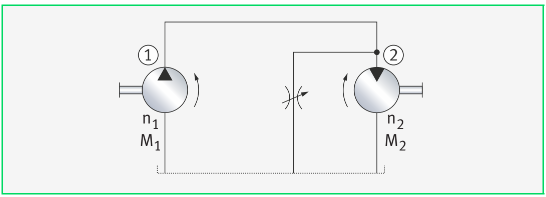


Figure 3
Transmission type III
Secondary flow throttle transmission

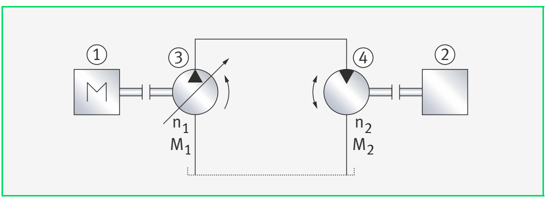
① Hydraulic pump
② Hydraulic motor



The following image shows a **closed circuit**:
the hydraulic pump is adjustable and reversible, the hydraulic motor is not adjustable.

Figure 4
Transmission type V to VII

① Drive unit
② Operating unit
③ Hydraulic pump
④ Hydraulic motor



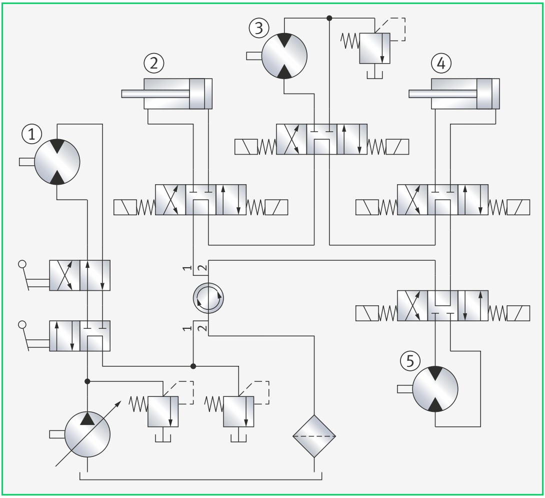
Hydraulic oil systems

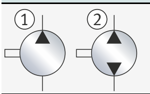
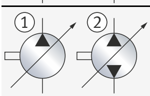
The following tables give an overview of symbols and names in hydraulic oil systems. The hydraulic symbols correspond to DIN ISO 1219-1 (December 2007) “Fluid power systems and components – Graphic symbols for conventional use and data-processing applications”.

The associated diagram shows an example of a complete, hydraulic oil system. The idle position of the system is always shown.

Figure 5
Industrial crane

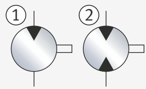
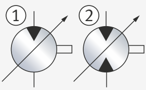
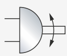
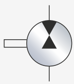
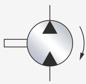
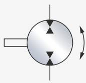
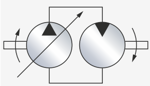
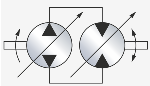
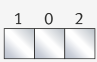
① Travel
② Whipping
③ Lift
④ Push
⑤ Rotate



Symbol	Name and explanation
Hydraulic pump	
	**Pump** With constant displacement volume ① With one flow direction ② With two flow directions
	**Pump** With variable displacement volume ① With one flow direction ② With two flow directions

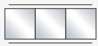
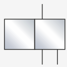
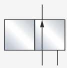
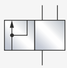
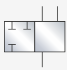
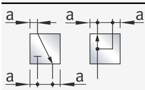
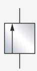
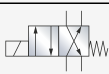
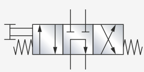
Continuation of table, see Page 188.

Continuation of table, Hydraulic oil systems, from Page 187.

Symbol	Name and explanation
Hydraulic motor	
	**Motor** With constant displacement volume 1 With one flow direction 2 With two flow directions
	**Motor** With variable displacement volume 1 With one flow direction 2 With two flow directions
	**Swivel motor** (with restricted swivel angle)
Hydraulic pump – hydraulic motor	
	**Pump motor** With constant displacement volume As pump in one flow direction As motor in the opposing direction
	**Pump motor** With constant displacement volume As pump or motor in one flow direction
	**Pump motor** With constant displacement volume As pump or motor in two flow directions
Compact hydraulic transmission	
	**Transmission** For one direction of output rotation with adjustment and constant motor for one delivery direction
	**Transmission** For two directions of output rotation with variable pump and variable motor for two delivery directions
Hydraulic valves (general)	
	The **valve** is represented by a square or rectangle
	Number of fields = number of valve settings, where the neutral position is arranged on the right if there are two boxes

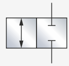
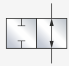
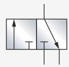
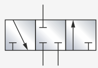
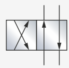
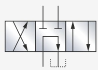
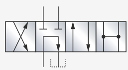
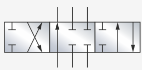
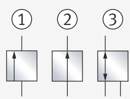
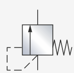
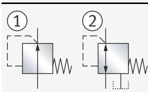
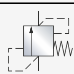
Continuation of table, see Page 189.

Continuation of table, Hydraulic oil systems, from Page 188.

Symbol	Name and explanation
Hydraulic valves (general)	
	In the case of valves with a constant functional transition between the switching positions, the boxes are framed by two lines above and below
	The ports, respectively inlet and outlet, are attached to the neutral position box
	Within the boxes, the lines and arrows indicate the direction of flow
	A connection between two paths within a valve is indicated by a dot. Where lines cross but a dot is not present, this indicates that the paths are not connected to each other
	Closed ports are indicated by perpendicular bars
	The respective positions of the paths and arrows (angled or straight) within the boxes correspond to the positions of the ports
	If a position is changed and the inlet or outlet remains connected to a port, the arrow is displaced relative to the ports
Hydraulic valve actuation	
The symbols for the actuation modes and auxiliary elements are arranged perpendicular to the ports and outside of the rectangle (for further actuation modes, see Actuation and drive modes, Page 195). The valves are drawn in the current-free starting position.	
	**$^4/_2$-way valve** (valve with 4 connections and 2 switching positions) With electromagnetic actuation and spring return
	**$^4/_3$-way valve** (valve with 4 connections and 3 switching positions) With manual actuation by pressing or pulling and spring centring at neutral position
Hydraulic directional control valves	
The description as a way valve is preceded by the number of connections and the number of switching positions; e. g. way valve with three controlled connections and two switching positions: 3/2-way valve (vocalised: three stroke two way valve).	

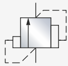
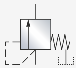
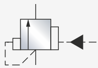
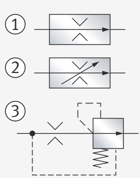
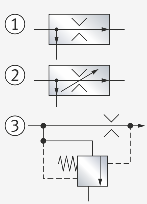
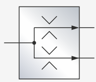
Continuation of table, see Page 190.

Continuation of table, Hydraulic oil systems, from Page 189.

Symbol	Name and explanation
Hydraulic directional control valves	
	**$^{2}/_{2}$-way valve** Locked in neutral position
	**$^{2}/_{2}$-way valve** With free flow in neutral position
	**$^{3}/_{2}$-way valve** Flow shut off in neutral position
	**$^{3}/_{3}$-way valve** With shut-off neutral position, forward and reverse settings
	**$^{4}/_{2}$-way valve** With forward and reverse settings
	**$^{4}/_{3}$-way valve** With recirculating neutral position, forward and reverse settings
	**$^{4}/_{4}$-way valve** As 4/3, but with floating position after forward setting
	**$^{6}/_{3}$-way valve** In neutral position, 1 inlet free, 2 inlets locked
Hydraulic pressure valves	
	**Pressure valve (general)** ① Single edge valve with closed neutral position ② Single edge valve with open neutral position ③ Double edge valve, three controlled ports
	**Pressure relief valve** Pressure limited at inlet by opening the outlet against a return force
	**Pressure control valve** Maintenance of constant outlet pressure ① Without outlet port = pressure reducing valve ② With outlet port = pressure control valve
	**Pressure drop valve** Reduction of outlet pressure by a fixed amount compared to the inlet pressure

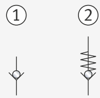
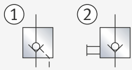
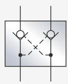
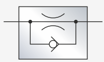
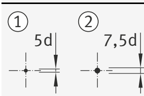
Continuation of table, see Page 191.

Continuation of table, Hydraulic oil systems, from Page 190.

Symbol	Name and explanation
Hydraulic pressure valves	
	**Proportional pressure valve** Reduction of outlet pressure in a fixed ratio to the inlet pressure
	**Sequence valve** Opening of path to further devices upon reaching the inlet pressure defined by the spring force
	**Proportional pressure relief valve** Restriction of inlet pressure to a value proportional to the pilot pressure
Hydraulic flow control valves	
	**Choke** Valve with integral, constant constriction; flow rate and pressure drop are dependent on viscosity
	**Orifice** Sharp-edged constriction, substantially independent of viscosity and effective in both directions
	**Choke valve** Adjustable constriction, effective in both directions
	**2-way flow control valve** 1 2-way flow limiting valve 2 2-way flow adjustment valve 3 Flow control valve maintains constant outlet flow by automatic closure
	**3-way flow control valve** 1 3-way flow limiting valve 2 3-way flow adjustment valve 3 Flow control valve maintains constant outlet flow by automatic opening of outlet (bypass valve)
	**Flow divider** Valves for dividing or combining several outlet or inlet flows. Substantially independent of pressure

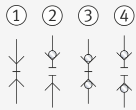
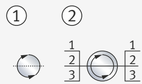
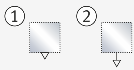
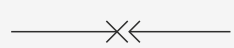
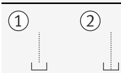
Continuation of table, see Page 192.

Continuation of table, Hydraulic oil systems, from Page 191.

Symbol	Name and explanation
Hydraulic shut-off valves	
	**Shut-off valves** Shut-off of flow in one direction and release of flow in the opposing direction ① Check valve: shut-off if the outlet pressure is greater than the inlet pressure, ② Shut-off if the outlet pressure is greater than or equal to the inlet pressure (with spring)
	**Check valve** ① Shut-off can be deactivated ② Flow can be shut off
	**Delockable double check valve** With 2 check valves for 2 separate flows, automatic locking of which is alternately deactivated by the inlet pressure
	**Choke check valve** With flow in one direction and choke in the other direction
Hydraulic pipelines and accessories	
	**Working pipeline** Pipeline and energy transfer
	Control pipeline, oil leakage pipeline, venting pipeline, flushing pipeline For transfer of control energy, for setting and control, for discharge of leaking fluids
	**Flexible pipeline** Pipeline being flexible in operation, rubber hose, corrugated tube etc.
	**Pipeline connection** Rigid connection, for example welded, soldered or connected by screws (including fittings) ① Within a symbol ② Outside a symbol

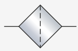
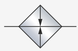
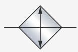
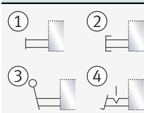
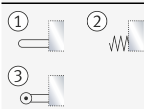
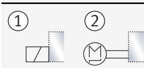
Continuation of table, see Page 193.

Continuation of table, Hydraulic oil systems, from Page 192.

Symbol	Name and explanation
Hydraulic pipelines and accessories	
	**Pipeline crossover** Crossover of pipelines that are not connected with each other
	**Quick release coupling** ① Connected without mechanically opened check valve ② Disconnected, with one check valve ③ Connected, with two check valves ④ Disconnected, with two check valves
	**Rotating connection** Pipeline connection that can be rotated during operation, for example a pivot ① With one path ② With three paths
	**Electrical line**
	**Outlet** ① Outlet opening ② With thread for pipe connector
	**Pressure port** Point for any connection of a device
	**Blind port** Marking for a closed port on a device or pipeline
	**Reservoir** Open, connected to the atmosphere ① With pipe end above the fluid level ② With pipe end below the fluid level
	**Hydraulic reservoir** For storage of hydraulic energy

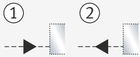
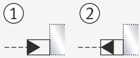
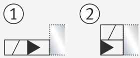
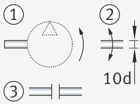
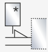
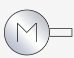
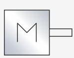
Continuation of table, see Page 194.

Continuation of table, Hydraulic oil systems, from Page 193.

Symbol	Name and explanation
Hydraulic pipelines and accessories	
	**Filter or mesh** For separation of contaminant particles
	**Heating system** Arrows indicate the supply of heat
	**Cooling system** Arrows indicate the dissipation of heat
	**Manometer**
	**Thermometer**
	**Pressure switch** Electromechanical, adjustable
	**Shut-off valve**
Actuation and drive modes	
	**Manual actuation modes** 1 General 2 By pressing 3 By lever 4 By pedal with detent
	**Mechanical actuation modes** 1 By plunger 2 By spring 3 By sensing roller
	**Electric actuation** 1 Electromagnet 2 Electric motor

Continuation of table, see Page 195.

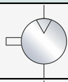
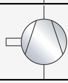
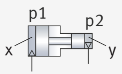
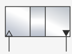
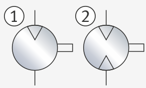
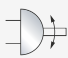
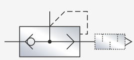
Continuation of table, Hydraulic oil systems, from Page 194.

Symbol	Name and explanation
Actuation and drive modes	
	**Pressure actuation (direct)** ① By application of pressure ② By withdrawal of pressure
	**Pressure actuation (indirect)** ① By application of pressure on control line ② By withdrawal of pressure on control line
	**Combined actuation** ① Electromagnet and pilot directional valve ② Electromagnet or pilot directional valve
	**Shaft** ① Pump drive shaft in one direction of rotation (right hand rotation from viewpoint of shaft end) ② In two directions of rotation ③ With shaft coupling
	**Detent** Maintenance of an engaged position
	**Locking device** Locking of a position or direction
	**Snap catch** Device passes beyond a dead point
	**Pressure source** Hydraulic energy
	**Pressure source** Pneumatic energy
	**Electric motor** with almost constant speed
	**Heat engine**

Pneumatics

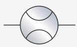
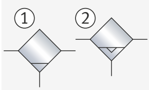
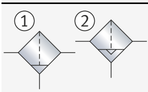
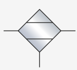
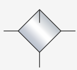
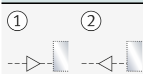
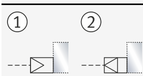
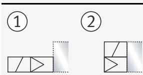
Pneumatic systems

The following table gives an overview of symbols and names in pneumatic systems.

Symbol	Name and explanation
Pneumatics – compressor	
	**Compressor** With constant displacement volume, only one flow direction
	**Vacuum pump** For removing gaseous media from a low pressure area
	**Pressure converter** Comprising two different pressure chambers x and y
	**Pressure medium converter** Changeover from one pressure medium to another while maintaining the same pressure
Pneumatics – motor	
	**Pneumatic motor** With constant displacement volume 1 With one flow direction 2 With two flow directions
	**Swivel motor**
Pneumatic valve	
	**Quick ventilation valve** Shut-off valve in which the outlet line is vented to the outside when the inlet line is in the vented stage (with noise damper in the outlet pipeline)
The remaining symbols are identical to those used for hydraulic oil systems	

Continuation of table, see Page 197.

Continuation of table, Pneumatic systems, from Page 196.

Symbol	Name and explanation
Pneumatic pipelines and accessories	
	**Noise damper** For reduction of resulting noise
	**Compressed air reservoir**
	**Flow measurement device** (also for use with hydraulic systems) Volume flow measurement device
Maintenance unit	
	**Water separator** Separation and drainage of condensation water from the system ① Manually actuated ② Automatic drainage
	**Filter** With water separator ① Manually actuated ② Automatic drainage
	**Outlet** without pipe connector
	**Dryer** For drying of air by means of chemicals
	**Oiler** For adding a small quantity of oil to the air flow
Actuations	
	**Pressure actuation (direct)** ① By application of pressure ② By withdrawal of pressure
	**Pressure actuation (indirect)** ① By application of pressure ② By withdrawal of pressure
	**Combined actuation** ① Electromagnet and directional valve ② Electromagnet or directional valve

Mechatronics

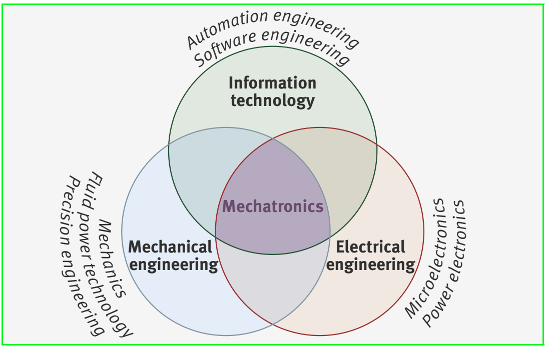
Definition

The development and production of mechatronic products involves an integrated partnership between the disciplines of mechanics, hydraulics, pneumatics, electrical engineering and electronics, control technology and information technology.

While mechatronics was defined in its earliest days (approx. 1970) as simply the integration of mechanics and electronics, the term now has a wider scope:
"Mechatronics is a branch of engineering which seeks to enhance the functionality of technical systems by close integration of mechanical, electronic and data processing components."
(Bosch, Automotive Handbook 1995).

A similar definition was issued by the IEEE/ASME in 1996 and was also used as a basis for the VDI Guideline 2206 "Design methodology for mechatronic systems":
"Mechatronics is the synergetic integration of mechanical engineering with electronic and intelligent computer control in the design and manufacturing of industrial products and processes."

Figure 1
Mechatronics as intersection between various technical disciplines
Source: VDI Guideline 2206

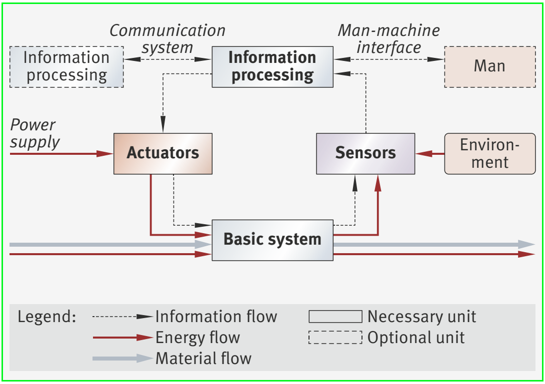


Mechatronics can therefore achieve synergies through the integration of mechanical engineering, electrical engineering and information technology, leading to improvements in the functionality of technical systems.

The spectrum of mechatronic products extends from simple implements through household appliances ("white goods, brown goods"), automotive engineering and medical technology to aviation and aerospace engineering.

If the mechatronic system is regarded as a "black box", it can be represented as follows as a reference architecture:

Figure 2
Reference architecture of a mechatronic system
Source: VDI Guideline 2206



The input and output values are material, energy and signal flows. The technical component, the basic system, represents the supporting basis system in practical terms and comprises elements taken from mechanics, electrical engineering, hydraulics and pneumatics. It is controlled on the open loop or closed loop principle by means of the information processing component. The sensors convert the signals from the technical component into information elements for the information processing component and the actuators intervene by modifying the parameters of the technical component.

Depending on the complexity of the system, the following tasks can be undertaken in technical systems:

- open loop control, closed loop control
- monitoring and error diagnosis
- coordination
- management.

They thus cover the entire spectrum ranging from reactive to cognitive activities.

Motivation and indicators for use

The motives and reasons for the implementation of mechatronic systems can be as follows:

- functional transfer, in other words the distribution at optimum cost of the main function to various disciplines
- the implementation of new functions
- increasing the precision of motion under real time conditions
- robustness against mechanical malfunctions
- capability of adaptation to changes in environmental conditions
- autodiagnosis, autocorrection
- improvement of operational security
- increasing autonomy
- automatic learning
- compensation of mechanical deficiencies.

In spite of the advantages stated, the blind use of components from different technical disciplines can lead to a more complicated problem and a more expensive solution. Mechatronic solutions should be applied in such a way that they give an increased benefit.

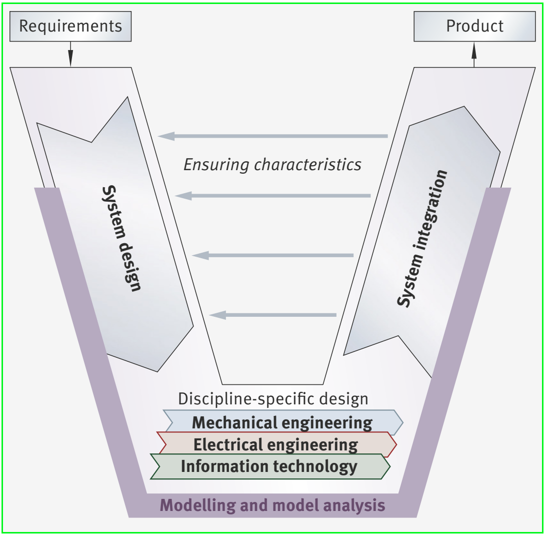
Development process for mechatronic systems

The development of mechatronic systems requires ways of thinking and working that are holistic and interdisciplinary.

Since the development methods and processes in the individual disciplines will demonstrate some similarities but will also show differences in decisive respects, it is vital to achieve good communication in the interdisciplinary team in order to ensure an effective method of working across the disciplines. A decisive role is played here by the issues of model building and orientation to the functional concept.

In VDI Guideline 2206, a process model in a V shape is presented for the development of mechatronic systems. It can be interpreted as a reference model that is used to propose an iterative process in design for ensuring product characteristics.

Figure 3
V model for development of mechatronic systems
Source: VDI Guideline 2206



In relation to the quality of the resulting products, it has become clear that there are three definitive aspects: the integration of the technical disciplines, a holistic approach and correct modelling (as a basis for simulation of the complete system).

It is important to prepare a process landscape in which mechatronic engineers can apply the most suitable method. Employees in such a team must be able to “see the big picture” and see themselves as system architects who can, as necessary, draw on the competences of the technical specialists.

It is important that powerful software tools are provided that can be used, by means of simulations, to secure the necessary characteristics in the various stages of development.

Examples of mechatronic systems

ABS system (antilock braking system)

This system ensures, through the interaction of mechanics, electronics and software, that a vehicle retains steerability and driveability under braking. Locking of the wheels is prevented.

The vehicle is the complete system and also acts as the supporting technical component (basic system). The electronic control unit is the information technology component. Each wheel contains sensors for measurement of speed and actuators to actuate the braking cylinder. The main cylinder is a central actuator.

ESP system (vehicle dynamics control)

Electronic stability programs are – like ABS systems – practically standard equipment in modern cars. They ensure stable and thus safe driving around bends, since braking is applied in a targeted manner to the individual wheels in order to counteract swerving of the vehicle.

In order to maintain the directional stability of the vehicle in all circumstances, the current state of the vehicle is detected by means of sensors measuring the wheel speed, yaw rate, lateral acceleration, steering wheel angle and pre-pressure sensor. An electronic control unit (= information technology component) processes these data and, if necessary, sends control signals to the hydraulic unit (actuator). Without the intervention of the driver, hydraulic pressure can be built up in the brakes for the individual wheels, which can then be decelerated in a targeted manner and independently of each other.

Magnet bearings

These bearings are suitable for extremely high speeds (in applications such as machine tool spindles) where conventional bearing arrangements are susceptible to failure. The rotor is held without contact and without friction in a magnetic field. The bearing forces are held in equilibrium by means of magnetic forces and these are controlled so that the rotor floats in a stable manner.

Mechatronics in rolling bearings

Functional expansion of rolling bearings

Rolling bearings are essentially designed to facilitate rotations about an axis or displacements along an axis with little friction, but prevent certain other movements through the support of forces and moments. In the course of development, bearing components have been designed such that they can take on additional mechanical functions such as guidance, support or mounting, such as in the case of flanged bearings.

The integration of sensors and actuators offers the possibility of expanding even further the functional scope of bearings.

Sensors detect operating parameters and bearing conditions and forward this information, normally in the form of electrical signals.

The values detected include:

- rotational angle (rotary bearings)
- position (linear bearings)
- velocity and rotational speed
- axial and radial force
- torque and tilting moment
- temperature
- lubricant condition
- vibrations, running noise
- wear and damage.

Actuators convert other forms of energy into mechanical energy, giving forces or torques. In combination with a rolling bearing, they can fulfil the following functions:

- driving
- braking, locking
- setting of operating clearance or rigidity
- damping of vibrations
- relubrication.

Inversely, generators can also convert mechanical energy into electrical energy. In many cases, the motion of the bearing can act as a source here.

If the activity of the actuator (such as driving) is coupled with an appropriately selected sensor signal (such as speed), the control loop can be closed and the system state can be influenced in the required direction (stabilisation of speed). The closed loop control is now normally achieved by means of microcontrollers; some of these are so small that they can even be integrated in the bearing. They collect and assess the sensor data and control the actuator in accordance with their programming.

In order that the mechatronic bearing can fulfil its task, the sensor and actuator functions must be matched to each other.

The operating parameters detected are electronically processed in a closed loop control system. Microcontrollers collect these data and initiate actions. If the microprocessor is controlling a motor in a rolling bearing, this can induce acceleration or deceleration in the direction of motion of the bearing.

The objective is to create a mechatronic unit in which the components are optimally matched to each other, in order to make best possible use of the advantages of mechanical and electronic components.

In contrast to solutions comprising individual components that have to be installed retrospectively, mechatronic rolling bearings offer the advantages of integrated solutions, such as reduced design envelope, easier mounting and a smaller number of components.

Sensor bearings

Rolling bearings are generally fitted with sensors in order to fulfil one of the following three objectives:

- detection of actual operating conditions such as loads, shocks and temperature. The detected data are used in order to achieve well-grounded bearing design for identical or similar applications. Such measurement initiatives often prevent unexpected bearing damage or failures. Once the load conditions have been clarified, the design can be adapted. Further measurements are normally no longer required in regular operation
- continuous condition monitoring of the rolling bearing and other adjacent machine elements such as gears. The sensors generally record several measurement values relevant to bearings. Automatic signal assessment is often carried out by means of defined algorithms. If necessary, an optical or acoustic alarm is triggered for the operating personnel
- online measurement of operating data for the open or closed loop control of the system in which the bearing is fitted. Applications range from electric motors (commutation) through machine tools (load calculation), vehicles such as cars (ABS, steer-by-wire), e-bikes (pedal torque measurement) or railways (braking control) to stationary production plant (overload protection) and household appliances.

Matching of measurement values and measurement purpose

Depending on the purpose of the measurement, different physical measurement values are recorded for the bearing. The sensors required for this measurement are selected accordingly. The table shows the relationship between the measurement values and the purpose.

Measurement value	Purpose		
	Design	Condition monitoring	Closed loop control of complete system
Position, velocity	–	–	■
Axial and radial force	■	–	■
Torque and tilting moment	■	–	■
Temperature	■	■	■
Lubricant condition	–	■	–
Vibration, noise	–	■	–
Wear, damage	–	■	–

Examples of measurement methods for some selected values are given below that play a role in rolling bearing technology.

Measurement of position and velocity

Recording of the rotational speed is, due to its use in antilock braking systems in cars, the most widely used form of integrated sensor technology used for rolling bearings. In general, it is used to determine the position and velocity of a moving bearing component relative to a stationary component.

A fundamental distinction is made between:

- absolute positional measurement, where the relative position between fixed reference points is present as a datum at any point in time immediately after switching on, and
- incremental measurement, where primary information is generated on the basis of changes in position and direction.

If an index is added, an incremental measurement system can be developed further in the direction of an absolute measurement system. This requires appropriate signal processing, especially recognition of the direction of motion. An orientation movement is always required after switching on, however, in order to arrive at the index point.

The typical design of a rotary bearing comprises an encoder, the scale, applied to the rotating bearing ring. Its position is measured by one or more sensors connected to the stationary ring. The signal created is transmitted by cable. Such bearings are often greater in width than the original types but match these in terms of the other mounting dimensions and performance data, see Figure 4.

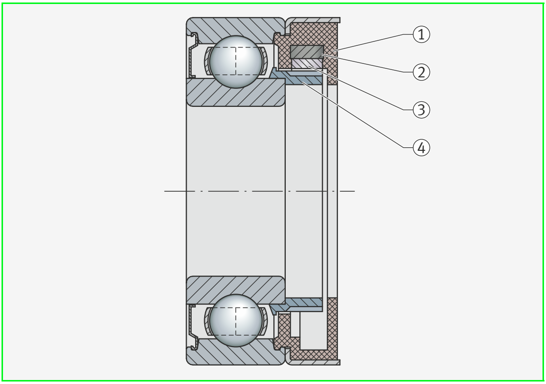


Figure 4
Rotary rolling bearing with measurement system

① Sensor housing
② Magnet
③ Hall IC (sensor)
④ Pulse emitter ring (encoder)

In the case of linear bearings, the encoder is normally applied to the profiled guideway as a linear scale. The sensor is mounted on the carriage. Encoders are used to measure the position and velocity of the carriage on the guideway, see Figure 5.

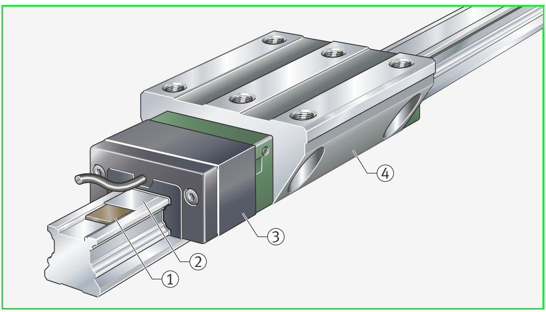


Figure 5
Linear rolling element guidance system with measurement system

① Guideway with integrated linear encoder
② Covering strip
③ Adaptive measuring head
④ Carriage

Operating principles

Sensors may use different operating principles. For rolling bearings, the sensors used so far are optical, inductive, capacitive and magnetic types.

By far the largest number of applications so far have been based on magnetic methods.

Measurement of force and torque

For determining rotational speed, some other components such as the shaft or gear are equally as suitable as the bearing. For measuring force, however, the bearing is often the only machine element that can be used.

Directly measurable forces

Where forces act and are presented with resistance by a rolling bearing, several effects occur that can in principle be used for measurement:

- an increase in the pressures at internal and external contact points
- stresses and thereby elongations at the location of particular volume elements and surface contours
- changes in distance between certain reference points. The bearing undergoes deflection.

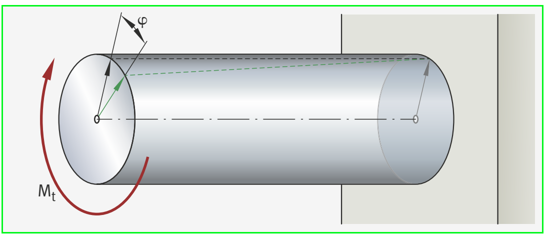
Torque determination

None of these effects occur, however, if a torque is generated along the rotational axis provided by a rotary bearing. Direct torque measurement within a bearing is therefore difficult.

In arrangements with two bearings at both ends of a shaft subjected to torsion by a torque, the difference in the bearing rotational angle can be used as a measurement value, see Figure 6. Gears ultimately convert the torque into reaction forces with the result that, in transmissions, the torque can be determined indirectly from the bearing forces.

Figure 6
Torsion of a shaft

φ = rotational angle
M_t = torsional moment



Selection of the measurement method

For selection of the measurement method, one of the decisive questions is whether it must be possible to measure the forces while the bearing is stationary. If this is not necessary, signals can be used that vary in a periodic manner with the passage of rolling elements and whose amplitude can be used as an indicator of the force. Long term drift and temperature influence have a much less disruptive effect than in the other case.

Measurement of pressures by means of sensor coatings

At the points where forces are transmitted, namely shaft/inner ring, inner ring/rolling element, rolling element/outer ring and outer ring/housing, there is normally no space for sensors unless these are very thin. Piezoelectric and piezoresistive coating systems fulfil this requirement. The former generate electric charges if there is a change in the active force and therefore their thickness; the latter develop a different electric resistance as a function of the specific load. Both methods are suitable for dynamic measurements while piezoresistive systems are preferable for static measurements.

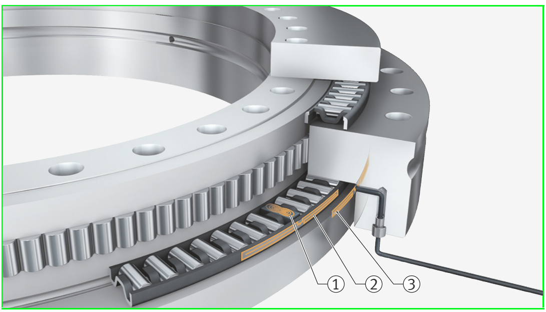
Condition monitoring

In addition to classical condition monitoring, bearings can also be subjected to structural health monitoring. Sensors for non-destructive inspection are integrated in the bearing.

One of these inspection methods is eddy current testing of the raceways, see Figure 7. The probes are located in one or more cage pockets and can check for cracks or spalling during rotation of the inner or outer ring. Supply of energy and transmission of signals is carried out wirelessly.

Figure 7
Inductive damage monitoring of rolling bearings

① Inductive sensor
② Antenna on cage
③ Antenna on outer ring



Actuator bearings

There is a wide range of operating principles that can be integrated in the rolling bearing or at least mounted on the bearing in a space-saving form.

Those with a purely mechanical function include:

- one way clutch for locking of one direction of rotation
- brake disc on the rotating ring and brake calliper on the static ring
- passive damping elements for reducing vibration.

The use of active elements with a thermal or electrical activation function offers further possibilities for equipping bearings with additional functions. Some have already been realised, while others are still at the development stage.

The following table shows the relationship between the actuator principle and the possible applications.

Actuator principle	Application				
	Drive	Braking Locking	Positioning	Damping	Relubrication
Electric motor	■	■	–	–	■
Magnetic coil	–	■	■	–	■
Piezo element	■	■	■	■	■
Magnetostriction	–	■	–	■	–
Rheological liquid	–	■	–	■	–
Shape memory alloy	–	■	■	–	■
Dielectric polymer	–	■	–	–	■

Drive and braking

Where an electric motor is required to generate a particular torque, the rotor and stator must be of an appropriate size. If the bearing required for guidance of the shaft is significantly smaller than the electric motor, integration is not advisable.

Integration of electric motors

For the integration of electric motors, bearings with a relatively large diameter are more suitable if this is determined not as a result of high loads but of other design requirements. Such bearings are, despite their size, not particularly large in section or heavy.

They are used, for example, in conveying equipment and computer tomographs, see Figure 8. If the generation of torque is distributed to the circumference by means of a ring motor, solutions can be realised that give space savings and run quietly.

Figure 8
Bearing for computer tomographs with integrated ring motor

Positioning and damping

These two functions are similar in that only small actuation distances of between 1 μm and 100 μm are necessary, but in some cases considerable forces are involved. The setting of bearing parameters such as rigidity, preload or internal clearance does not require actuators that operate particularly quickly. In many cases, it is only necessary to compensate for temperature influences. In active vibration reduction, the actuator frequency response must however be matched to the excitation spectrum present.

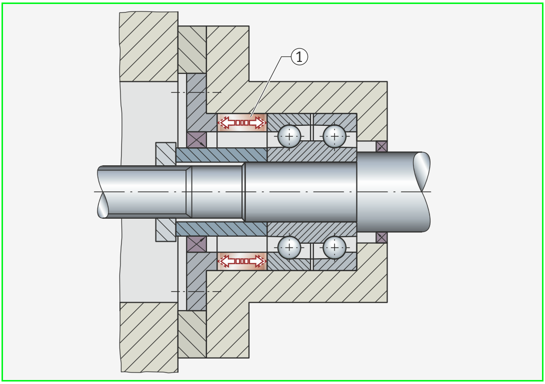
Change in preload

Spindle bearings are characterised by particularly precise shaft guidance. In order to ensure zero backlash, they are preloaded. If the thermal influence is excessively high, there is a danger that the preload will become excessively large, which will thus reduce the operating life.

With an axially active piezo element, the preload can be held constant independently of the temperature, see Figure 9.

Figure 9
Spindle bearing arrangement, preload by means of piezo tensioning elements

① Piezo tensioning element



Energy supply and data transmission

Sensors such as actuators normally require electrical energy in order to function. The rolling bearing can be used here as a source of mechanical energy.

Energy must be supplied, depending on the application, to the following:

- sensors
- signal processing and evaluation units
- cableless data transmitters
- electronic controllers
- actuators.

Data transmission is more difficult to achieve than energy supply. Cables can be used for both but cause problems in mounting in many cases and are easily damaged. Plugs on the bearing are expensive if they are required to fulfil the normal requirements for mechanical stability and leak tightness. There are arguments in favour of wireless data transmission.

Development potential of mechatronic rolling bearings

Although rolling bearings have a history stretching back 120 years, they are still a long way from reaching the end point in their evolution. One branch of this development, the integration of electronic components, has been sketched out in this chapter.

The solutions presented here reflect the current status of development in mechatronic bearings. There is still room for optimisation and possibly also completely new ideas. The rolling bearing will continue to evolve and move from being a component to (also hopefully) a reliable and robust mechatronic system.

Strength of materials

Terms, values and definitions

Strength theory

The strength theory provides us with the basis for calculating the mechanical stress and dimensioning technical designs.

The following two questions are answered here:

- distribution of the inner forces on the cross-sectional surface of a stressed machine part
- the changes in shape that are caused by these cross-sectional values.

The answer to the first question gives the mechanical stress that is acting on the design. The answer to the second question provides information about the elastic deformations of a machine part which are associated with the load, such as changes in length or deflections for example.

Stress in area elements

The external forces and moments acting on a body are balanced by corresponding reaction forces inside the body. If a homogeneous mass distribution is assumed, the occurring inner reaction forces are distributed over a large area. The force density (quotient of internal force and effective area) prevailing in every area element is the **stress**. It mostly changes its size and direction from one point to another.

To describe the stress state in a cross section, the stresses are split:

- **normal stress** σ = components perpendicular to the cross-sectional plane
- **shear stresses** τ = two components in the cross-sectional plane.

If a cross-sectional plane is placed in such a way that both shear stresses become zero, then the normal stress reaches an extreme value, which is referred to as the **principal stress**.

Deformation

Stresses are always linked with deformations. A distinction is made between two kinds of deformation:

- elastic deformations
- plastic deformations.

Elastic deformation

Elastic deformations disappear again once the imposed external load has been removed. They often follow Hooke's Law.

Changes in the length of a line element per unit of length are described as elongations. Length changes are caused by **normal stresses**.

According to Hooke's Law, elongations are proportional to the stresses accompanying them:

Equation 1

$$\sigma = E \cdot \varepsilon \qquad \varepsilon = \text{elongations}$$

Here, the proportionality constant is the material parameter modulus of elasticity E.

Shear stresses result in angle changes. The change in an angle that was originally a right angle is, relative to this, described as torsional shear strain or shear γ.

Torsional shear strain (dislocation) is proportional to the shear stress:

Equation 2

$$\tau = G \cdot \gamma \qquad \gamma = \text{torsional shear strains (dislocation)}$$

Here, the proportionality constant is the material parameter modulus of rigidity (shear) G.

The following relationship exists between the two material parameters:

Equation 3

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \text{Poisson's ratio}$$

Plastic deformation

If the external forces on a component and thus the inner stresses exceed a certain limit that is intrinsic to the material, either plastic deformations occur, which remain after removal of the external load or the component breaks. The theory of yield, strain hardening and fracture applies.

As a rule, only the elastic range of the material is utilised in the design of components. For this reason, we will only be considering the elastic behaviour of materials here and will be ignoring plastic deformation.

Values and units The following table shows a selection of values and units associated with the strength of materials.

Value	Unit	Designation	Comments
x, y, z	mm	Cartesian coordinates	Right-handed trihedron
u, v, w	mm	Deformation in x, y, z direction	
a	mm	Distance, lever arm, major elliptical semi-axis	In contrast to the SI system, the basic unit metre (m) is not used in mechanical engineering, but the derived unit millimetre (mm) instead.
b	mm	Width, minor elliptical semi-axis	
d, D	mm	Diameter	
r, R	mm	Radius	
f	mm	Deflection, sagging	
h	mm	Height	
l	mm	Length	$1\ \text{mm} = 10^{-3}\ \text{m}$
A	mm^2	Area, cross-sectional area	$1\ \text{mm}^2 = 10^{-6}\ \text{m}^2$
E	N/mm^2	Modulus of elasticity	$E = \sigma/\epsilon$
F	N	Force	$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$
F_G	N	Weight	$F_G = m \cdot g$
g	mm/s^2	Gravitational acceleration	$g = 9{,}806\,65\ \text{m/s}^2$
G	N/mm^2	Shear modulus	$G = \tau/\gamma$
H	mm^3	First moment of area	$H_y = \int z\, dA$
I_a	mm^4	Axial second moment of area	$I_y = \int z^2\, dA$
I_p	mm^4	Polar second moment of area	$I_p = \int r^2\, dA$
I_t	mm^4	Torsional area moment	–
m	kg	Mass	SI base unit
M_b	$\text{N} \cdot \text{mm}$	Bending moment	Cross-sectional value
M_t	$\text{N} \cdot \text{mm}$	Torsional moment	Cross-sectional value
F_N, N	N	Normal force	Cross-sectional value
p	N/mm^2	Pressure, Hertzian pressure	–
Q	N	Transverse force	Cross-sectional value

Continuation of table, see Page 215.

Continuation of table, Values and units, from Page 214.

Value	Unit	Designation	Comments
R_e	N/mm^2	Yield point	see material tables
R_m	N/mm^2	Tensile strength, breaking strength	
$R_{p0,2}$	N/mm^2	0,2 proof stress	
T	K	Temperature	SI base unit
W_a	mm^3	Axial section modulus	W_x, W_y, W_z
W_p	mm^3	Polar section modulus	$W_p = I_p/R$ (circle)
W_t	mm^3	Torsional section modulus	–
W_i	$N \cdot mm$	Internal deformation work	of the internal stresses
$W_ä$	$N \cdot mm$	External deformation work	of the forces, moments
α	1/K	Coefficient of linear thermal expansion	$\Delta l = \alpha \cdot l \cdot \Delta T$
α_k	1	Stress concentration factor, notch concentration factor	–
β	1/K	Coefficient of cubical thermal expansion	$\beta = 3\,\alpha$
β_k	1	Fatigue notch factor	–
γ	1	(Torsional) shear strain	$\gamma = \tau/G$
ϵ	1	Elongation	$\epsilon = \Delta l/l$
ϵ_q	1	Transverse elongation	$\epsilon_q = \Delta d/d = -\nu\,\epsilon$
ϵ_m	1	Elongation at fracture	–
Θ	rad/mm	Twist	$\Theta = \varphi/l$
ν	1	Poisson's ratio	$\nu = 0{,}3$ (for most metallic materials) Further designations for ν: $1/m$, ν_E
ρ	kg/mm^3	Density, mass density	–
σ	N/mm^2	Normal stress (tensile stress, compressive stress)	$\sigma = F_N/A$
σ_W	N/mm^2	Fatigue strength under reversed stresses	see Smith diagram
σ_{Sch}	N/mm^2	Fatigue strength under pulsating stresses	
σ_A	N/mm^2	Amplitude strength	
σ_D	N/mm^2	Fatigue limit (general)	
τ	N/mm^2	Shear stress; Torsional shear stress	$\tau = Q/A$; $\tau = M_t/W_p$
φ	rad	Angle, torsional angle	–

Material characteristics The following table lists a number of important material characteristics.

Material	Modulus of elasticity[1) E kN/mm^2 = GPa	Poisson's ratio ν	Coefficient of linear expansion α 10^{-6}/K	Density ρ kg/dm^3	Tensile strength[2) R_m N/mm^2 = MPa
Metals					
Aluminium	72,2	0,34	23,9	2,7	40 ... 160
Aluminium alloys	59 ... 78	0,33 ... 0,34	18,5 ... 24,0	2,6 ... 2,9	300 ... 700
Brass	78 ... 123	0,35	17,5 ... 19,1	8,3 ... 8,7	140 ... 780
Brass (60% Cu)	100	0,36	18	8,5	200 ... 740
Bronze	108	0,35	16,8 ... 18,8	7,2 ... 8,9	300 ... 320
Cast iron	64 ... 81	0,24 ... 0,29	9 ... 12	7,1 ... 7,4	140 ... 490
Copper	125	0,35	16,86	8,93	200 ... 230
Gold	79	0,42	14,2	19,3	130 ... 300
Iron	206	0,28	11,7	7,86	300
Lead	16	0,44	29,1	11,34	10 ... 20
Magnesium	44	0,33	26,0	1,74	150 ... 200
Nickel	167	0,31	13,3	8,86	370 ... 800
Nickel alloys	158 ... 213	0,31	11 ... 14	7,8 ... 9,2	540 ... 1 275
Platinum	170	0,22	9,0	21,5	220 ... 380
Silver	80	0,38	19,7	10,5	180 ... 350
Steel, alloyed	186 ... 216	0,2 ... 0,3	9 ... 19	7,8 ... 7,86	500 ... 1 500
Steel, unalloyed	210	0,3	12	7,85	300 ... 700
X5CrNi18-10	190	0,27	16	7,9	500 ... 700
100Cr6, hardened	208	0,30	12	7,85	2 000 ... 2 400
Tin	55	0,33	21,4	7,29	15 ... 30
Titanium	105	0,33	8,35	4,5	300 ... 740
Zinc	94	0,25	29	7,14	100 ... 150

Continuation of table, see Page 217.

1) The following relationship applies between the modulus of elasticity E and the shear modulus G of the materials:

$$G = \frac{E}{2(1+\nu)}$$

2) Differentiated values for the tensile strength R_m and the yield point R_e of materials can be found in the relevant DIN standards and in the Construction materials chapter of this Pocket Guide.

Continuation of table, Material characteristics, from Page 216.

Material	Modulus of elasticity[1) E kN/mm² = GPa	Poisson's ratio ν	Coefficient of linear expansion α 10^{-6}/K	Density ρ kg/dm³	Tensile strength[2) R_m N/mm² = MPa
Non-metallic materials (inorganic)					
Brick	10 ... 40	0,20 ... 0,35	8 ... 10	1,7 ... 1,9	–
Concrete	22 ... 39	0,15 ... 0,22	5,4 ... 14,2	2,0 ... 2,8	10 ... 40
Construction glass	62 ... 86	0,25	9	2,4 ... 2,7	30 ... 90
Glass (general)	39 ... 98	0,10 ... 0,28	3,5 ... 5,5	2,2 ... 6,3	30 ... 90
Granite	50 ... 60	0,13 ... 0,26	3 ... 8	2,6 ... 2,8	10 ... 20
Marble	60 ... 90	0,25 ... 0,30	5 ... 16	1,8 ... 2,7	–
Porcelain	60 ... 90	–	3 ... 6,5	2,2 ... 2,5	15 ... 40
Quartz glass	62 ... 76	0,17 ... 0,25	0,5 ... 0,6	2,21	30 ... 90
Non-metallic materials (organic)					
Araldite	3,2	0,33	50 ... 70	–	–
Plexiglas® (PMMA)	2,6 ... 3,2	0,35	70 ... 100	1,18	40 ... 70
Polyamide (Nylon®)	1,3 ... 1,7	–	70 ... 100	1,01 ... 1,14	40 ... 80
Polyethylene (HDPE)	0,15 ... 1,6	–	150 ... 200	0,91 ... 0,97	25 ... 30
Polyvinyl chloride	1 ... 3	–	70 ... 100	1,2 ... 1,7	45 ... 60

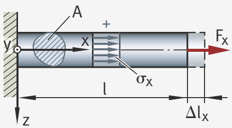
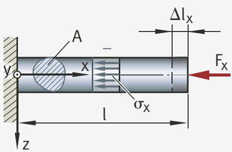
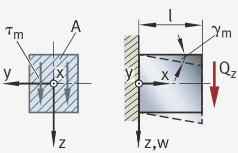
1) The following relationship applies between the modulus of elasticity E and the shear modulus G of the materials:

$$G = \frac{E}{2(1+\nu)}$$

2) Differentiated values for the tensile strength R_m and the yield point R_e of materials can be found in the relevant DIN standards and in the Construction materials chapter of this Pocket Guide.

Load types

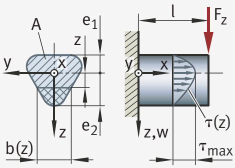
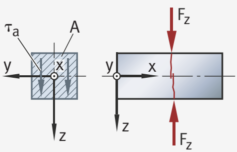
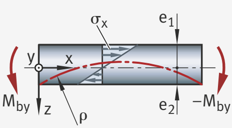
Load types The most important load types, complete with occurring stresses and associated deformations, are presented below.

Load type	Stress	Deformation
Tensile loading 	**Normal stress** $\sigma_x = \frac{F_x}{A}$ **Tensile loading** F_x is positive σ_x is positive **Compressive loading** F_x is negative σ_x is negative	**Elongation (compression[1])** $\varepsilon_x = \frac{\Delta l_x}{l} = \frac{\sigma_x}{E} = \frac{F_x}{E \cdot A}$ **Increase (decrease[1]) in length** $\Delta l_x = \frac{F_x \cdot l}{E \cdot A}$ **Transverse contraction (transverse compression)** $\epsilon_y = -\nu \cdot \epsilon_x$ $\epsilon_z = -\nu \cdot \epsilon_x$
Compressive loading 		
Shear loading (average) 	**Shear stress** (average) $\tau_m = \frac{Q_z}{A}$	**Torsional shear strain (dislocation)** (average) $\gamma_m = \frac{\tau_m}{G} = \frac{Q_z}{G \cdot A}$ $w(l) = \gamma_m \cdot l$

Continuation of table, see Page 219.

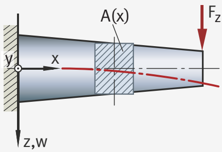
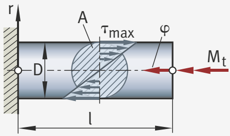
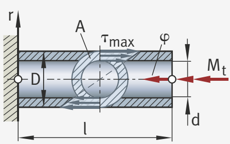
[1] Depending on sign of F_x.

Continuation of table, Load types, from Page 218.

Load type	Stress	Deformation
Shear loading (actual)  $Q_z = F_z$	**Shear stress distribution** $\tau(z) = \frac{Q_z H_y(z)}{I_y b(z)}$ with static moment $H_y(z) = \int_z^{e_2} z\, b(z)\, dz$ $\tau_{max} = \frac{Q_z H_y(z=0)}{I_y(z=0)}$	**Deflection of the beam** (only as the result of shear stress) $w(x) = k \frac{Q_z}{G \cdot A} \cdot x < \frac{\tau_{max}}{G} \cdot x$ $w(l) = k \frac{Q_z}{G \cdot A} \cdot l$ k = cross-sectional factor
Direct shear loading  	**Average shear stress** $\tau_a = \frac{F_z}{A}$	Shearing occurs when the material's shear strength is exceeded
Bending without transverse force  M_{by} = const. I_y = const. $e_1 = e_{max}$ I_y = axial second moment of area about the y axis	**Bending stress** Distribution $\sigma = \frac{M_{by}}{I_y} z$ Maximum value $\sigma_{max} = \frac{M_{by}}{I_y} e_{max} = \frac{M_{by}}{W_y}$	**Curvature** $k = \frac{1}{\rho} = \frac{M_{by}}{E I_y}$ ρ = curvature radius **Differential equation of the elastic curve** $w''(x) = \frac{M_{by}}{E I_y}$

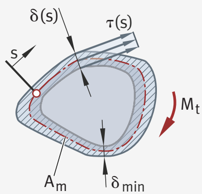
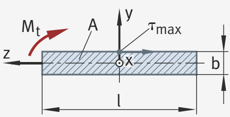
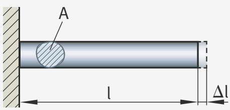
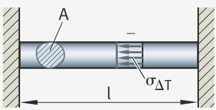
Continuation of table, see Page 220.

Continuation of table, Load types, from Page 219.

Load type	Stress	Deformation
Bending (general)  $I_y(x) \neq$ const. $M_{by}(x) \neq$ const. (influence of transverse force is ignored, justified for long, slender beams)	**Bending stress** Distribution $\sigma_b(x, z) = \frac{M_{by}(x)}{I_y(x)} z$ Maximum value $\sigma_{b_{max}}(x) = \frac{M_{by}(x)}{W_y(x)}$	**Differential equation of the elastic curve** $w''(x) = \frac{M_{by}(x)}{E I_y(x)}$
Torsion of circular solid cross sections  I_p = polar second moment of area	**Torsional stress** Distribution $\tau(r) = \frac{M_t}{I_p} r$ Maximum value $\tau_{max} = \frac{M_t}{I_p} \cdot \frac{D}{2} = \frac{M_t}{W_p}$	**Twist** $\vartheta = \frac{\varphi}{l} = \frac{M_t}{G I_p}$ **Angle of twist** $\varphi = \frac{M_t\, l}{G I_p}$
Torsion of circular hollow cross sections (tubes) 	**Torsional stress** Maximum value $\tau_{max} = \frac{M_t}{W_p}$ $W_p = \frac{I_p(D) - I_p(d)}{D/2}$	**Angle of twist** $\varphi = \frac{M_t\, l}{G I_p}$ $I_p = I_p(D) - I_p(d)$

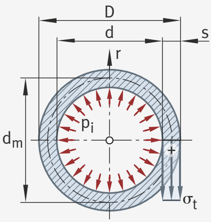
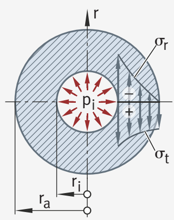
Continuation of table, see Page 221.

Continuation of table, Load types, from Page 220.

Load type	Stress	Deformation
Torsion of thin-walled, closed hollow cross sections  	**Shear stress** Distribution over circumference $\tau(s) = \frac{M_t}{2\,A_m\,\delta(s)}$ Maximum value $\tau_{max} = \frac{M_t}{W_t} = \frac{M_t}{2A_m \cdot \delta_{min}}$	**Twist** $\vartheta = \frac{M_t}{G \cdot I_t} = \frac{M_t}{G}\frac{\oint \frac{ds}{\delta(s)}}{4\,A_m^2}$ $I_t = \frac{4\,A_m^2}{\oint \frac{ds}{\delta(s)}}$
Torsion of narrow rectangular cross sections  	**Shear stress** Distribution $\tau = \frac{2\,M_t}{I_t}\,y$ Maximum value $\tau_{max} = \frac{M_t}{W_t} = \frac{2\,M_t}{I_t}\frac{b}{2} = \frac{3\,M_t}{b^2\,h}$	**Twist** $\vartheta = \frac{M_t}{G \cdot I_t} = \frac{M_t}{G}\frac{3}{b^3\,h}$ $I_t = \frac{b^3\,h}{3}$
Heating of a bar with free elongation  	**No stresses** Elongation occurs without stresses.	**Length change** $\Delta l = l \cdot \alpha \cdot \Delta T$ α = coefficient of linear thermal expansion ΔT = temperature change
Heating of a bar, clamped at both ends  	**Thermal stress** $\sigma\Delta_T = -E \cdot \alpha \cdot \Delta T$ α = coefficient of linear thermal expansion ΔT = temperature change	**No deformation** An increase in length is not possible on account of the clamping operation and must be absorbed by compression in the bar.

Continuation of table, see Page 222.

Continuation of table, Load types, from Page 221.

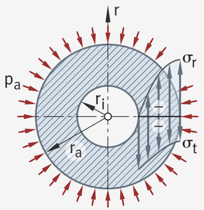
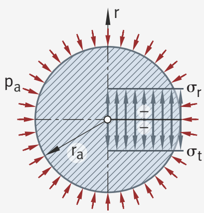
Load type	Stress	Deformation
Thin-walled tube under internal pressure p_i  $D/d < 1{,}2$	**Tangential stress** (Barlow's formula) $\sigma_t = \frac{p_i\, d_m}{2\, s}$ **Axial stress** (Barlow's formula) $\sigma_a = \frac{p_i\, d_m}{4 \cdot s}$ where $d_m = \frac{D + d}{2}$ $s = \frac{D - d}{2}$	**Diameter change** $\Delta d_m = \frac{d_m\, \sigma_t}{E}$ **Length change** $\Delta l = \frac{l\, \sigma_a}{E}$
Thick-walled tube under internal pressure p_i[1)]  Ratio: $Q = \frac{r_i}{r_a} = \frac{d_i}{d_a}$	**Tangential stress** $\sigma_t = p_i \frac{(r_a / r)^2 + 1}{(r_a / r_i)^2 - 1}$ **Radial stress** $\sigma_r = -p_i \frac{(r_a / r)^2 - 1}{(r_a / r_i)^2 - 1}$ **Axial stress**[2)] $\sigma_a = p_i \frac{1}{(r_a / r_i)^2 - 1}$	**Radial displacement** $u(r) = \frac{p_i}{E} \frac{\left[(1-\nu)\, Q^2\, r + (1+\nu)\, r_i^2 / r\right]}{1 - Q^2}$ **Diameter change** $\Delta d_a = \frac{p_i\, d_a}{E} \frac{2\, Q^2}{1 - Q^2}$ $\Delta d_i = \frac{p_i\, d_i}{E} \left[\frac{1 + Q^2}{1 - Q^2} + \nu\right]$

Continuation of table, see Page 223.

1) In the presence of p_i and p_a, the relationships for the stresses and deformations can be superimposed. The results of FEM calculation may deviate by up to 10% from the results determined using these equations.

2) System with axially impeded expansion.

Continuation of table, Load types, from Page 222.

Load type	Stress	Deformation
Thick-walled tube under external pressure p_a[1)]  $r_i \leq r \leq r_a$	**Tangential stress** $\sigma_t = -p_a \dfrac{(r_a/r_i)^2 + (r_a/r)^2}{(r_a/r_i)^2 - 1}$ **Radial stress** $\sigma_r = -p_a \dfrac{(r_a/r_i)^2 - (r_a/r)^2}{(r_a/r_i)^2 - 1}$ **Axial stress**[2)] $\sigma_a = -p_a \dfrac{(r_a/r_i)^2}{(r_a/r_i)^2 - 1}$	**Radial displacement** $u(r) = -\dfrac{p_a}{E} \dfrac{\left[(1-\nu)\,r + (1+\nu)\,r_i^2/r\right]}{(1-Q^2)}$ **Diameter change** $\Delta d_a = -\dfrac{p_a\,d_a}{E}\left[\dfrac{1+Q^2}{1-Q^2} - \nu\right]$ $\Delta d_i = -\dfrac{p_a\,d_i}{E}\,\dfrac{2}{1-Q^2}$
Solid shaft under external pressure p_a  $0 \leq r \leq r_a$	**Tangential stress** $\sigma_t = -p_a = \text{const.}$ **Radial stress** $\sigma_t = -p_a = \text{const.}$ **Axial stress**[2)] $\sigma_a = -p_a = \text{const.}$	**Radial displacement** $u(r) = -\dfrac{p_a\,r}{E}(1-\nu)$ **Diameter change** $\Delta d_a = -\dfrac{p_a\,d_a}{E}(1-\nu)$ ν = Poisson's ratio

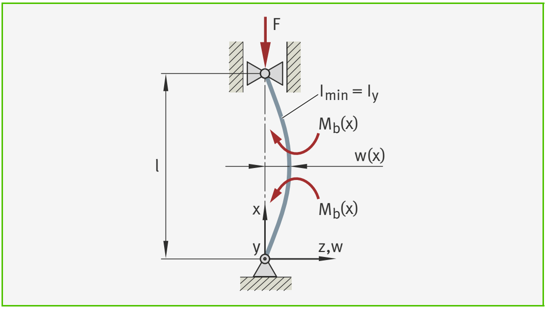
[1)] In the presence of p_i and p_a, the relationships for the stresses and deformations can be superimposed. The results of FEM calculation may deviate by up to 10% from the results determined using these equations.

[2)] System with axially impeded expansion.

Buckling of slender bars

A buckling load represents a limit case of compressive loading, as occurs, for example, with long spindles, articulated columns of slip-on gears, frame members and in other similar cases. Slender bars, when subjected to a compressive load, move out of the unbent (unstable) equilibrium state into an adjacent bent (stable) state when a critical compressive stress is reached.

Figure 1
Buckling of a slender bar



With the aid of compressive stress:

Equation 4

$$\sigma_d = \frac{F}{A}$$

this gives the following for the buckling stress (in the Euler range):

Equation 5

$$\sigma_K = \frac{F_K}{A} = \frac{\pi^2 \cdot E \cdot I_y}{A \cdot l^2} \qquad I_y = I_{min}$$

Buckling in the elastic (Euler) range

If we consider the deformed equilibrium state of the bar shown (see Figure 1), the differential equation for buckling around the major cross-sectional axis y (with I_y as the smallest second moment of area), in the case of small deflections w(x), is:

Equation 6

$$E \cdot I_y \cdot w''(x) = -M_b(x) = -F \cdot w(x)$$

$$w''(x) + \alpha^2 \cdot w(x) = 0 \qquad \text{where } \alpha = \sqrt{\frac{F}{E \cdot I_y}}$$

The solution to this differential equation is:

Equation 7

$$w(x) = c_1 \cdot \sin(\alpha \cdot x) + c_2 \cdot \cos(\alpha \cdot x)$$

From the marginal conditions for the bar shown:

Equation 8

$$w(x = 0) = 0 \qquad \text{and } w(x = l) = 0$$

it follows that $c_2 = 0$ and $\sin(\alpha \cdot l) = 0$ (eigenvalue equation) with the eigenvalues:

Equation 9

$$\alpha_K = \frac{n \cdot \pi}{l} \qquad n = 1, 2, 3 \ldots$$

This gives the buckling load:

Equation 10

$$F_K = \alpha_K{}^2 \cdot E \cdot I_y = n^2 \cdot \pi^2 \cdot E \cdot I_y / l^2$$

and the smallest buckling load:

Equation 11

$$F_K = \frac{\pi^2 \cdot E \cdot I_y}{l^2} \qquad n = 1$$

Appropriate eigenvalues are produced for other bearing arrangement cases which, however, can all be ascribed to Euler's buckling load l_K with the reduced buckling length.

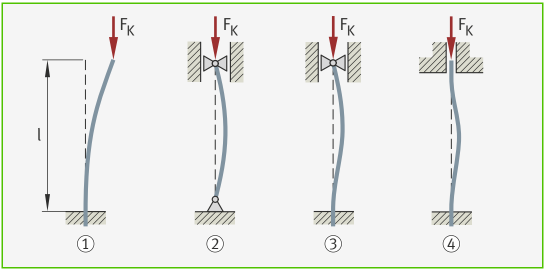
Euler's four buckling cases

The following diagram shows the four buckling cases for slender bars in accordance with Euler:

Figure 2

Euler's buckling cases

① Case 1: $l_K = 2\,l$
② Case 2: $l_K = l$
③ Case 3: $l_K = 0{,}7\,l$
④ Case 4: $l_K = 0{,}5\,l$



With the radius of gyration:

Equation 12

$$i_y = \sqrt{I_y / A}$$

and the slenderness ratio:

Equation 13

$$\lambda = l_K / i_y = l_K \Big/ \sqrt{\left(I_y / A\right)}$$

the following is given for the buckling stress:

Equation 14

$$\sigma_K = F_K / A = \pi^2 \cdot E \big/ \lambda^2$$

These relationships for F_K and σ_K apply only in the linear, elastic material range, i.e. as long as the following applies:

Equation 15

$$\sigma_K = \pi^2 \cdot E \big/ \lambda^2 < R_p \qquad \text{or} \qquad \lambda > \sqrt{\pi^2 \cdot E / R_p}$$

The transition from the elastic to the inelastic (plastic) range lies at the limit slenderness:

Equation 16

$$\lambda_0 = \sqrt{\pi^2 \cdot E / R_p}$$

In this case, R_p is the proportional limit of the material.

Buckling in the inelastic (plastic) range

For smaller slenderness ratios than the limit slenderness, Euler's hyperbola is replaced by Tetmajer's line (see Figure 3, Page 227), which has the following form:

Equation 17

$$\sigma_K = a - b \cdot \lambda$$

The values for limit slenderness λ_0 and for a and b are presented in the following table for a number of materials.

Material	Old designation	E N/mm^2	λ_0	a	b
S235JR	St 37	$2{,}1 \cdot 10^5$	104	310	1,14
E295, E335	St 50, St 60	$2{,}1 \cdot 10^5$	89	335	0,62
5% Ni steel		$2{,}1 \cdot 10^5$	86	470	2,30
Flake graphite cast iron		$1{,}0 \cdot 10^5$	80	$\sigma_K = 776 - 12 \cdot \lambda + 0{,}053 \cdot \lambda^2$ [1)]	
Softwood		$1{,}0 \cdot 10^4$	100	29,3	0,194

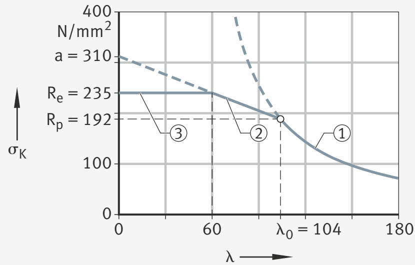
1) No longer a line, rather Engesser's hyperbola.

Tetmajer's line runs from the intersection with Euler's hyperbola to the intersection with the yield point R_e of the material used.

Consequently, three areas exist as a function of the slenderness ratio:

Figure 3
Buckling stress diagram for S235

① Euler's hyperbola
② Tetmajer's line
③ Yield point



In the example "Buckling stress diagram for material S235", we arrive at the following for the yield point R_e:

Equation 18

$$R_e = 235\ \mathrm{N/mm^2}$$

and for the proportional limit R_p:

Equation 19

$$R_p = 0{,}8 \cdot R_e = 188\ \mathrm{N/mm^2}$$

Uniaxial and multi-axial stress states

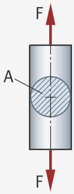
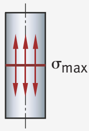
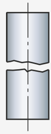
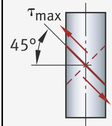
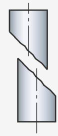
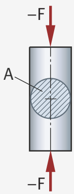
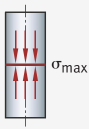
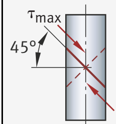
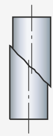
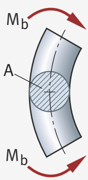
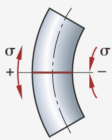
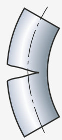
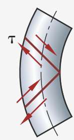
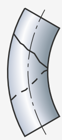
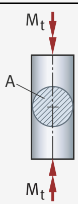
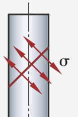
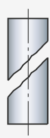
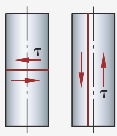
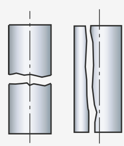
Fracture forms

The mechanism of fracture forms for the uniaxial stress state is presented below.

The most important types of failures under mechanical loading are:

Type of failure	Crucial strength characteristic
Start of yield	Yield point, 0,2 proof stress
Cleavage fracture	Breaking strength
Fatigue fracture	Fatigue strength for present dynamic load case

The following fracture forms can be described for the uniaxial stress state:

External load	Brittle materials[1]		Tough materials	
	Maximum normal stress	Cleavage fracture	Maximum shear stress	Shear or torsional shear deformation
Tension 	 $\sigma_{max} = \frac{F}{A}$		 $\tau_{max} = \frac{\sigma_{max}}{2}$	
Compression 	 $\sigma_{max} = -\frac{F}{A}$	Cleavage fracture not possible	 $\tau_{max} = \frac{\sigma_{max}}{2}$	
Bending 	 $\sigma_{max} = \frac{M_b}{W_a}$		 $\tau_{max} = \frac{\sigma_{max}}{2}$	
Torsion 	 $\sigma_{max} = 2\,\tau_{max}$		 $\tau_{max} = \frac{M_t}{W_p}$	
Applicable fracture hypothesis	Normal stress hypothesis		Shear stress hypothesis, distortion energy hypothesis	

[1] In accordance with the FKM guideline, materials with an elongation at fracture of < 8–12% are regarded as brittle.

Strength hypotheses The following important strength hypotheses can be formulated for the multi-axial stress state:

Failure by		Cleavage fracture	Deformation, ductile fracture	
Strength hypothesis		**Normal stress hypothesis (NSH)**	**Shear stress hypothesis (SSH)**	**Distortion energy hypothesis (DEH)**
Stress state		**Equivalent stress σ_v**		
$\sigma_1, \sigma_2, \sigma_3$ $\sigma_1 \geqq \sigma_2 \geqq \sigma_3$	3-axial	σ_1	$\sigma_1 - \sigma_3 = 2\,\tau_{max}$	$\frac{1}{\sqrt{2}} \cdot \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$
$\sigma_1, \sigma_2,$ $\sigma_3 = 0$	2-axial	σ_1	$\sigma_1 = 2\,\tau_{max}$	$\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2}$
$\sigma_1, \sigma_3,$ $\sigma_2 = 0$	2-axial	σ_1	$\sigma_1 - \sigma_3 = 2\,\tau_{max}$	$\sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1 \sigma_3}$
σ_x, σ_y, τ	2-axial	$\frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2} \cdot \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$	$\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau^2}$ 1) $\frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2} \cdot \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$ 2)	$\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\,\tau^2}$
$\sigma_x, \tau,$ $\sigma_y = 0$ 3)	2-axial	$\frac{1}{2}\sigma_x + \frac{1}{2}\sqrt{\sigma_x^2 + 4\,\tau^2}$	$\sqrt{\sigma_x^2 + 4\,\tau^2}$	$\sqrt{\sigma_x^2 + 3\,\tau^2}$

1) The following applies:

$$\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau^2} > \sigma_x + \sigma_y$$

2) The following applies:

$$\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau^2} < \sigma_x + \sigma_y$$

3) Bending and torsion:
Application of the distortion energy hypothesis (DEH) has proven itself in cases of superimposed bending and torsion. For dynamic loads (for example, alternating bending load with superimposed torsion load), the correction factor α_0 must be taken into account:

$$\sigma_v = \sqrt{\sigma_x^2 + 3 \cdot (\alpha_0 \cdot \tau)^2}$$

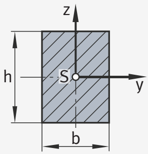
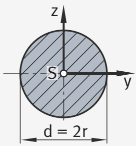
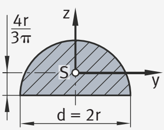
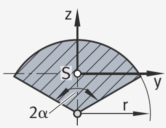
In the case of alternating bending and static torsion: $\alpha_0 = 0{,}7$ 4)
In the case of alternating bending and alternating torsion: $\alpha_0 = 1{,}0$ 4)
In the case of static bending and alternating torsion: $\alpha_0 = 1{,}5$ 4)

4) Approximation values only valid for unnotched components made from general construction steel.

Area moments and section moduli

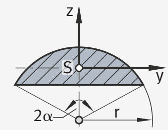
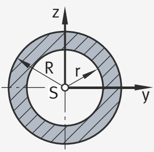
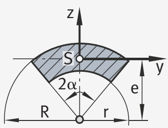
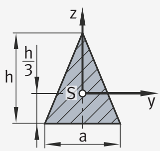
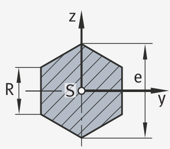
Axial area moments and section moduli

Axial 2nd moments of area and section moduli are calculated using:

Rectangle 	$I_y = \frac{b\,h^3}{12} = A\,\frac{h^2}{12}$ $I_z = \frac{h\,b^3}{12} = A\,\frac{b^2}{12}$	$W_y = \frac{b\,h^2}{6} = A\,\frac{h}{6}$ $W_z = \frac{h\,b^2}{6} = A\,\frac{b}{6}$
Circle 	$I_y = I_z = \frac{\pi\,d^4}{64} = \frac{\pi\,r^4}{4} = \frac{A\,r^2}{4}$	$W_y = W_z = \frac{\pi\,d^3}{32} = \frac{\pi\,r^3}{4} = \frac{A\,r}{4}$
Semicircle 	$I_y = \left(\frac{\pi}{8} - \frac{8}{9\,\pi}\right) r^4$ $I_z = \frac{\pi\,r^4}{8} = \frac{A\,r^2}{4}$	$W_y = 0{,}1902\,r^3$ $W_z = \frac{\pi^3}{8} = \frac{A\,r}{4}$
Sector of a circle 	$I_y = r^4 \left[\frac{2\,\alpha + \sin 2\,\alpha}{8} - \frac{2\left(1 - \cos 2\,\alpha\right)}{9\,\alpha}\right]$ $I_z = \frac{r^4}{8}\left(2\,\alpha - \sin 2\,\alpha\right)$	$W_y = \frac{I_y}{\lvert z_{max} \rvert}$ $z = \frac{2\,r \sin \alpha}{3\,\alpha}$ α in radians

Continuation of table, see Page 231.

Continuation of table, Axial area moments and section moduli, from Page 230.

Segment of a circle 	$I_y = r^4 \left[\frac{4\alpha - \sin 4\alpha}{16} - \frac{8}{9} \cdot \frac{\sin^6 \alpha}{2\alpha - \sin 2\alpha} \right]$ $I_z = \frac{r^4}{48} \left[12\alpha - 8 \sin 2\alpha + \sin 4\alpha \right]$	$W_y = \frac{I_y}{\lvert z_{max} \rvert}$ $z = \frac{4 r \sin^3 \alpha}{3(2\alpha - \sin 2\alpha)}$ α in radians
Annulus 	$I_y = I_z = \frac{\pi}{4}\left(R^4 - r^4\right)$	$W_y = W_z = \frac{\pi}{4} \frac{\left(R^4 - r^4\right)}{R}$
Annular sector 	$I_y = \frac{R^4 - r^4}{8} \cdot \left(2\alpha + \sin 2\alpha\right) - e^2 \alpha \left(R^2 - r^2\right)$ $I_z = \frac{R^4 - r^4}{8} \cdot \left(2\alpha - \sin 2\alpha\right)$	$W_y = \frac{I_y}{R - e}$ $e = \frac{2}{3} \cdot \frac{\left(R^3 - r^3\right) \sin \alpha}{\left(R^2 - r^2\right) \alpha}$ α in radians
Triangle 	$I_y = \frac{a h^3}{36} = \frac{A h^2}{18}$ $I_z = \frac{h a^3}{48} = \frac{A a^2}{24}$	$W_y = \frac{a h^2}{24} = \frac{A h}{12}$ $W_z = \frac{h a^2}{24} = \frac{A a}{12}$
Hexagon 	$I_y = I_z = \frac{5\sqrt{3}}{16} R^4 = \frac{5\sqrt{3}}{256} e^4$	$W_y = \frac{5\sqrt{3}}{16} R^3 = \frac{5\sqrt{3}}{128} e^3$ $W_z = \frac{5}{8} R^3 = \frac{5}{64} e^3$

Continuation of table, see Page 232.

Continuation of table, Axial area moments and section moduli, from Page 231.

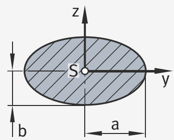
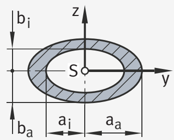
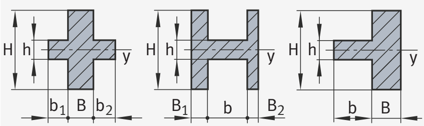
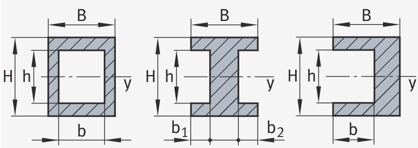
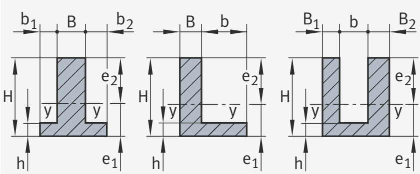
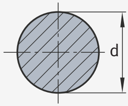
Profile	Axial area moment	Section modulus
Ellipse  	$I_y = \dfrac{\pi\, a\, b^3}{4} = \dfrac{A\, b^2}{4}$ $I_z = \dfrac{\pi\, a^3\, b}{4} = \dfrac{A\, a^2}{4}$	$W_y = \dfrac{\pi\, a\, b^2}{4} = \dfrac{A\, b}{4}$ $W_z = \dfrac{\pi\, a^2\, b}{4} = \dfrac{A\, a}{4}$
Hollow ellipse  	$I_y = \dfrac{\pi\left(a_a\, b_a^3 - a_i\, b_i^3\right)}{4}$ $I_z = \dfrac{\pi\left(a_a^3\, b_a - a_i^3\, b_i\right)}{4}$	$W_y = \dfrac{\pi\left(a_a\, b_a^3 - a_i\, b_i^3\right)}{4\, b_a}$ $W_z = \dfrac{\pi\left(a_a^3\, b_a - a_i^3\, b_i\right)}{4\, a_a}$
Cross, H profile, T profile  		$I_y = \dfrac{B\,H^3 + b\,h^3}{12}$ $W_y = \dfrac{B\,H^3 + b\,h^3}{6\,H}$ where $B = B_1 + B_2$ and $b = b_1 + b_2$
Rectangular hollow profile, I profile, C profile  		$I_y = \dfrac{B\,H^3 - b\,h^3}{12}$ $W_y = \dfrac{B\,H^3 - b\,h^3}{6\,H}$ where $b = b_1 + b_2$
T profile, L profile, U profile  		$I_y = \dfrac{B\,H^3 + b\,h^3}{3} - \left(B\,H + b\,h\right) e_1^2$ where $B = B_1 + B_2$ and $b = b_1 + b_2$ $W_{y1,\,2} = \dfrac{I_y}{e_{1,\,2}}$ $e_1 = \dfrac{1}{2}\,\dfrac{B\,H^2 + b\,h^2}{B\,H + b\,h}$ $e_2 = H - e_1$

Table: Values for circular sections

For circular sections with diameter d, we arrive at the following 2nd moments of area and section moduli:

	Area moment		Section modulus	
	axial	polar	axial	polar
	$I_a = \frac{\pi d^4}{64}$	$I_p = 2 \cdot I_a$	$W_a = \frac{\pi d^3}{32}$	$W_p = 2 \cdot W_a$

d mm	I_a mm^4	W_a mm^3	d mm	I_a mm^4	W_a mm^3	d mm	I_a mm^4	W_a mm^3
1	0,0491	0,0982	31	45 333	2 925	61	679 651	22 284
2	0,7854	0,7854	32	51 472	3 217	62	725 332	23 398
3	3,976	2,651	33	58 214	3 528	63	773 272	24 548
4	12,57	6,283	34	65 597	3 859	64	823 550	25 736
5	30,68	12,27	35	73 662	4 209	65	876 240	26 961
6	63,62	21,21	36	82 448	4 580	66	931 420	28 225
7	117,9	33,67	37	91 998	4 973	67	989 166	29 527
8	201,1	50,27	38	102 354	5 387	68	1 049 556	30 869
9	322,1	71,57	39	113 561	5 824	69	1 112 660	32 251
10	490,9	98,17	40	125 664	6 283	70	1 178 588	33 674
11	718,7	130,7	41	138 709	6 766	71	1 247 393	35 138
12	1 018	169,6	42	152 745	7 274	72	1 319 167	36 644
13	1 402	215,7	43	167 820	7 806	73	1 393 995	38 192
14	1 886	269,4	44	183 984	8 363	74	1 471 963	39 783
15	2 485	331,3	45	201 289	8 946	75	1 553 156	41 417
16	3 217	402,1	46	219 787	9 556	76	1 637 662	43 096
17	4 100	482,3	47	239 531	10 193	77	1 725 571	44 820
18	5 153	572,6	48	260 576	10 857	78	1 816 972	46 589
19	6 397	673,4	49	282 979	11 550	79	1 911 967	48 404
20	7 854	785,4	50	306 796	12 272	80	2 010 619	50 265
21	9 547	909,2	51	332 086	13 023	81	2 113 051	52 174
22	11 499	1 045	52	358 908	13 804	82	2 219 347	54 130
23	13 737	1 194	53	387 323	14 616	83	2 329 605	56 135
24	16 286	1 357	54	417 393	15 459	84	2 443 920	58 189
25	19 175	1 534	55	449 180	16 334	85	2 562 392	60 292
26	22 432	1 726	56	482 750	17 241	86	2 685 120	62 445
27	26 087	1 932	57	518 166	18 181	87	2 812 205	64 648
28	30 172	2 155	58	555 497	19 155	88	2 943 748	66 903
29	34 719	2 394	59	594 810	20 163	89	3 079 853	69 210
30	39 761	2 651	60	636 172	21 206	90	3 220 623	71 569

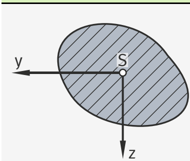
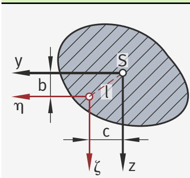
Continuation of table, see Page 234.

Continuation of table, Table: Values for circular sections, from Page 233.

d mm	I_a mm^4	W_a mm^3	d mm	I_a mm^4	W_a mm^3	d mm	I_a mm^4	W_a mm^3
91	3 366 165	73 982	111	7 451 813	134 267	131	14 456 235	220 706
92	3 516 586	76 448	112	7 723 997	137 929	132	14 902 727	225 799
93	3 671 992	78 968	113	8 003 571	141 656	133	15 359 483	230 970
94	3 832 492	81 542	114	8 290 666	145 450	134	15 826 658	236 219
95	3 998 198	84 173	115	8 585 417	149 312	135	16 304 411	241 547
96	4 169 220	86 859	116	8 887 958	153 241	136	16 792 899	246 954
97	4 345 671	89 601	117	9 198 425	157 238	137	17 292 282	252 442
98	4 527 664	92 401	118	9 516 956	161 304	138	17 802 721	258 010
99	4 715 315	95 259	119	9 843 689	165 440	139	18 324 378	263 660
100	4 908 738	98 175	120	10 178 763	169 646	140	18 857 416	269 392
101	5 108 055	101 150	121	10 522 320	173 923	141	19 401 999	275 206
102	5 313 378	104 184	122	10 874 501	178 271	142	19 958 294	281 103
103	5 524 830	107 278	123	11 235 450	182 690	143	20 526 466	287 083
104	5 742 532	110 433	124	11 605 311	187 182	144	21 106 684	293 148
105	5 966 604	113 650	125	11 984 229	191 748	145	21 699 116	299 298
106	6 197 171	116 928	126	12 372 350	196 387	146	22 303 933	305 533
107	6 434 357	120 268	127	12 769 824	201 100	147	22 921 307	311 855
108	6 678 287	123 672	128	13 176 799	205 887	148	23 551 409	318 262
109	6 929 087	127 139	129	13 593 424	210 751	149	24 194 414	324 757
110	7 186 886	130 671	130	14 019 852	215 690	150	24 850 496	331 340

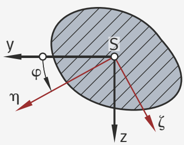
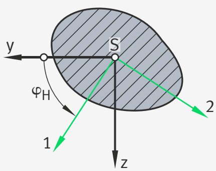
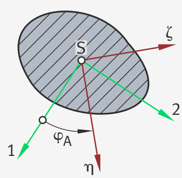
Area moments for various reference axes

The following table lists a number of 2nd moments of area for various reference axes.

Second moment of area for	Axial second moment of area	Centrifugal moment	Polar second moment of area
Any vertical centre-of-gravity axes yz perpendicular to each other			
	$I_z = \int_A z^2 \cdot dA$ $I_y = \int_A y^2 \cdot dA$	$I_{yz} = \int_A y \cdot z \cdot dA$	$I_{ps} = \int_A r^2 \cdot dA$ $I_{ps} = \int_A \left(y^2 + z^2\right) \cdot dA$ $= I_y + I_z$
Axes that are offset in parallel with the yz axes			
	$I_\eta = I_y + b^2 \cdot A$ $I_\zeta = I_z + c^2 \cdot A$	$I_{\eta\zeta} = I_{yz} + b \cdot c \cdot A$	$I_p = I_{ps} + l^2 \cdot A$ $= I_{ps} + (b^2 + c^2) \cdot A$ $= I_\eta + I_\zeta$

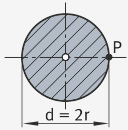
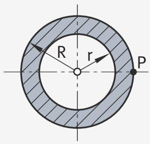
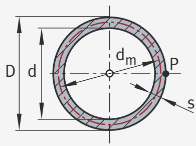
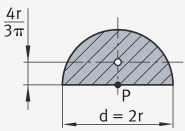
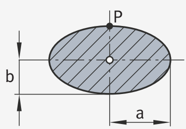
Continuation of table, see Page 235.

Continuation of table, Area moments for various reference axes, from Page 234.

Second moment of area for	Axial second moment of area	Centrifugal moment	Polar second moment of area
Axes which are rotated in the positive sense about the angle φ in relation to the yz axes			
	$I_\eta = \frac{I_y + I_z}{2} + \frac{I_y - I_z}{2} \cdot \cos 2\varphi - I_{yz} \cdot \sin 2\varphi$ $I_\zeta = \frac{I_y + I_z}{2} - \frac{I_y - I_z}{2} \cdot \cos 2\varphi - I_{yz} \cdot \sin 2\varphi$	$I_{\eta\zeta} = \frac{I_y - I_z}{2} \cdot \sin 2\varphi - I_{yz} \cdot \cos 2\varphi$	$I_{ps} = I_\eta + I_\zeta$ $= I_1 + I_2$
Principal inertia axes which are rotated in the positive sense about the angle φ_H in relation to the yz axes			
	$\sin 2\varphi_H = \frac{I_{yz}}{\sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}}$ $\cos 2\varphi_H = \frac{\frac{I_y + I_z}{2}}{\sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}}$ $I_{1,\,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} = 0$		$I_{ps} = I_1 + I_2$ $= I_h + I_z$
Axes which are rotated in the positive sense about the angle φ_A in relation to the principal inertia axes			
	$I_\eta = \frac{I_1 + I_2}{2} + \frac{I_1 - I_2}{2} \cdot \cos 2\varphi_A$ $I_\zeta = \frac{I_1 + I_2}{2} - \frac{I_1 - I_2}{2} \cdot \cos 2\varphi_A$	$I_{\eta\zeta} = \frac{I_1 - I_2}{2} \cdot \sin 2\varphi_A$	$I_{ps} = I_\eta + I_\zeta$ $= I_1 + I_2$

Torsional area moments and torsional section moduli

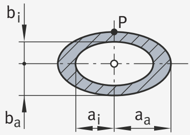
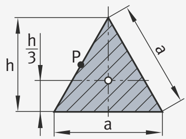
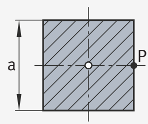
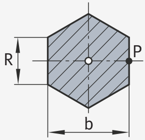
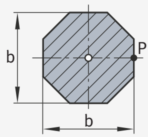
Torsional area moments and torsional section moduli can be calculated as follows (with "P" as the locations for τ_{max}):

Circle[1)]  	$I_p = \frac{\pi\, d^4}{32} = \frac{\pi\, r^4}{2}$ $W_p = \frac{\pi\, d^3}{16} = \frac{\pi\, r^3}{2}$
Annulus[1)]  	$I_p = \frac{\pi}{2}\left(R^4 - r^4\right)$ $W_p = \frac{\pi}{2} \frac{\left(R^4 - r^4\right)}{R}$
Thin-walled annulus[1)]  	$\frac{D}{d} < 1{,}2$ $\quad d_m = \frac{D+d}{2}$ $I_p = \frac{\pi}{4} d_m{}^3\, s$ $W_p = \frac{\pi}{2} d_m{}^2\, s$
Semicircle  	$I_t = 0{,}296\, r^4$ $W_t = 0{,}348\, r^3$
Ellipse[2)]  	$I_t = \pi \frac{a^3\, b^3}{a^2 + b^2}$ $W_t = \frac{\pi}{2}\, a\, b^2$

Continuation of table, see Page 237.

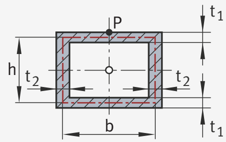
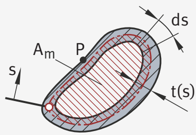
1) τ_{max} prevails over entire circumference.
2) P only applies if $a/b \geqq 1$.

Continuation of table, Torsional area moments and torsional section moduli, from Page 236.

Hollow ellipse 	$a_a : b_a = a_i : b_i = c$ $I_t = \pi \dfrac{c^3 \left(b_a{}^4 - b_i{}^4\right)}{1 + c^2}$ $W_t = \dfrac{\pi\, c \left(b_a{}^4 - b_i{}^4\right)}{2\, b_a}$
Equilateral triangle 	$I_t = \dfrac{a^4}{46{,}19} \approx \dfrac{h^4}{26}$ $W_t = \dfrac{a^3}{20} \approx \dfrac{h^3}{13}$
Square 	$I_t = 0{,}141\ a^4$ $W_t = 0{,}208\ a^3$
Hexagon 	$I_t = 0{,}115\ b^4 = 1{,}037\ R^4$ $W_t = 0{,}188\ b^3 = 0{,}977\ R^3$
Octagon 	$I_t = 0{,}108\ b^4$ $W_t = 0{,}185\ b^3$

Continuation of table, see Page 238.

Continuation of table, Torsional area moments and torsional section moduli, from Page 237.

Thin-walled box section 	$t_1 < t_2 \ll b, h$ $I_t = \dfrac{2(b\,h)^2}{\dfrac{b}{t_1} + \dfrac{h}{t_2}}$ $W_t = 2\,b\,h\,t_{min}$
Thin-walled hollow section 	$I_t = \dfrac{4\,A_m{}^2}{\int \dfrac{ds}{t(s)}}$ $W_t = 2\,A_m t_{min}$ A_m = content of the area enclosed by the centre line

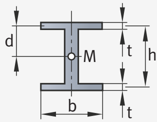
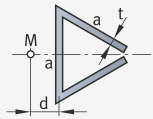
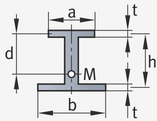
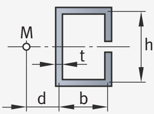
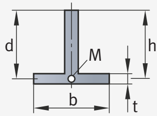
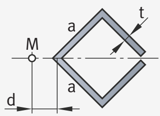
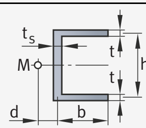
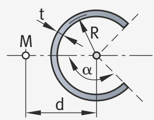
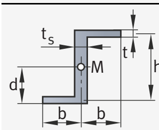
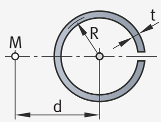
Bending due to transverse force

Shear centres of thin-walled profiles

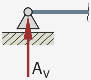
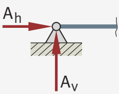
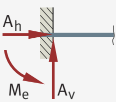
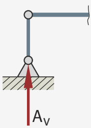
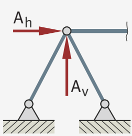
When symmetrical profiles are bent by a transverse force, the cross-section is devoid of torsion. The shear centre (transverse force centre) lies on the symmetry plane. If the profile cross-section has two axes of symmetry, the shear centre falls within the symmetry point, i.e. in the centre of gravity.

In general, this is no longer the case if the load plane does not coincide with a plane of symmetry belonging to the profile. Torsional loading of the profile can be avoided by shifting the load plane appropriately. This is purely dependent on the type of profile, but not on the size of the load (profile constant).

The positions d of the shear centres M are given below for a number of profiles.

Profile	Position d	Profile	Position d
	$d = \frac{h}{2}$		$d = \frac{a\sqrt{3}}{6}$
	$d = \frac{h\,b^3}{a^3 + b^3}$		$d = \frac{b}{2} \cdot \frac{3\,b + 2\,h}{3\,b + h}$
	$d = h$		$d = \frac{a\sqrt{2}}{4}$
	$d = \frac{3\,t\,b^2}{h\,t_s + 6\,b\,t}$		$d = 2R\,\frac{\sin\alpha - \alpha\cos\alpha}{\alpha - \sin\alpha\cos\alpha}$ α in radians
	$d = \frac{h}{2}$		$d = 2\,R$

Flat support types The following possible bearing reactions exist for flat support types:

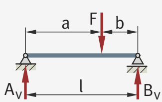
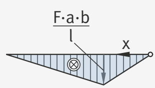
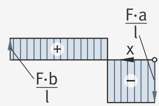
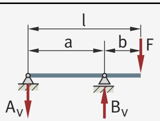
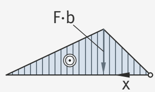
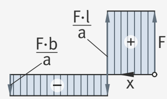
Support type	Degrees of freedom	Bearing reactions	Explanation
Movable pivoting bearing 	2	1	The beam coupled to the movable pivoting bearing can be moved in a horizontal direction and can be rotated about the pivot point. Consequently, it has two degrees of freedom. The bearing can only apply one reaction force acting perpendicular to the sliding direction.
Fixed pivoting bearing 	1	2	In the case of a fixed pivoting bearing, the beam cannot be moved in any direction and can only be rotated about the joint. The effect that the fixed pivoting bearing has on the beam can generally be represented by a force in any direction that can be split into two independent components.
Fixed restraint 	0	3	A securely clamped beam can neither be moved nor rotated. It has no degrees of freedom. The bearing arrangement can be loaded by forces and moments in any direction. The effect of the fixed restraint on the beam can therefore be represented by two forces and one restraining moment.
Articulated column 	2	1	The effect of an articulated column on the beam coupled to it is equivalent to that of a movable pivoting bearing. Perpendicular to the articulated column, the beam can be moved and rotated about the joint. A reaction force can only be transmitted to the beam in the direction of the support.
Three-hinged support 	1	2	The effect of the three-hinged support corresponds to that of the fixed pivoting bearing. It prevents every translational movement in the plane clamped by the supports. Only one degree of freedom remains for rotation about the pivot point. The effect on the beam is covered by two independent forces.

Intermediate elements The following possible intermediate conditions exist for intermediate elements:

Intermediate element	Intermediate conditions	Intermediate reactions	Explanation
Joint 	$M_b = 0$	$Q \neq 0$ $N \neq 0$	A joint supplies the intermediate condition that the bending moment on the joint must disappear if freedom from friction of the beam connection is presumed. Therefore, in the event of a section through the joint, only one transverse force and one normal force occur as cross-sectional values or intermediate reactions.
Sliding sleeve 	$N = 0$	$Q \neq 0$ $M_b \neq 0$	A sliding sleeve cannot transmit a normal force. The disappearance of the normal force on it can be evaluated as an intermediate condition. Transverse force and bending moment can be transmitted as intermediate reactions. Once again, it is presumed that the connection is devoid of friction.

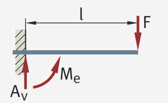
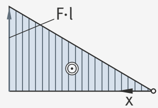
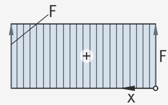
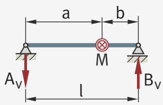
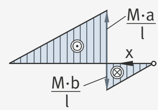
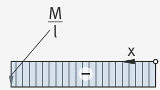
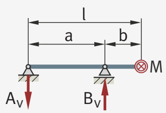
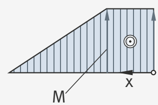
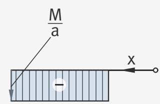
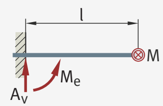
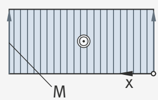
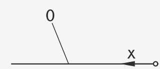
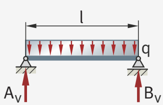
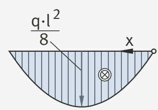
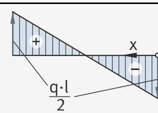
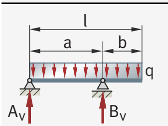
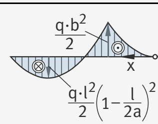
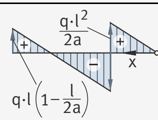
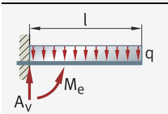
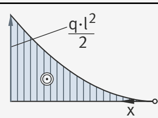
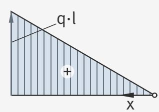
Bending due to transverse force for simple loaded beams It is possible to calculate bearing reactions, moment and transverse force distributions for simple loaded beams and formulate equations for the elastic curve.

Bearing reactions, moment and transverse force distributions The bearing reactions, moment and transverse force distributions are as follows for simple loaded beams:

System	Bearing reactions	Bending moment distribution	Transverse force distribution
	$A_v = F \cdot \frac{b}{l}$ $B_v = F \cdot \frac{a}{l}$		
	$A_v = F \cdot \frac{b}{a}$ $B_v = F \cdot \frac{l}{a}$		

Continuation of table, see Page 242.

Continuation of table, Bearing reactions, moment and transverse force distributions, from Page 241.

System	Bearing reactions	Bending moment distribution	Transverse force distribution
	$A_v = F$ $M_e = F \cdot l$		
	$A_v = \frac{M}{l}$ $B_v = \frac{M}{l}$		
	$A_v = \frac{M}{a}$ $B_v = \frac{M}{a}$		
	$A_v = 0$ $M_e = M$		
	$A_v = \frac{q \cdot l}{2}$ $B_v = \frac{q \cdot l}{2}$		
	$A_v = q \cdot l \left(1 - \frac{l}{2\,a}\right)$ $B_v = \frac{q \cdot l^2}{2\,a}$		
	$A_v = q \cdot l$ $M_e = \frac{q \cdot l^2}{2}$		

Elastic curve equation

If the elastic curve equations w(x) are formulated for simple loaded beams, the following applies:

Equation 20

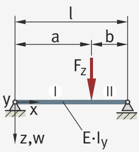
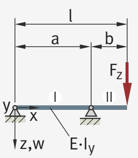
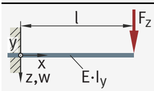
$$w''(x) = -\frac{M_{by}(x)}{E \cdot I_y(x)}$$

and:

Equation 21

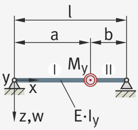
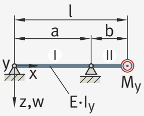
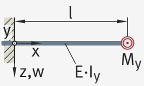
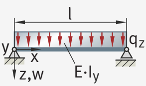
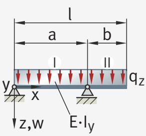
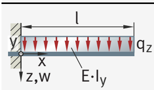
$$I_y(x) = \text{const.}$$

This gives the following:

System	Elastic curve equation w(x)	w_{max}
	$w_I = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(2\frac{a \cdot x}{l^2} - 3\frac{a^2 \cdot x}{l^3} + \frac{a^3 \cdot x}{l^4} + \frac{a \cdot x^3}{l^4} - \frac{x^3}{l^3}\right)$ $w_{II} = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(-\frac{a^3}{l^3} + 2\frac{a \cdot x}{l^2} + \frac{a^3 \cdot x}{l^4} - 3\frac{a \cdot x^2}{l^3} + \frac{a \cdot x^3}{l^4}\right)$	$w(a) = \frac{F_z \cdot a^2 \cdot b^2}{3\,E \cdot I_y \cdot l}$ $a > b$ $w_{I,\,max}$ where $x = l \cdot \sqrt{\frac{2a}{3l} - \frac{a^2}{3l^2}}$ $a < b$ $w_{II,\,max}$ where $x = l - l\sqrt{\frac{1}{3} - \frac{a^2}{3l^2}}$
	$w_I = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(\frac{a \cdot x}{l^2} - \frac{a^2 \cdot x}{l^3} - \frac{x^3}{a \cdot l^2} + \frac{x^3}{l^3}\right)$ $w_{II} = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(\frac{a^2}{l^2} - 4\frac{a \cdot x}{l^2} + \frac{a^2 \cdot x}{l^3} + 3\frac{x^2}{l^2} - \frac{x^3}{l^3}\right)$	$w_{I,\,max} = \frac{\sqrt{3} \cdot F_z \cdot a^2 \cdot b}{27\,E \cdot I_y}$ $w_{II,\,max} = \frac{F_z\, l \cdot b^2}{3\,E \cdot I_y}$
	$w = \frac{F_z \cdot l^3}{6\,E \cdot I_y}\left(3\frac{x^2}{l^2} - \frac{x^3}{l^3}\right)$	$w_{max} = \frac{F_z \cdot l^3}{3\,E \cdot I_y}$

Continuation of table, see Page 244.

Continuation of table, Elastic curve equation, from Page 243.

System	Elastic curve equation w(x)	w_{max}
	$w_I = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(6\,\frac{a \cdot x}{l^2} - 2\,\frac{x}{l} - 3\,\frac{a^2 \cdot x}{l^3} - \frac{x^3}{l^3}\right)$ $w_{II} = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(-3\,\frac{a^2}{l^2} + 2\,\frac{x}{l} + 3\,\frac{a^2 \cdot x}{l^3} - 3\,\frac{x^2}{l^2} + \frac{x^3}{l^3}\right)$	$w_{I,\,max} = \frac{M_y \cdot l^2}{3\,E \cdot I_y} \cdot \left(-\frac{2}{3} + \frac{2\,a}{l} - \frac{a^2}{l^2}\right)^{\frac{3}{2}}$ $w_{II,\,max} = \frac{M_y \cdot l^2}{3\,E \cdot I_y}\left(\frac{1}{3} - \frac{a^2}{l^2}\right)^{\frac{3}{2}}$
	$w_I = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(\frac{a \cdot x}{l^2} - \frac{x^3}{a \cdot l^2}\right)$ $w_{II} = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(\frac{a^2}{l^2} - 4\,\frac{a \cdot x}{l^2} + 3\,\frac{x^2}{l^2}\right)$	$w_{I,\,max} = \frac{\sqrt{3} \cdot M_y \cdot a^2}{27\,E \cdot I_y}$ $w_{II,\,max} = \frac{M_y \cdot l^2}{6\,E \cdot I_y}\left(3 - \frac{4\,a}{l} + \frac{a^2}{l^2}\right)$
	$w = -\frac{M_y}{2\,E \cdot I_y}\,x^2$	$w_{max} = \frac{M_y \cdot l^2}{2\,E \cdot I_y}$
	$w = \frac{q_z \cdot l^4}{24\,E \cdot I_y}\left(\frac{x}{l} - 2\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right)$	$w_{max} = \frac{5\,q_z \cdot l^4}{384\,E \cdot I_y}$
	$w_I = \frac{q_z \cdot l^4}{24\,E \cdot I_y}\left(-2\,\frac{a \cdot x}{l^2} + 4\,\frac{a^2 \cdot x}{l^3} - \frac{a^3 \cdot x}{l^4} + 2\,\frac{x^3}{a \cdot l^2} - 4\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right)$ $w_{II} = \frac{q_z \cdot l^4}{24\,E \cdot I_y}\left(2\,\frac{a^2}{l^2} - 8\,\frac{a \cdot x}{l^2} + 4\,\frac{a^2 \cdot x}{l^3} - \frac{a^3 \cdot x}{l^4} + 6\,\frac{x^2}{l^2} - 4\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right)$	$w_{II,\,max} = \frac{q_z \cdot l^4}{24\,E \cdot I_y} \cdot \left(3 - 8\,\frac{a}{l} + 6\,\frac{a^2}{l^2} - \frac{a^3}{l^3}\right)$
	$w = \frac{q_z \cdot l^4}{24\,E \cdot I_y}\left(6\,\frac{x^2}{l^2} - 4\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right)$	$w_{max} = \frac{q_z \cdot l^4}{8\,E \cdot I_y}$

Principle of passive deformation work

Area of application

The principle of passive deformation work is used to:

- calculate deformation (deflection, torsion) in systems with a statically determinate bearing arrangement
- calculate bearing reactions in externally statically indeterminate systems
- calculate cross-sectional values in systems with a statically indeterminate internal structure.

Checking

In every case, it is first necessary to check whether the system has an external statically determinate bearing arrangement and a statically determinate internal structure.

Example 1: Statically determinate system

If the system is both externally and internally statically determinate, the following guidelines apply:

- Apply a unit force “1” (for deflection) and a unit moment “M” (for torsion) to the system, in the direction of the sought-after deformation, as an external load, at the point in the system where the sought-after deformation occurs (deflection or torsion). During the loading sequence, the system is only loaded using the unit value to start with and only then are the effective external forces applied (point 1)
- Determine normal force, transverse force (mostly negligible) and moment distributions (bending moment, torsion moment) separately for each external load, including the unit values. The same running co-ordinate x must be retained in order to determine all cross-sectional values. The cross-sectional values resulting from the unit values are identified by a dash (point 2)
- Specify the entire passive internal deformation work occurring in the system. Thus:
  external passive deformation work = internal passive deformation work (point 3):

Equation 22

$$\begin{matrix} ① & \text{“}1\text{”} \cdot w = \\ ② & \text{“}M\text{”} \cdot \varphi = \end{matrix} \int \frac{M_b \cdot \bar{M}_b}{E \cdot I_a} \cdot dx + \int \frac{M_t \cdot \bar{M}_t}{G \cdot I_p} \cdot dx + \int \frac{N \cdot \bar{N}}{E \cdot A} \cdot dx + ③$$

① Deflection
② Torsion
③ Passive spring work (transverse forces are ignored)

- Evaluate the integrals with the aid of the integral tables or by mathematical calculation (observe the signs of the cross-sectional values) (point 4)
- Divide the resulting relationship by the unit value and calculate the sought-after value (point 5).

Example 2: Statically indeterminate system

If the system is statically indeterminate, it may:
- have a statically indeterminate bearing arrangement
- have a statically indeterminate internal structure.

System with statically indeterminate bearing arrangement

If the system has a statically indeterminate bearing arrangement, the following guidelines apply:
- Make the system statically determinate by detaching surplus support joints and by applying the bearing reactions to the system as external imposed forces at these points. At the same time, a marginal condition must also be specified for this, which identifies the initial state (point 1)
- At the point where the marginal condition exists, apply a unit force "1" (for deflection) or a unit moment "M" (for torsion) (point 2)
- See points 2 to 4 for statically determinate systems (point 3)
- Apply the marginal condition, divide the relationship by the unit value and calculate the unknown bearing reaction (point 4).

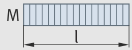
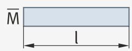
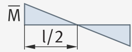
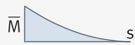
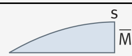
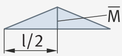
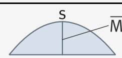
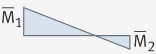
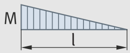
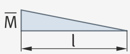
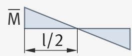
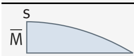
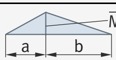
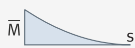
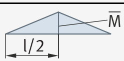
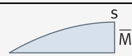
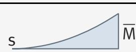
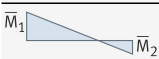
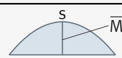
System with a statically indeterminate internal structure

If the system has a statically indeterminate internal structure, the following guidelines apply:
- Make the system statically determinate by installing joints or movable sleeves, or by guiding sections etc. Apply cross-sectional values as external imposed forces and define the marginal conditions
- See points 1 to 4 for systems with statically indeterminate bearing arrangements.

In current practice, systems with statically indeterminate bearing arrangements are calculated numerically.

Tables of integrals The following table shows a selection of sample calculations for the integral: $\int M \cdot \bar{M} \cdot dx$

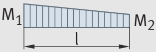
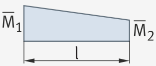
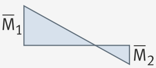
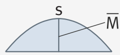
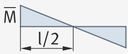
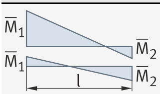
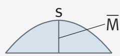
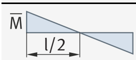
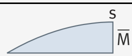
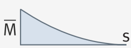
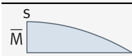
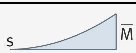
Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$
The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).

			
	$M \cdot \bar{M} \cdot l$		0
	$\frac{1}{2} M \cdot \bar{M} \cdot l$		$\frac{2}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot \bar{M} \cdot l$		$\frac{1}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot \bar{M} \cdot l$		$\frac{2}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot \bar{M} \cdot l$		$\frac{1}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot \left(\bar{M}_1 + \bar{M}_2\right) \cdot l$		$\frac{2}{3} M \cdot \bar{M} \cdot l$
	$\frac{1}{2} M \cdot \left(\bar{M}_1 + \bar{M}_2\right) \cdot l$		
			
	$\frac{1}{3} M \cdot \bar{M} \cdot l$		$\frac{1}{6} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot \bar{M} \cdot l$		$\frac{5}{12} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot \bar{M} \cdot l \cdot \left(1 + \frac{b}{l}\right)$		$\frac{1}{4} M \cdot \bar{M} \cdot l$
	$\frac{1}{4} M \cdot \bar{M} \cdot l$		$\frac{1}{4} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot \left(2\bar{M}_1 + \bar{M}_2\right) \cdot l$		$\frac{1}{12} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot \left(2\bar{M}_1 + \bar{M}_2\right) \cdot l$		$\frac{1}{3} M \cdot \bar{M} \cdot l$

Continuation of table, see Page 248.

Continuation of table, Tables of integrals $\int M \cdot \bar{M} \cdot dx$, from Page 247.

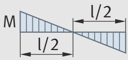
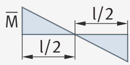
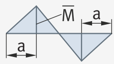
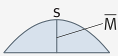
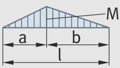
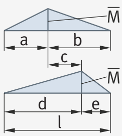
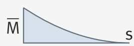
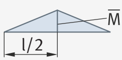
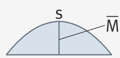
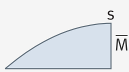
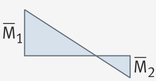
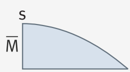
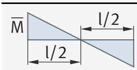
Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$
The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).

			
	$\frac{1}{6}\left[M_1\left(2\bar{M}_1+\bar{M}_2\right)+M_2\left(2\bar{M}_2+\bar{M}_1\right)\right]\cdot l$		$\frac{1}{12}\bar{M}\cdot(M_1+3M_2)\cdot l$
	$\frac{1}{6}\left[2\left(M_1\bar{M}_1+M_2\bar{M}_2\right)+M_2\bar{M}_1+M_1\bar{M}_2\right]\cdot l$		$\frac{1}{3}\bar{M}\cdot(M_1+M_2)\cdot l$
	$\frac{1}{6}\bar{M}\cdot(M_1+M_2)\cdot l$		$\frac{1}{12}\bar{M}\cdot(3M_1+5M_2)\cdot l$
	$\frac{1}{12}\bar{M}\cdot(3M_1+M_2)\cdot l$		$\frac{1}{12}\bar{M}\cdot(5M_1+3M_2)\cdot l$
			
	$\frac{1}{6}\cdot\left[2\left(M_1\bar{M}_1+M_2\bar{M}_2\right)+M_2\bar{M}_1+M_1\bar{M}_2\right]\cdot l$		$\frac{1}{3}\bar{M}\cdot(M_1+M_2)\cdot l$
	$\frac{1}{6}\bar{M}\cdot(M_1-M_2)\cdot l$		$\frac{1}{12}\bar{M}\cdot(3M_1+5M_2)\cdot l$
	$\frac{1}{12}\bar{M}\cdot(3M_1+M_2)\cdot l$		$\frac{1}{12}\bar{M}\cdot(5M_1+3M_2)\cdot l$
	$\frac{1}{12}\bar{M}\cdot(M_1+3M_2)\cdot l$		

Continuation of table, see Page 249.

Continuation of table, Tables of integrals $\int M \cdot \bar{M} \cdot dx$, from Page 248.

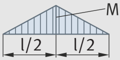
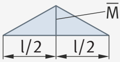
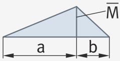
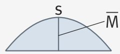
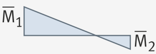
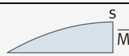
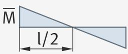
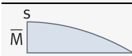
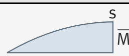
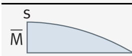
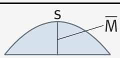
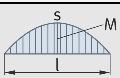
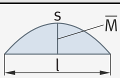
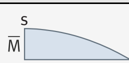
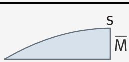
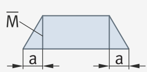
Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$
The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).

			
	$\frac{1}{3} M \cdot \bar{M} \cdot l$		$-\frac{1}{6} M \cdot \bar{M} \cdot l$
	$\frac{1}{6} M \cdot \bar{M} \cdot l$		$\frac{1}{6} M \cdot \bar{M} \cdot l$
	$-\frac{1}{6} M \cdot \bar{M} \cdot l$		$\frac{1}{3} M \cdot \bar{M} \cdot l \cdot \left(1 - \frac{a}{l}\right)$
	0		
			
	$\frac{1}{3} M \cdot \bar{M} \cdot l$		$\frac{1}{12} M \cdot \bar{M} \cdot l \cdot \left(3 - 3\frac{a}{l} + \frac{a^2}{l^2}\right)$
	$\frac{1}{6} M \cdot \bar{M} \cdot l \cdot \left(2 - \frac{c^2}{d \cdot b}\right)$		$\frac{1}{12} M \cdot \bar{M} \cdot l \cdot \left(3 - 3\frac{b}{l} + \frac{b^2}{l^2}\right)$
	$\frac{1}{3} M \cdot \bar{M} \cdot l \cdot \left(\frac{3\,l}{4\,b} - \frac{a^2}{l \cdot b}\right)$		$\frac{1}{3} M \cdot \bar{M} \cdot l \cdot \left(1 + \frac{a}{l} - \frac{a^2}{l^2}\right)$
	$\frac{1}{6} M \left[\bar{M}_1 \left(1 + \frac{b}{l}\right) + \bar{M}_2 \left(1 + \frac{a}{l}\right)\right] \cdot l$		$\frac{1}{12} M \cdot \bar{M} \cdot l \cdot \left(5 - \frac{b}{l} - \frac{b^2}{l^2}\right)$
	$\frac{1}{6} M \left[\bar{M}_1 \left(1 + \frac{b}{l}\right) + \bar{M}_2 \left(1 + \frac{a}{l}\right)\right] \cdot l$		$\frac{1}{12} M \cdot \bar{M} \cdot l \cdot \left(5 - \frac{a}{l} - \frac{a^2}{l^2}\right)$
	$\frac{1}{6} M \cdot \bar{M} \cdot l \cdot \left(1 - 2\frac{a}{l}\right)$		

Continuation of table, see Page 250.

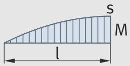
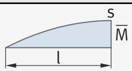
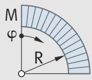
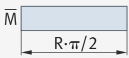
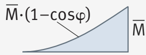
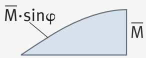
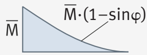
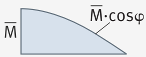
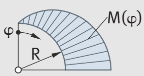
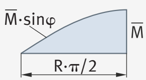
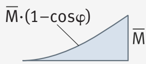
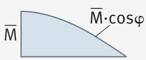
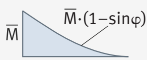
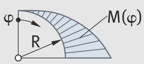
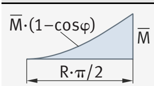
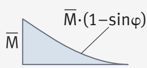
Continuation of table, Tables of integrals $\int M \cdot \bar{M} \cdot dx$, from Page 249.

Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$
The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).

			
	$\frac{1}{3} M \cdot \bar{M} \cdot l$		$\frac{7}{48} M \cdot \bar{M} \cdot l$
	$\frac{1}{3} M \cdot \bar{M} \cdot l \cdot \left(\frac{3\,l}{4\,b} - \frac{a^2}{l \cdot b}\right)$		$\frac{7}{48} M \cdot \bar{M} \cdot l$
	$\frac{1}{4} M \cdot (\bar{M}_1 + \bar{M}_2) \cdot l$		$\frac{5}{12} M \cdot \bar{M} \cdot l$
	$\frac{1}{4} M \cdot (\bar{M}_1 + \bar{M}_2) \cdot l$		$\frac{17}{48} M \cdot \bar{M} \cdot l$
	0		$\frac{17}{48} M \cdot \bar{M} \cdot l$
			
	$\frac{1}{5} M \cdot \bar{M} \cdot l$		$\frac{2}{15} M \cdot \bar{M} \cdot l$
	$\frac{1}{30} M \cdot \bar{M} \cdot l$		$\frac{3}{10} M \cdot \bar{M} \cdot l$
	$\frac{1}{5} M \cdot \bar{M} \cdot l$		
			
	$\frac{8}{15} M \cdot \bar{M} \cdot l$		$\frac{7}{15} M \cdot \bar{M} \cdot l$
	$\frac{7}{15} M \cdot \bar{M} \cdot l$		$\frac{2}{3} M \cdot \bar{M} \cdot l \cdot \left(1 - 2\frac{a^2}{l^2} + \frac{a^3}{l^3}\right)$

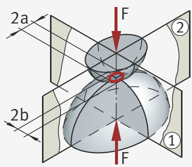
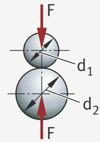
Continuation of table, see Page 251.

Continuation of table, Tables of integrals $\int M \cdot \bar{M} \cdot dx$, from Page 250.

Sample calculations for the integral $\int M \cdot \bar{M} \cdot dx$ The moments M and $\bar{M}$ are interchangeable. Their amounts are specified in the tables, their signs must be observed during the evaluation (s = parabola vertex).			
			
	$\frac{8}{15} M \cdot \bar{M} \cdot l$		$\frac{1}{30} M \cdot \bar{M} \cdot l$
Constant and harmonic moment distributions on circular arc beams			
	$dx = R \cdot d\varphi$ $M = \text{const.}$		
	$M \cdot \bar{M} \cdot R \cdot \frac{\pi}{2}$		$M \cdot \bar{M} \cdot R \left(\frac{\pi}{2} - 1\right)$
	$M \cdot \bar{M} \cdot R$		$M \cdot \bar{M} \cdot R \left(\frac{\pi}{2} - 1\right)$
	$M \cdot \bar{M} \cdot R$		
	$M(\varphi) = \sin \varphi$		
	$\frac{1}{4} M \cdot \bar{M} \cdot R \cdot \pi$		$\frac{1}{2} M \cdot \bar{M} \cdot R$
	$\frac{1}{2} M \cdot \bar{M} \cdot R$		$M \cdot \bar{M} \cdot R \cdot \left(1 - \frac{\pi}{4}\right)$
	$M(\varphi) = 1 - \cos \varphi$		
	$M \cdot \bar{M} \cdot R \left(\frac{3}{4}\pi - 2\right)$		$M \cdot \bar{M} \cdot R \left(\frac{\pi}{2} - \frac{3}{2}\right)$

Hertzian contact and pressure

Calculating contact pairs The equations for calculating a number of important Hertzian contact pairs are:

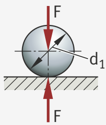
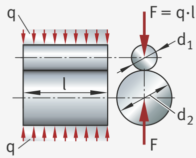
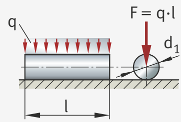
Contact type	General	Contact between curved surfaces	Point contact ball/ball
Hertzian pressure (general formula)	p_{max}	$p_{max} = \frac{3}{2} \cdot \frac{F}{\pi \cdot a\,b}$	$p_{max} = \frac{3}{2} \cdot \frac{F}{\pi \cdot a^2}$
Solid body combination Solid body 1, solid body 2[1]	$\sum\rho = \rho_{11} + \rho_{12} + \rho_{21} + \rho_{22}$ Calculation of the curvature total, see Page 254.	①, ② Principal planes of curvature 	Ball/ball 
Main axes of the "elliptical" contact area	a, b a = major semi-axis b = minor semi-axis	$a = \xi \sqrt[3]{\frac{3F\left(1-\nu^2\right)}{E \cdot \sum\rho}}$ $b = \eta \sqrt[3]{\frac{3F\left(1-\nu^2\right)}{E \cdot \sum\rho}}$ $\xi, \eta = f(\cos\tau)$[2]	$a = b = \sqrt[3]{\frac{3F\left(1-\nu^2\right)}{4E\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}}$
Maximum Hertzian pressure	p_{max}	$p_{max} = \frac{1}{\xi \cdot \eta} \sqrt[3]{\frac{3F \cdot E^2 \left(\sum\rho\right)^2}{8\pi^3\left(1-\nu^2\right)^2}}$ $\xi, \eta = f(\cos\tau)$[2]	$p_{max} = \sqrt[3]{\frac{6F \cdot E^2}{\pi^3\left(1-\nu^2\right)^2}\left(\frac{1}{d_1}+\frac{1}{d_2}\right)^2}$
Convergence of both solid bodies	δ	$\delta = \frac{\psi}{\xi} \sqrt[3]{\frac{9F^2 \sum\rho \left(1-\nu^2\right)^2}{8E^2}}$ $\psi/\xi = f(\cos\tau)$[2]	$\delta = \sqrt[3]{\frac{9F^2\left(1-\nu^2\right)^2}{2E^2}\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}$
Hertzian pressure for values from column "General"	p_{max} in N/mm² for $\nu_1 = \nu_2 = 0{,}3$ $E_1 = E_2 = 2{,}1 \cdot 10^5$ N/mm² d, l in mm, F in N	$p_{max} = \frac{864}{\xi \cdot \eta} \cdot \sqrt[3]{F\left(\sum\rho\right)^2}$ $\xi \cdot \eta = f(\cos\tau)$[2]	$p_{max} = 2176 \sqrt[3]{F\left(\frac{1}{d^1}+\frac{1}{d^2}\right)^2}$

Continuation of table, see Page 253.

[1] If the solid bodies are composed of different materials with the elastic constants E_1 and ν_1 or E_2 and ν_2, the term $\frac{1-\nu^2}{E}$ is replaced in all equations by $\frac{1}{2}\left(\frac{1-\nu_1{}^2}{E_1}+\frac{1-\nu_2{}^2}{E_2}\right)$

[2] Auxiliary value $\cos\tau$ see Page 254.

Continuation of table, Calculating contact pairs, from Page 252.

Contact type	Point contact ball/plane	Line contact	
Hertzian pressure (general formula)	$p_{max} = \frac{3}{2} \cdot \frac{F}{\pi \cdot a^2}$	$p_{max} = \frac{4}{\pi} \cdot \frac{F}{2\,b \cdot l} = \frac{2}{\pi} \cdot \frac{F}{b \cdot l}$	
Solid body combination Solid body 1, solid body 2[1)]	Ball/plane 	Cylinder/cylinder 	Cylinder/plane 
Main axes of the "elliptical" contact area	$a = b = \sqrt[3]{\frac{3\,F\left(1-\nu^2\right)d_1}{4\,E}}$	$a = l$ $b = \sqrt{\frac{4\,F\left(1-\nu^2\right)}{\pi \cdot E \cdot l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}}$	$a = l$ $b = \sqrt{\frac{4\,F\left(1-\nu^2\right)d_1}{\pi \cdot E \cdot l}}$ Contact area = rectangle
Maximum Hertzian pressure	p_{max} $= \sqrt[3]{\frac{6\,F \cdot E^2}{\pi^3\left(1-\nu^2\right)^2 \cdot d_1^{\,2}}}$	p_{max} $= \sqrt{\frac{F \cdot E}{\pi \cdot l\left(1-\nu^2\right)}\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}$	$p_{max} = \sqrt{\frac{F \cdot E}{\pi \cdot d_1 \cdot l \cdot \left(1-\nu^2\right)}}$
Convergence of both solid bodies	$\delta = \sqrt[3]{\frac{9\,F^2\left(1-\nu^2\right)^2}{2\,E^2 \cdot d_1}}$	$\delta = \frac{2\,F}{\pi\,l}\left[\frac{1-\nu_1^{\,2}}{E_1}\left(\ln\frac{d_1}{b}+0{,}407\right) + \frac{1-\nu_2^{\,2}}{E_2}\left(\ln\frac{d_2}{b}+0{,}407\right)\right]$	$\delta = \frac{3{,}97}{10^5} \cdot \frac{F^{0{,}9}}{l^{0{,}8}}$ for steel/steel
Hertzian pressure for values from column "General"	$p_{max} = 2176\sqrt[3]{\frac{F}{d_1^{\,2}}}$	$p_{max} = 271\sqrt{\frac{F}{d_1 \cdot l}\left(1+\frac{d_1}{d_2}\right)}$	$p_{max} = 271\sqrt{\frac{F}{d_1 \cdot l}}$

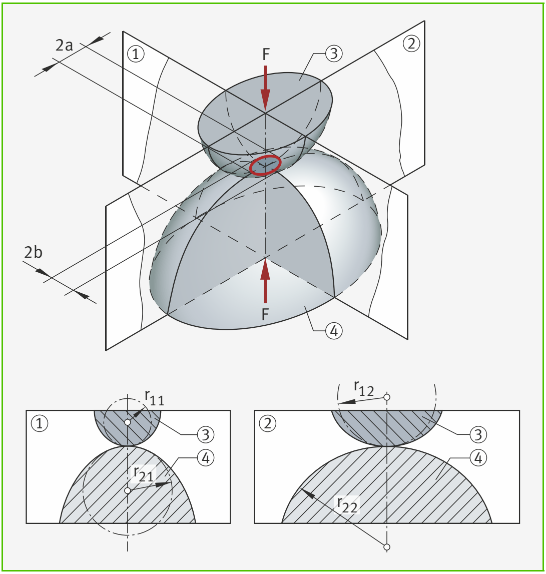
[1)] If the solid bodies are composed of different materials with the elastic constants E_1 and ν_1 or E_2 and ν_2, the term $\frac{1-\nu^2}{E}$ is replaced in all equations by $\frac{1}{2}\left(\frac{1-\nu_1^{\,2}}{E_1}+\frac{1-\nu_2^{\,2}}{E_2}\right)$

Hertzian coefficients for curved surfaces

Values for the coefficients ξ, η, $\xi \cdot \eta$, ψ/ξ according to Hertz are calculated below for contact between curved surfaces under load.
The Hertzian coefficients ξ, η and ψ/ξ are also designated μ, ν and $2K/(\pi\,\mu)$, see publication Ball and Roller Bearings WLP.

Figure 4
Contact between curved surfaces under load

① Principal curvature plane 1
② Principal curvature plane 2
③ Solid body 1
④ Solid body 2



Calculation

Equation 23

For the curvature ρ, the following applies:

$$\rho_{ij} = \frac{1}{r_{ij}} > 0 \quad \text{(convex)} \qquad \rho_{ij} = \frac{1}{r_{ij}} < 0 \quad \text{(concave)}$$

$$\sum\rho = \rho_{11} + \rho_{12} + \rho_{21} + \rho_{22} = \frac{1}{r_{11}} + \frac{1}{r_{12}} + \frac{1}{r_{21}} + \frac{1}{r_{22}}$$

and for the auxiliary value $\cos\tau$:

Equation 24

$$\cos\tau = \frac{\left|\rho_{11} - \rho_{12} + \rho_{21} - \rho_{22}\right|}{\sum\rho}$$

The axes a, b of the contact ellipse are calculated using:

Equation 25

$$a = \xi \sqrt[3]{\frac{3F\left(1-\nu^2\right)}{E \cdot \sum \rho}} \qquad b = \eta \sqrt[3]{\frac{3F\left(1-\nu^2\right)}{E \cdot \sum \rho}}$$

For the maximum Hertzian pressure, the following applies:

Equation 26

$$p_{max} = \frac{1}{\xi \cdot \eta} \sqrt[3]{\frac{3F \cdot E^2 \left(\sum \rho\right)^2}{8\pi^3 \left(1-\nu^2\right)^2}}$$

and for the convergence of solid bodies:

Equation 27

$$\delta = \frac{\psi}{\xi} \sqrt[3]{\frac{9F^2 \cdot \sum \rho \left(1-\nu^2\right)^2}{8E^2}}$$

Table: Hertzian coefficients

The coefficients ξ, η, $\xi \cdot \eta$, ψ/ξ according to Hertz for contact between curved surfaces under load are thus:

cos τ	ξ	η	$\xi \cdot \eta$	ψ/ξ
0,9995	23,95	0,163	3,91	0,171
0,9990	18,53	0,185	3,43	0,207
0,9985	15,77	0,201	3,17	0,230
0,9980	14,25	0,212	3,02	0,249
0,9975	13,15	0,220	2,89	0,266
0,9970	12,26	0,228	2,80	0,279
0,9965	11,58	0,235	2,72	0,291
0,9960	11,02	0,241	2,65	0,302
0,9955	10,53	0,246	2,59	0,311
0,9950	10,15	0,251	2,54	0,320
0,9945	9,77	0,256	2,50	0,328
0,9940	9,46	0,260	2,46	0,336
0,9935	9,17	0,264	2,42	0,343
0,9930	8,92	0,268	2,39	0,350
0,9925	8,68	0,271	2,36	0,356
0,9920	8,47	0,275	2,33	0,362
0,9915	8,27	0,278	2,30	0,368
0,9910	8,10	0,281	2,28	0,373
0,9905	7,93	0,284	2,25	0,379
0,9900	7,76	0,287	2,23	0,384
0,9895	7,62	0,289	2,21	0,388
0,9890	7,49	0,292	2,19	0,393
0,9885	7,37	0,294	2,17	0,398
0,9880	7,25	0,297	2,15	0,402
0,9875	7,13	0,299	2,13	0,407
0,9870	7,02	0,301	2,11	0,411
0,9865	6,93	0,303	2,10	0,416
0,9860	6,84	0,305	2,09	0,420
0,9855	6,74	0,307	2,07	0,423
0,9850	6,64	0,310	2,06	0,427
0,9845	6,55	0,312	2,04	0,430
9,9840	6,47	0,314	2,03	0,433
0,9835	6,40	0,316	2,02	0,437
0,9830	6,33	0,317	2,01	0,440
0,9825	6,26	0,319	2,00	0,444
0,9820	6,19	0,321	1,99	0,447
0,9815	6,12	0,323	1,98	0,450
0,9810	6,06	0,325	1,97	0,453
0,9805	6,00	0,327	1,96	0,456
0,9800	5,94	0,328	1,95	0,459
0,9795	5,89	0,330	1,94	0,462
0,9790	5,83	0,332	1,93	0,465
0,9785	5,78	0,333	1,92	0,468
0,9780	5,72	0,335	1,92	0,470
0,9775	5,67	0,336	1,91	0,473
0,9770	5,63	0,338	1,90	0,476
0,9765	5,58	0,339	1,89	0,478
0,9760	5,53	0,340	1,88	0,481
0,9755	5,49	0,342	1,88	0,483
0,9750	5,44	0,343	1,87	0,486

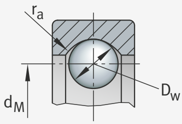
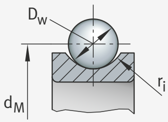
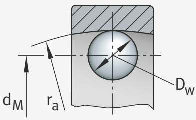
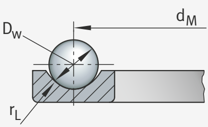
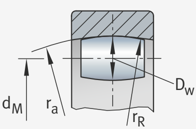
Continuation of table, see Page 256.

Continuation of table, Table: Hertzian coefficients, from Page 255.

cos τ	ξ	η	ξ · η	ψ/ξ
0,9745	5,39	0,345	1,86	0,489
0,9740	5,35	0,346	1,85	0,491
0,9735	5,32	0,347	1,85	0,493
0,9730	5,28	0,349	1,84	0,495
0,9725	5,24	0,350	1,83	0,498
0,9720	5,20	0,351	1,83	0,500
0,9715	5,16	0,353	1,82	0,502
0,9710	5,13	0,354	1,81	0,505
0,9705	5,09	0,355	1,81	0,507
0,9700	5,05	0,357	1,80	0,509
0,9690	4,98	0,359	1,79	0,513
0,9680	4,92	0,361	1,78	0,518
0,9670	4,86	0,363	1,77	0,522
0,9660	4,81	0,365	1,76	0,526
0,9650	4,76	0,367	1,75	0,530
0,9640	4,70	0,369	1,74	0,533
0,9630	4,65	0,371	1,73	0,536
0,9620	4,61	0,374	1,72	0,540
0,9610	4,56	0,376	1,71	0,543
0,9600	4,51	0,378	1,70	0,546
0,9590	4,47	0,380	1,70	0,550
0,9580	4,42	0,382	1,69	0,553
0,9570	4,38	0,384	1,68	0,556
0,9560	4,34	0,386	1,67	0,559
0,9550	4,30	0,388	1,67	0,562
0,9540	4,26	0,390	1,66	0,565
0,9530	4,22	0,391	1,65	0,568
0,9520	4,19	0,393	1,65	0,571
0,9510	4,15	0,394	1,64	0,574
0,9500	4,12	0,396	1,63	0,577
0,9480	4,05	0,399	1,62	0,583
0,9460	3,99	0,403	1,61	0,588
0,9440	3,94	0,406	1,60	0,593
0,9420	3,88	0,409	1,59	0,598
0,9400	3,83	0,412	1,58	0,603
0,9380	3,78	0,415	1,57	0,608
0,9360	3,73	0,418	1,56	0,613
0,9340	3,68	0,420	1,55	0,618
0,9320	3,63	0,423	1,54	0,622
0,9300	3,59	0,426	1,53	0,626
0,928	3,55	0,428	1,52	0,630
0,926	3,51	0,431	1,51	0,634
0,924	3,47	0,433	1,50	0,638
0,922	3,43	0,436	1,50	0,642
0,920	3,40	0,438	1,49	0,646
0,918	3,36	0,441	1,48	0,650
0,916	3,33	0,443	1,47	0,653
0,914	3,30	0,445	1,47	0,657
0,912	3,27	0,448	1,46	0,660
0,910	3,23	0,450	1,45	0,664
0,908	3.20	0,452	1,45	0,667
0,906	3,17	0,454	1,44	0,671
0,904	3,15	0,456	1,44	0,674
0,902	3,12	0,459	1,43	0,677
0,900	3,09	0,461	1,42	0,680
0,895	3,03	0,466	1,41	0,688
0,890	2,97	0,471	1,40	0,695
0,885	2,92	0,476	1,39	0,702
0,880	2,86	0,481	1,38	0,709
0,875	2,82	0,485	1,37	0,715
0,870	2,77	0,490	1,36	0,721
0,865	2,72	0,494	1,35	0,727
0,860	2,68	0,498	1,34	0,733
0,855	2,64	0,502	1,33	0,739
0,850	2,60	0,507	1,32	0,745
0,840	2,53	0,515	1,30	0,755
0,830	2,46	0,523	1,29	0,765
0,820	2,40	0,530	1,27	0,774
0,810	2,35	0,537	1,26	0,783
0,800	2,30	0,544	1,25	0,792
0,750	2,07	0,577	1,20	0,829
0,700	1,91	0,607	1,16	0,859
0,650	1,77	0,637	1,13	0,884
0,600	1,66	0,664	1,10	0,904
0,550	1,57	0,690	1,08	0,922
0,500	1,48	0,718	1,06	0,938
0,450	1,41	0,745	1,05	0,951
0,400	1,35	0,771	1,04	0,962
0,350	1,29	0,796	1,03	0,971
0,300	1,24	0,824	1,02	0,979
0,250	1,19	0,850	1,01	0,986
0,200	1,15	0,879	1,01	0,991
0,150	1,11	0,908	1,01	0,994
0,100	1,07	0,938	1,00	0,997
0,050	1,03	0,969	1,00	0,999
0	1	1	1	1

Hertzian pressure in rolling bearings

The following auxiliary values are used to calculate Hertzian pressure in rolling bearings:

Deep groove ball bearings ball – outer ring 	[1] $\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ $r_a \approx 0{,}53 \cdot D_w$	$\sum\rho = \frac{2}{D_w} \cdot \left(2 - \frac{\gamma}{1+\gamma} - \frac{D_w}{2 \cdot r_a}\right)$ $\cos\tau = \frac{-\frac{\gamma}{1+\gamma} + \frac{D_w}{2 \cdot r_a}}{2 - \frac{\gamma}{1+\gamma} - \frac{D_w}{2 \cdot r_a}}$
Deep groove ball bearings ball – inner ring 	[1] $\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ $r_i \approx 0{,}52 \cdot D_w$	$\sum\rho = \frac{2}{D_w} \cdot \left(2 + \frac{\gamma}{1-\gamma} - \frac{D_w}{2 \cdot r_i}\right)$ $\cos\tau = \frac{\frac{\gamma}{1-\gamma} + \frac{D_w}{2 \cdot r_i}}{2 + \frac{\gamma}{1-\gamma} - \frac{D_w}{2 \cdot r_i}}$
Self-aligning ball bearings ball – outer ring 	[1] $\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ $r_a = \frac{d_M + D_w}{2}$	$\sum\rho = \frac{4}{D_w}\left(\frac{1}{1+\gamma}\right)$ $\cos\tau = 0$
Axial ball bearings ball – bearing ring 	[1] $\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ $r_L \approx 0{,}54 \cdot D_w$	$\sum\rho = \frac{2}{D_w}\left(2 - \frac{D_w}{2 \cdot r_L}\right)$ $\cos\tau = \frac{1}{4 \cdot \frac{r_L}{D_w} - 1}$
Barrel roller bearings, spherical roller bearings roller – outer ring 	[1] $\gamma = \frac{D_w}{d_M} \cdot \cos\alpha$ $r_a = \frac{d_M + D_w}{2}$	$\sum\rho = \frac{1 + \gamma \cdot \left(\frac{r_A}{r_R} - 1\right)}{1+\gamma}$ $\cos\tau = \frac{1 - \gamma \cdot \left(\frac{r_a}{r_R} - 1\right)}{1 + \gamma \cdot \left(\frac{r_a}{r_R} - 1\right)}$

[1] α is the nominal contact angle of the bearing.

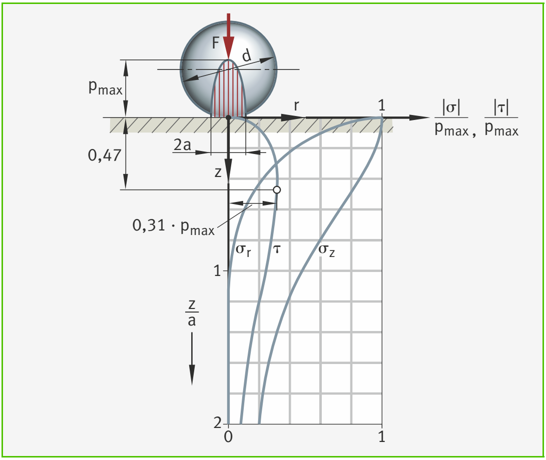
Stress state under Hertzian contact

According to Hertz's theory, stresses and deformations occur due to the effect of compressive forces where there is contact (point type or linear) between two solid bodies.

Point contact ball – plane

In the event of ball – plane point contact, the following relationships apply:

Figure 5
Point contact ball – plane



For the radius of the contact area, the following applies:

Equation 28

$$a = \sqrt[3]{\frac{3F\left(1-\nu^2\right)\cdot d}{4E}}$$

The stress state (for r = 0) is calculated using:

Equation 29

$$\frac{\sigma_z}{p_{max}} = -\frac{1}{\left(\frac{z}{a}\right)^2 + 1}$$

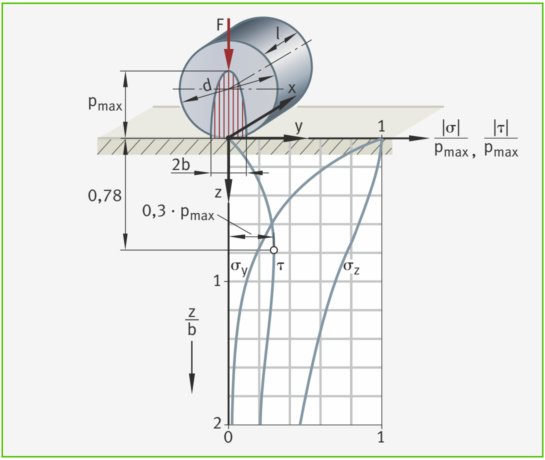
$$\frac{\sigma_r}{p_{max}} = -\left(1+\nu\right)\left[1-\frac{z}{a}\arctan\left(\frac{a}{z}\right)\right] + \frac{1}{2\left[\left(\frac{z}{a}\right)^2+1\right]}$$

$$\frac{\tau}{p_{max}} = -\frac{3}{4}\cdot\frac{1}{\left(\frac{z}{a}\right)^2+1} + \frac{1+\nu}{2}\left[1-\frac{z}{a}\arctan\left(\frac{a}{z}\right)\right]$$

Line contact roller – plane

In the event of roller – plane line contact (**planar stress state**), the following relationships apply:

Figure 6
Line contact roller – plane (planar stress state)



For the half width of the contact area, the following applies:

Equation 30

$$b = \sqrt{\frac{4\,F\left(1-\nu^2\right)\cdot d}{\pi \cdot E \cdot l}}$$

The stress state (for y = 0) is calculated using:

Equation 31

$$\frac{\sigma_z}{p_{max}} = -\frac{1}{\sqrt{1+\left(\frac{z}{b}\right)^2}}$$

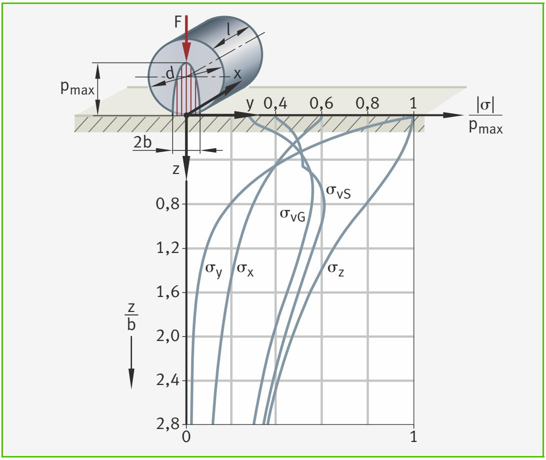
$$\frac{\sigma_y}{p_{max}} = 2\left(\frac{z}{b}\right) - \frac{1+2\left(\frac{z}{b}\right)^2}{\sqrt{1+\left(\frac{z}{b}\right)^2}}$$

$$\frac{\tau}{p_{max}} = \frac{z}{b} - \frac{\left(\frac{z}{b}\right)^2}{\sqrt{1+\left(\frac{z}{b}\right)^2}}$$

Line contact roller – plane

In the event of roller – plane line contact (**spatial stress state**), the following relationships apply:

Figure 7
Line contact roller – plane (spatial stress state)



The stress state (for x = y = 0) is calculated using:

Equation 32

$$\frac{\sigma_x}{p_{max}} = -2\nu\left[\sqrt{1+\left(\frac{z}{b}\right)^2} - \left(\frac{z}{b}\right)\right]$$

$$\frac{\sigma_y}{p_{max}} = -\left[\frac{1+2\left(\frac{z}{b}\right)^2}{\sqrt{1+\left(\frac{z}{b}\right)^2}} - 2\left(\frac{z}{b}\right)\right]$$

$$\frac{\sigma_z}{p_{max}} = -\frac{1}{\sqrt{1+\left(\frac{z}{b}\right)^2}}$$

These equations represent the maximum stresses for the coordinates x = y = 0. They are based on the assumption of a plane deformation state ($\epsilon_x = 0$).

Equivalent stress

As strength hypotheses for calculating an equivalent stress, the following hypotheses have generally gained acceptance and proved effective:

- **shear stress hypothesis** according to **Tresca** and **St. Venant**
- **distortion energy hypothesis** according to **Hencky** and **von Mises**.

Shear stress hypothesis

According to the shear stress hypothesis, it is assumed that the material begins to flow when the maximum shear stress at any one point reaches a critical value.

The equivalent stress is calculated as:

Equation 33

$$\sigma_{vS} = 2\,\tau_{max} = \max\begin{bmatrix}\sigma_z - \sigma_y \\ \sigma_z - \sigma_x \\ \sigma_y - \sigma_x\end{bmatrix}$$

Distortion energy hypothesis

By contrast, according to the distortion energy hypothesis, plastic deformation sets in when the distortion energy that can be absorbed elastically in a volume element is exceeded.

The equivalent stress is calculated as:

Equation 34

$$\sigma_{vG} = \sqrt{\frac{1}{2}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2\right]}$$

with the principal normal stresses σ_x, σ_y, σ_z.

Material strength

According to the shear stress hypothesis, the maximum equivalent stress within the material is:

Shear stress hypothesis	Distortion energy hypothesis
$\sigma_{vS\,max} = 0{,}60 \cdot p_{max}$ at a depth $z = 0{,}78 \cdot b$	$\sigma_{vG\,max} = 0{,}56 \cdot p_{max}$ at a depth $z = 0{,}71 \cdot b$

If the equivalent stress according to the shear stress hypothesis is assumed, the material strain is:

Equation 35

$$\sigma_{vS\,max} = 0{,}60 \cdot p_{max}$$

In order to prevent plastic deformation, according to the shear stress hypothesis, in the material under static loading (constant strength over the whole cross-section), the following must be fulfilled:

Equation 36

$$\sigma_{vS\,max} < R_{p0{,}2}$$

In a given material with the yield point R_e or substitute proof stress $R_{p0,2}$, this gives a permissible maximum Hertzian pressure of:

Equation 37

$$p_{max\,per} < 1{,}67 \cdot R_{p0,2}$$

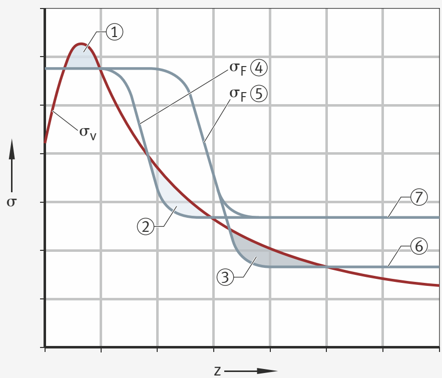
For case, flame or induction hardened materials, an adequate hardening depth must be ensured. The hardening depth according to DIN EN ISO 2639:2002 is the depth of hardened surface zone which has a hardness of at least 550 HV. The hardness curve to the core of the material must also be such that the strength and yield point of the material which can be derived from the hardness is above the equivalent stress curve at all points.

Figure 8 shows where deformation zones can form in the material when comparing the maximum equivalent stress with the yield point of the material:

Figure 8
Deformation zones

σ_V = equivalent stress
σ_F = yield stress
z = surface distance

①, ②, ③ Deformation zones
④ For small case hardening depth
⑤ For large case hardening depth
⑥ Low core strength
⑦ High core strength



- In zone ①, the yield point of the material is exceeded in the area of maximum stress in a material with constant strength, respectively through hardening, as well as surface layer hardening. This deformation occurs at sufficiently high Hertzian pressure in all materials and for all hardening processes.
- In zone ②, the material deforms plastically if the chosen hardening depth is too low.
- In zone ③, plastic deformation occurs if the hardness or the yield stress of the core material is too low.

A steep hardness gradient, which may occur particularly with flame or induction hardening, leads under the same nominal hardening depth to an expansion of the deformation zones.

Hardening depth

Case, flame or induction hardened raceways must have a surface hardness of 670 HV to 840 HV and a sufficiently large hardening depth (for case hardening: case hardening depth CHD; for flame or induction hardening: surface hardening depth SHD).

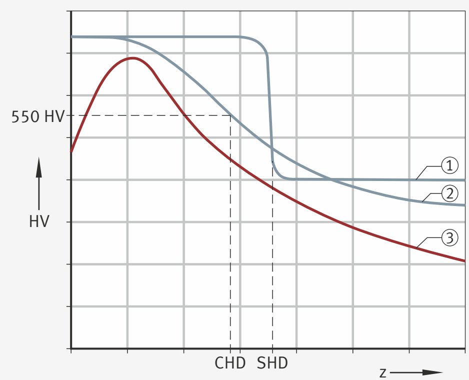
The hardness curves are shown in Figure 9; the required hardness curve is determined by converting the equivalent stress curve into Vickers hardness (see conversion table in the chapter Construction materials, Page 303).

The following curves are produced for surface hardness:

Figure 9
Hardening depth

HV = Vickers hardness
z = Distance from surface
CHD = case hardening depth
SHD = surface hardening depth

1 Flame or induction hardening
2 Case hardening
3 Required hardness



The required minimum hardening depth depends essentially on the rolling element diameter, material loading, core strength and the hardening process.

For raceways loaded up to the static load carrying capacity C_0, in which a Hertzian pressure of $p_{max} = 4000\ N/mm^2$ is present for line contact, the hardening depths can be calculated from the following relationships.

The case hardening depth for **case hardening** is:

Equation 38

$$CHD \geq 0{,}052 \cdot D_w$$

D_w = rolling element diameter

The surface hardening depth for **flame or induction hardening** is:

Equation 39

$$SHD \geq 140 \cdot D_w / R_{p0,2}$$

Material selection for rolling bearing raceways

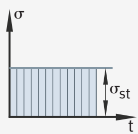
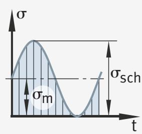
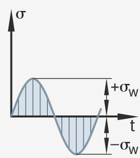
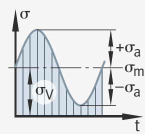
When selecting materials for rolling bearing raceways, it is necessary to bear in mind that in order to achieve the full load carrying capacity of the bearing location, a surface hardness of 670 HV to 840 HV, an adequate hardening depth and a degree of cleanliness corresponding to that of normal steels must be present.

The following are particularly suitable for rolling elements and rolling bearing rings:

- **through hardening steels**
  (for example 100Cr6 to DIN EN ISO 683-17). Surface layer hardening of these rolling bearing steels is also feasible in special cases
- **case hardening steels**
  (for example 17MnCr5 to DIN EN ISO 683-17 or 16MnCr5 to DIN EN 10084). The selection process must take account of both hardenability and core strength. For case hardening, a fine-grained structure is required and the case hardening depth must be selected accordingly
- **steels for flame or induction hardening**
  (for example C56E2 to DIN EN ISO 683-17). For flame or induction hardening, only the parts of the machine component used as raceways for the rolling elements must be hardened. Once again, hardenability is also an important precondition in material selection. The material should be in a quenched and tempered state for hardening.

Dynamic loading – geometrical stability

Component loading The stress distributions for component loading are as follows:

Load case	I static	II pulsating	III alternating	I + III general oscillating
				
Maximum stress	σ_{st} = const.	$\sigma_o = \sigma_{sch}$	$\sigma_o = +\sigma_W$	$\sigma_o = \sigma_m + \sigma_a$
Mean stress	–	$\sigma_m = \sigma_{sch}/2$	$\sigma_m = 0$	$\sigma_m = \sigma_V$ (preload)
Minimum stress	–	$\sigma_u = 0$	$\sigma_u = -\sigma_W$	$\sigma_u = \sigma_m - \sigma_a$
Strength characteristic of the material that is decisive for component calculation				
	Breaking strength R_m (brittle material)	Fatigue strength under pulsating stresses σ_{Sch}	Fatigue strength under reversed stresses σ_W	Amplitude strength σ_A
	Yield point R_e; $R_{p0,2}$ (tough material)	Fatigue limit characteristic σ_D (general) or low cycle fatigue strength for low cycle fatigue design, for example σ_{Sch-N}, σ_{W-N}		

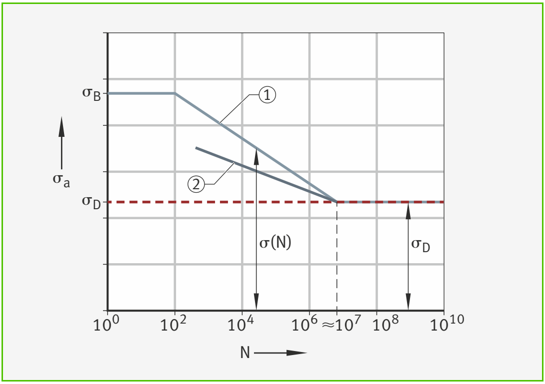
Wöhler's diagram

The following diagram shows Wöhler's diagram for the example of tension – compression.

Figure 10
Wöhler's diagram

σ_a = stress amplitude
N = load cycles
σ_D = fatigue limit
$\sigma(N)$ = low cycle fatigue strength
σ_B = stress at fracture

① Wöhler curve
② Damage line



If the load falls below the damage line, the material will not be subjected to any preliminary damage.

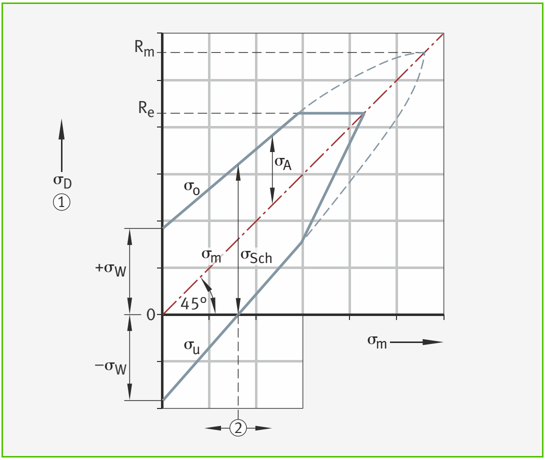
Fatigue strength diagram in accordance with Smith

Fatigue strength is illustrated in the diagram according to Smith.

Figure 11
Fatigue strength diagram in accordance with Smith

σ_D = fatigue limit
σ_m = mean stress
σ_W = fatigue strength under reversed stresses
σ_A = amplitude strength
σ_{Sch} = fatigue strength under pulsating stresses
σ_o = maximum stress
σ_u = minimum stress
R_m = breaking strength
R_e = yield point

① Strength characteristics
② Reverse stress domain, pulsating stress domain



Influence of size and surface

The influence of size and surface on component strength (≠ material strength) is illustrated in the diagrams in Figure 12 and Figure 13. The following applies:

Equivalent stress on the component		Permissible load		Geometrical stability of the shape
σ_v	$\leqq$	σ_{per}	=	$\dfrac{\sigma_D \cdot b_o \cdot b_d}{S_{min} \cdot \beta_k}$

where

σ_D = decisive fatigue limit value of the material

b_o = surface factor ($\leqq 1$)

b_d = size factor ($\leqq 1$)

β_k = fatigue notch factor ($\geqq 1$)

S_{min} = minimum safety (1,2 ... 2)

Figure 12
Influence of size

b_d = size factor
d = component diameter

① For bending and torsion
② For tension/compression

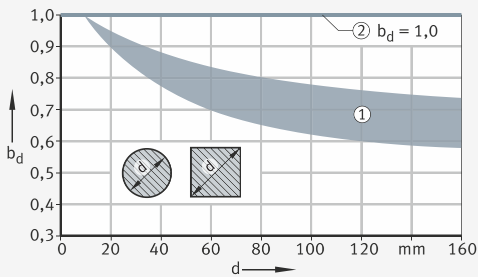
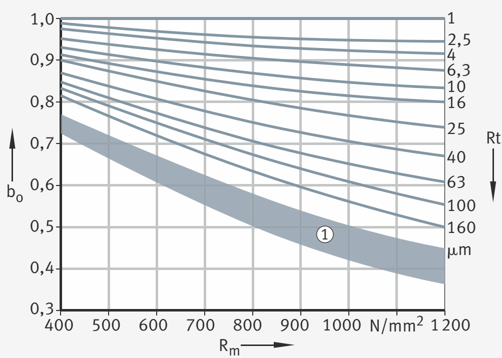


Figure 13
Influence of surface quality on fatigue strength

b_o = surface factor
Rt = roughness depth
R_m = breaking strength of the material

① Surfaces with rolling scale



Fatigue strength diagrams for general construction steels

Fatigue strength diagrams for general construction steels (DIN 17100 and DIN EN 10025) are illustrated below.

The following applies:

- load case I: stationary loading
- load case II: pure pulsating loading
- load case III: pure alternating loading.

For cold drawn material (for example E295GC), the yield point values may be up to 50% higher, whereas the fatigue strength under pulsating and under completely reversed stress may only be assumed to be about 10% higher than the table values. Cold forming reduces the material's ability to stretch, and its sensitivity to cleavage fracture is higher.

Tensile and compressive loading

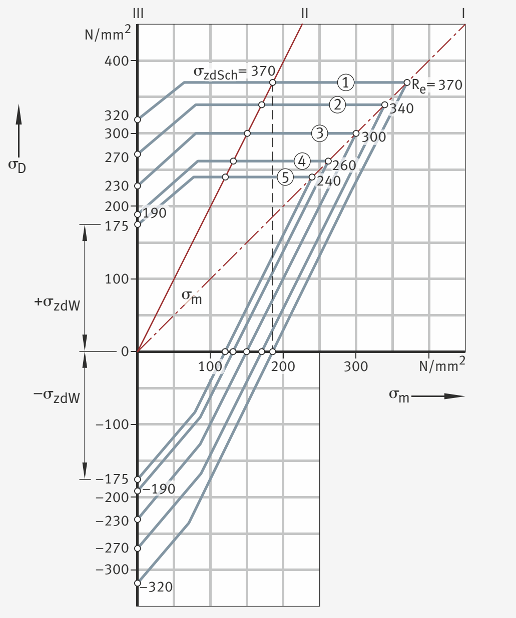
Under tensile and compressive loading we arrive at:

Figure 14

Fatigue strength of general construction steels: tensile and compressive loading

σ_D = fatigue limit
σ_m = mean stress
σ_{zdW} = fatigue strength under reversed tensile/compressive stresses
σ_{zdSch} = fatigue strength under pulsating tensile stresses
R_e = yield point

1. E360
2. E335
3. E295
4. S275
5. S235



Source: Steinhilper, W.; R. Röper: Maschinen- und Konstruktionselemente. Springer-Verlag, Berlin, Heidelberg, New York (2002).

Torsional loading

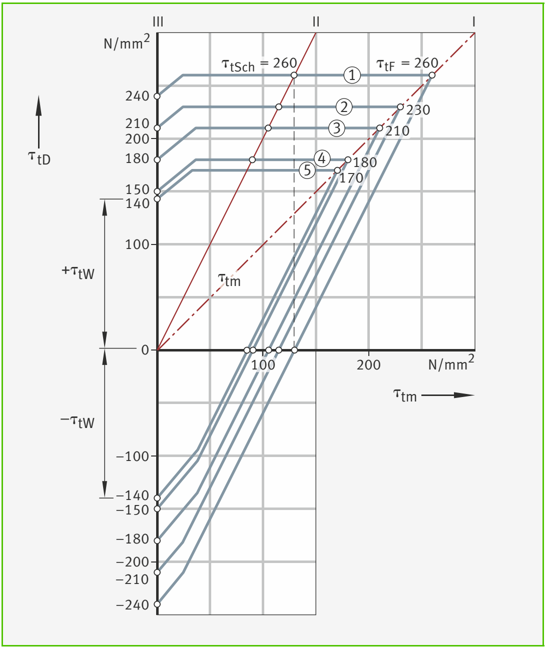
Figure 15

Fatigue strength of general construction steels: torsional loading

τ_{tD} = torsional fatigue limit
τ_{tm} = torsional mean stress
τ_{tW} = fatigue strength under reversed torsional stresses
τ_{tSch} = fatigue strength under pulsating torsional stresses
τ_{tF} = yield point under torsion

1 E360
2 E335
3 E295
4 S275
5 S235

Under torsional loading we arrive at:



Source: Steinhilper, W.; R. Röper: Maschinen- und Konstruktionselemente. Springer-Verlag, Berlin, Heidelberg, New York (2002).

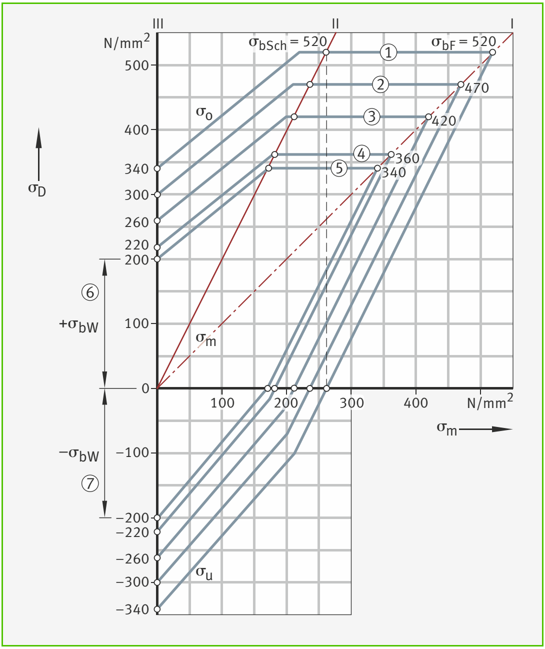
Bending loading

Figure 16
Fatigue strength of general construction steels: bending loading

σ_D = fatigue limit
σ_m = mean stress
σ_{bW} = fatigue strength under reversed bending stresses
σ_o = maximum stress
σ_u = minimum stress
σ_{bSch} = fatigue strength under pulsating bending stresses
σ_{bF} = yield point under bending

1 E360
2 E335
3 E295
4 S275
5 S235
6 Tensile area
7 Compressive area

Under bending loading we arrive at:



Source: Steinhilper, W.; R. Röper: Maschinen- und Konstruktionselemente. Springer-Verlag, Berlin, Heidelberg, New York (2002).

Construction materials

A selection of significant construction materials is presented on the following pages – starting with various steel grades (including their heat treatment), via cast iron, cast steel and non-ferrous metals through to plastics.

Steel

Steel grades

Various grades of steels are defined in DIN standards. A selection of steels is described in more detail below:

- unalloyed structural steels
- quenched and tempered steels
- case hardening steels
- stainless steels
- rolling bearing steels
- free-cutting steels.

System of material designations

The standard DIN EN 10027-1:2005 gives short names for steels. The whole standard is very extensive. For this reason, the system of engineering steels is presented as an example.

Main symbols					Additional symbols for steels	Additional symbols for steel products
G	E	n	n	n	an...	+an +an...[1)]

Main symbols		Additional symbols		
Letter	Mechanical properties	For steel		For steel products
		Group 1	Group 2	
G = Cast steel (if necessary) E = Engineering steels	nnn = Defined yield strength[2)] in MPa[3)] for the smallest thickness value	G = Other features, if necessary with one or two consecutive numbers or if notched bar impact properties are defined, in accordance with the following rules according to table Notched bar impact work, Page 271	C = Suitability for cold drawing	Symbols for the treated condition: see standard DIN EN 10027-1:2005, Table 18

1) n = number, a = letter, an = alphanumeric.

2) The term “yield strength” is defined, in accordance with the information in the relevant product standard, as the upper or lower yield strength (R_{eH} or R_{eL}) or the proof stress under non-proportional elongation (R_p) or proof stresss under complete elongation (R_t).

3) 1 MPa = 1 N/mm^2.

Notched bar impact work

The following table shows the notched bar impact work in accordance with EN 10027:2005 Table 1, Group 1.

Notched bar impact work J (Joule)			Test temperature °C
27 J	40 J	60 J	
JR	KR	LR	+20
J0	K0	L0	0
J2	K2	L2	–20
J3	K3	L3	–30
J4	K4	L4	–40
J5	K5	L5	–50
J6	K6	L6	–60

Examples of short names

Some examples of short names in accordance with EN 10027-1 are as follows:

Standard	Short name
EN 10025-2	E295
	E295GC
	E335
	E360

Standard	Short name
EN 10293	GE240
EN 10296-1	E355K2

Unalloyed structural steels

According to the standard DIN EN 10025 (February 2005), unalloyed structural steels are long and flat products made from hot-rolled, unalloyed, base and quality steels. They are characterised by their chemical composition and mechanical properties, see table, Mechanical properties, Page 272.

Unalloyed structural steels are used in building construction, civil engineering, bridge construction, hydraulic construction, tank and container construction and in vehicle and machine construction for example.

The steels referred to in this standard are not suitable for heat treatment. Stress relief annealing is permissible.

Mechanical properties The following table gives the mechanical properties of a number of unalloyed structural steels as an extract from the corresponding standard.

Steel grade designation		Tensile strength R_m [1] for nominal thickness values mm			Yield strength R_{eH} [1] for nominal thickness values mm					
Short name[2]	Material number[3]	< 3	> 3 ≦ 100	> 100 ≦ 150	≦ 16	> 16 ≦ 40	> 40 ≦ 63	> 63 ≦ 80	> 80 ≦ 100	> 100 ≦ 150
		N/mm²			min. N/mm²					
S185[4]	1.0035	310 ... 540	290 ... 510	–	185	175	–	–	–	–
S235JR[4] S235JRG[4]	1.0037 1.0036	360 ... 510	340 ... 470	–	235	225	–	–	–	–
S235JRG2 S235J2G3	1.0038 1.0116			340 ... 470	235	225	215	215	215	195
S275JR S275J2G3	1.0044 1.0144	430 ... 580	410 ... 560	400 ... 540	275	265	255	245	235	225
S355J2G3	1.0570	510 ... 680	490 ... 630	470 ... 630	355	345	335	325	315	295
E295[5]	1.0050	490 ... 660	470 ... 610	450 ... 610	295	285	275	265	255	245
E335[5]	1.0060	590 ... 770	570 ... 710	550 ... 710	335	325	315	305	295	275
E360[5]	1.0070	690 ... 900	670 ... 830	650 ... 830	360	355	345	335	325	305

For further mechanical and technological properties and information on the chemical composition of steels see DIN EN 10025 (March 1994).

[1] The values given in the table for the tensile test apply to longitudinal test pieces l. For strip, plate and wide flat steel with widths ≧ 600 mm, transverse test pieces t are applicable.

[2] In accordance with EN 10 027-1 and ECISS IC 10.

[3] In accordance with EN 10 027-2.

[4] Only available in nominal thicknesses ≦ 25 mm.

[5] These steel grades are not normally used for channels, angles and sections.

Quenched and tempered steels Standards DIN EN 10083-1/2 (October 2006) and DIN EN 10083-3 (January 2007, January 2009 (2009: corresponding revision) list the mechanical properties of steels in the quenched and tempered condition (+ QT).
Definition of the dimension limits does not mean that full martensitic quenching and subsequent tempering is possible up to the defined sampling point. The hardening depth is the result of the course of the end quenching curves.

Mechanical properties The following table gives the mechanical properties of a number of quenched and tempered steels as an extract from the corresponding standard.

Steel designation		Yield strength R_e[1] (0,2% proof stress) min. N/mm²			Tensile strength R_m N/mm²			Elongation at fracture[2] A min. %			Reduction in area at fracture Z min. %			Notched bar impact work (Charpy test piece) KV min. J		
Short name	Material number	Diameter mm														
		> – ≦ 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100	– 16	16 40	40 100
C22 C22E C22R	1.0402 1.1151 1.1149	340	290	–	500 … 650	470 … 620	–	20	22	–	50	50	–	– 50 50	– 50 50	–
C25 C25E C25R	1.0406 1.1158 1.1163	370	320	–	550 … 700	500 … 650	–	19	21	–	45	50	–	– 45 45	– 45 45	–
C30 C30E C30R	1.0528 1.1178 1.1179	400	350	300[3]	600 … 750	550 … 700	500 … 650[3]	18	20	21[3]	40	45	50[3]	– 40 40	– 40 40	– 40[3] 40
C35 C35E C35R	1.0501 1.1181 1.1180	430	380	320	630 … 780	600 … 750	550 … 700	17	19	20	40	45	50	– 35 35	– 35 35	– 35 35
C40 C40E C40R	1.0511 1.1186 1.1189	460	400	350	650 … 800	630 … 780	600 … 750	16	18	19	35	40	45	– 30 30	– 30 30	– 30 30
C45 C45E C45R	1.0503 1.1191 1.1201	490	430	370	700 … 850	650 … 800	630 … 780	14	16	17	35	40	45	– 25 25	– 25 25	– 25 25
C50 C50E C50R	1.0540 1.1206 1.1241	520	460	400	750 … 900	700 … 850	650 … 800	13	15	16	30	35	40	–	–	–
C55 C55E C55R	1.0535 1.1203 1.1209	550	490	420	800 … 950	750 … 900	700 … 850	12	14	15	30	35	40	–	–	–
C60 C60E C60R	1.0601 1.1221 1.1223	580	520	450	850 … 1 000	800 … 950	750 … 900	11	13	14	25	30	35	–	–	–

Continuation of table, see Page 274.

[1] R_e: upper yield strength or, if there is no marked yield strength, 0,2% proof stress $R_{p0,2}$.
[2] Elongation at fracture: initial length $L_0 = 565 \cdot \sqrt{S_0}$ (S_0 = original cross section).
[3] Valid for diameters up to 63 mm or for thicknesses up to 35 mm.

Continuation of table, Mechanical properties (quenched and tempered steels), from Page 273.

Steel designation		Yield strength R_e[1] (0,2% proof stress) min. N/mm²			Tensile strength R_m N/mm²			Elongation at fracture[2] A min. %			Reduction in area at fracture Z min. %			Notched bar impact work (Charpy test piece) KV min. J		
Short name	Material number	Diameter mm														
		> – ≤ 16	> 16 ≤ 40	> 40 ≤ 100	> – ≤ 16	> 16 ≤ 40	> 40 ≤ 100	> – ≤ 16	> 16 ≤ 40	> 40 ≤ 100	> – ≤ 16	> 16 ≤ 40	> 40 ≤ 100	> – ≤ 16	> 16 ≤ 40	> 40 ≤ 100
28Mn6	1.1170	590	490	440	800 … 950	700 … 850	650 … 800	13	15	16	40	45	50	35	40	40
38Cr2 38CrS2	1.7003 1.7023	550	450	350	800 … 950	700 … 850	600 … 750	14	15	17	35	40	45	35	35	35
46Cr2 46CrS2	1.7006 1.7025	650	550	400	900 … 1100	800 … 950	650 … 800	12	14	15	35	40	45	30	35	35
34Cr4 34CrS4	1.7033 1.7037	700	590	460	900 … 1100	800 … 950	700 … 850	12	14	15	35	40	45	35	40	40
37Cr4 37CrS4	1.7034 1.7038	750	630	510	950 … 1150	850 … 1000	750 … 900	11	13	14	35	40	40	30	35	35
41Cr4 41CrS4	1.7035 1.7039	800	660	560	1000 … 1200	900 … 1100	800 … 950	11	12	14	30	35	40	30	35	35
25CrMo4 25CrMoS4	1.7218 1.7213	700	600	450	900 … 1100	800 … 950	700 … 850	12	14	15	50	55	60	45	50	50
34CrMo4 34CrMoS4	1.7220 1.7226	800	650	550	1000 … 1200	900 … 1100	800 … 950	11	12	14	45	50	55	35	40	40
42CrMo4 42CrMoS4	1.7225 1.7227	900	750	650	1100 … 1300	1000 … 1200	900 … 1100	10	11	12	40	45	50	30	35	35
50CrMo4	1.7228	900	780	700	1100 … 1300	1000 … 1200	900 … 1100	9	10	12	40	45	50	30[3]	30[3]	30[3]

Continuation of table, see Page 275.

[1] R_e: upper yield strength or, if there is no marked yield strength, 0,2% proof stress $R_{p0,2}$.
[2] Elongation at fracture: initial length $L_0 = 565 \cdot \sqrt{S_0}$ (S_0 = original cross section).
[3] Preliminary values.

Continuation of table, Mechanical properties
(quenched and tempered steels), from Page 274.

Steel designation		Yield strength R_e[1] (0,2% proof stress) min. N/mm²			Tensile strength R_m N/mm²			Elongation at fracture[2] A min. %			Reduction in area at fracture Z min. %			Notched bar impact work (Charpy test piece) KV min. J		
Short name	Material number	Diameter mm														
	> / ≦	– / 16	16 / 40	40 / 100	– / 16	16 / 40	40 / 100	– / 16	16 / 40	40 / 100	– / 16	16 / 40	40 / 100	– / 16	16 / 40	40 / 100
36CrNiMo4	1.6511	900	800	700	1100 … 1300	1000 … 1200	900 … 1100	10	11	12	45	50	55	35	40	45
34CrNiMo6	1.6582	1000	900	800	1200 … 1400	1100 … 1300	1000 … 1200	9	10	11	40	45	50	35	45	45
30CrNiMo8	1.6580	1050	1050	900	1250 … 1450	1250 … 1450	1100 … 1300	9	9	10	40	40	45	30	30	35
36NiCrMo16	1.6773	1050	1050	900	1250 … 1450	1250 … 1450	1100 … 1300	9	9	10	40	40	45	30	30	35
51CrV4	1.8159	900	800	700	1100 … 1300	1000 … 1200	900 … 1100	9	10	12	40	45	50	30[3]	30[3]	30[3]

[1] R_e: upper yield strength or, if there is no marked yield strength, 0,2% proof stress $R_{p0,2}$.
[2] Elongation at fracture: initial length $L_0 = 565 \cdot \sqrt{S_0}$ (S_0 = original cross section).
[3] Preliminary values.

Case hardening steels

According to the standard DIN EN 10084 (June 2008), case hardening steels are structural steels with a relatively low carbon content. They are used for components whose surface zones are usually carburised or carbonitrided prior to hardening.

After hardening, these steels exhibit a high degree of hardness in the surface zone and good resistance to wear. The core zone primarily exhibits a high degree of toughness.

The standard DIN EN 10084 applies to semi-finished products, for example: cogs, roughed slabs, billets, hot-rolled wire, hot-rolled or forged steel bar (round, rectangular, hexagonal, octagonal and flat steel), hot-rolled wide flat steel, hot or cold-rolled sheet and strip, open die and drop forgings.

Brinell hardness The following table gives the Brinell hardness of a number of case hardening steels in various treated conditions as an extract from the corresponding standard.

Steel designation		Hardness in treated condition[1]			
Short name	Material number	+S[2] max. HB	+A[3] max. HB	+TH[4] HB	+FP[5] HB
C10E	1.1121	–	131	–	–
C10R	1.1207	–	131	–	–
C15E	1.1141	–	143	–	–
C15R	1.1140	–	143	–	–
17Cr3	1.7016	[6]	174	–	–
17CrS3	1.7014	[6]	174	–	–
28Cr4	1.7030	255	217	166 ... 217	156 ... 207
28CrS4	1.7036	255	217	166 ... 217	156 ... 207
16MnCr5	1.7131	[6]	207	156 ... 207	140 ... 187
16MnCrS5	1.7139	[6]	207	156 ... 207	140 ... 187
20MnCr5	1.7147	255	217	170 ... 217	152 ... 201
20MnCrS5	1.7149	255	217	170 ... 217	152 ... 201
20MoCr4	1.7321	255	207	156 ... 207	140 ... 187
20MoCrS4	1.7323	255	207	156 ... 207	140 ... 187
20NiCrMo2-2	1.6523	[6]	212	152 ... 201	145 ... 192
20NiCrMoS2-2	1.6526	[6]	212	152 ... 201	145 ... 192
17CrNiMo6-4	1.6566	255	229	179 ... 229	149 ... 201
17CrNiMoS6-4	1.6569	255	229	179 ... 229	149 ... 201
20CrNiMoS6-4	1.6571	255	229	179 ... 229	154 ... 207

[1] Hardness requirements for the products supplied in the following conditions.
[2] Treated to improve shearability.
[3] Soft-annealed.
[4] Treated for strength.
[5] Treated to ferrite-pearlite structure.
[6] Under suitable conditions, these steel grades are shearable in the untreated condition.

Stainless steels

The standard DIN EN 10088:2005 gives the chemical composition and mechanical properties of stainless steels.

Stainless steels are particularly resistant to chemically aggressive substances. They contain at least 10,5% Cr and no more than 1,2% C. In accordance with their essential use characteristics, they are further subdivided into:

- Corrosion-resistant steels:
  material numbers 1.40xx to 1.46xx
- Heat-resistant steels:
  material numbers 1.47xx to 1.48xx
- Creep-resistant steels:
  material numbers 1.49xx.

Stainless steels can also be classified in accordance with their microstructure into:

- Ferritic steels:
  good suitability for welding, creep-resistant, special magnetic properties, poor suitability for machining by cutting, suitable for cold forming, not resistant to intercrystalline corrosion, $E = 220\,000\ N/mm^2$
- Martensitic steels:
  hardenable, good suitability for machining by cutting, high strength, magnetic, weldable under certain conditions, $E = 216\,000\ N/mm^2$
- Precipitation hardening steels:
  hardenable by precipitation hardening, suitability for machining by cutting dependent on hardness, magnetic, $E = 200\,000\ N/mm^2$
- Austenitic steels:
  good suitability for welding, good suitability for cold forming, difficult to machine by cutting, non-magnetic, $E = 200\,000\ N/mm^2$
- Austenitic-ferritic steels (duplex steels):
  resistant to stress corrosion cracking, high erosion resistance and high fatigue strength, $E = 200\,000\ N/mm^2$.

Mechanical properties The following table gives the mechanical properties of a number of stainless steels as an extract from the corresponding standard.

Steel designation		Elongation at fracture A	Tensile strength for d_N[1)] R_{mN}	Yield strength, 0,2% proof stress for d_N[1)] R_{eN}, $R_{p0,2N}$	Examples of application
Short name	Material number	min. %	min. N/mm²	min. N/mm²	
Stainless steels in accordance with DIN EN 10088-3 (semi-finished products, bars and profiles)					Characterised by particularly high resistance to chemically aggressive substances; the resistance is based on the formation of covering layers due to the chemical attack
Treated condition	Ferritic steels: annealed (+A)				
	Martensitic steels: quenched and tempered (+QT, for example QT700)				
	Austenitic and austenitic-ferritic steels: solution annealed (+AT)				
Practically no influence on size due to technology					
Ferritic steels					
X2CrMoTiS18-2	1.4523	15	430	280	Acid-resistant parts in the textile industry
X6CrMoS17	1.4105	20	430	250	Free-cutting steels; bolts, fasteners
X6Cr13	1.4000	20	400	230	Chip carriers, cutlery, interior fittings
X6Cr17	1.4016	20	400	240	Connectors, deep drawn formed parts
Martensitic steels					
X20Cr13	1.4021	13	700	500	Armatures, flanges, springs, turbine parts
X39CrMo17-1	1.4122	12	750	550	Tubes, shafts, spindles, wear parts
X14CrMoS17	1.4104	12	650	500	Free-cutting steel; turned parts, apparatus fittings
X12CrS13	1.4005	12	650	450	Connectors, cutting tools, components subjected to wear
X3CrNiMo13-4	1.4313	15	780	620	
X17CrNi16-2	1.4057	14	800	600	

Continuation of table, see Page 279.

Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

[1)] d_N: Reference dimension (diameter, thickness) of semi-finished product in accordance with the relevant material standard.
R_{mN}, R_{eN}, $R_{p0,2N}$: standard values for tensile strength, yield strength and 0,2% proof stress measured relative to d_N.
Guide values for fatigue strength: $\sigma_{bW} \approx 0,5 \cdot R_m$, $\sigma_{zdW} \approx 0,4 \cdot R_m$, $\tau_{tW} \approx 0,3 \cdot R_m$.

Continuation of table, Mechanical properties (stainless steels), from Page 278.

Steel designation		Elongation at fracture A	Tensile strength for d_N[1)] R_m	Yield strength, 0,2% proof stress for d_N[1)] R_{eN}, $R_{p0,2N}$	Examples of application
Short name	Material number	min. %	min. N/mm^2	min. N/mm^2	
Austenitic steels					
X5CrNi18-10	1.4301	45	500	190	Universal use; building, vehicle construction, foodstuffs industry
X8CrNiS18-9	1.4305	35	500	190	Free-cutting steel; machine and connecting elements
X6CrNiTi18-10	1.4541	40	500	190	Household goods, photography industry, sanitary use
X2CrNiMo17-12-2 X2CrNiMoN17-13-3	1.4404 1.4429	40 40	520 580	220 280	Offshore engineering, pressure vessels, welded construction parts; pins, shafts
X2CrNiMo17-12-2	1.4401	40	500	200	Bleaching equipment, foodstuffs, oil and dyeing industry
X6CrNiMoTi17-12-2	1.4571	40	500	200	Containers (tanker trucks), heating vessels, synthetic resin and rubber industry
All austenitic grades cold hardened					
Tensile strength step	C700 C800	20 12	700 800	350 500	Load-bearing components
Austenitic-ferritic steels (duplex steels)					
X2CrNiMoN22-5-3	1.4462	25	650	450	Components for high chemical and mechanical loading; water and wastewater engineering, offshore engineering, pulp and chemicals industry, tank construction, centrifuges, conveying equipment
X2CrNiN23-4	1.4362	25	600	400	
X2CrNiMoCuWN25-7-4	1.4501	25	730	530	

Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

[1)] d_N: Reference dimension (diameter, thickness) of semi-finished product in accordance with the relevant material standard.
R_{mN}, R_{eN}, $R_{p0,2N}$: standard values for tensile strength, yield strength and 0,2% proof stress measured relative to d_N.
Guide values for fatigue strength: $\sigma_{bW} \approx 0,5 \cdot R_m$, $\sigma_{zdW} \approx 0,4 \cdot R_m$, $\tau_{tW} \approx 0,3 \cdot R_m$.

Rolling bearing steels According to the standard DIN EN ISO 683-17 (April 2000), rolling bearing steels are steels for components of rolling bearings. They are subjected first and foremost to high local alternating stresses and wear during operation. In the used condition they exhibit hardening microstructure (at least in the surface zone).

Hardness The following table gives the hardness in the usual conditions of delivery for a number of rolling bearing steels as an extract from the corresponding standard.

Steel designation		Hardness in the delivered condition						Previous designation
Short name	Material number	+S max. HB	+A max. HB	+HR HB	+AC[1] max. HB	+AC[1] +C max. HB	+FP HB	
Through hardening rolling bearing steels								
–	1.3501	[2]	–	–	207	241[3][4]	–	100 Cr 2
100Cr6	1.3505	[2]	–	–	207	241[3][4]	–	100 Cr 6
100CrMnSi6-4	1.3520	[2]	–	–	217	251[4]	–	100 CrMn 6
100CrMo7	1.3537	[2]	–	–	217	251[4]	–	100 CrMo 7
100CrMo7-3	1.3536	[2]	–	–	230	–	–	100 CrMo 7 3
100CrMnMoSi8-4-6	1.3539	[2]	–	–	230	–	–	100 CrMnMo 8
Case hardening rolling bearing steels								
17MnCr5	1.3521	[5]	207	156 ... 207	170	[6]	140 ... 187	17 MnCr 5
19MnCr5	1.3523	255	217	170 ... 217	180	[6]	152 ... 201	19 MnCr 5
–	1.3531	255	–	179 ... 227	180	[6]	–	16 CrNiMo 6
18NiCrMo14-6	1.3533	255	–	–	241	[6]	–	17 NiCrMo 14

[1] For case hardening steels, this condition is applied if cold forming operations are intended. For through hardening, stainless and high-temperature rolling bearing steels, this condition is also used if the steel is processed by machining operations.

[2] If this condition is necessary, the maximum hardness value and the requirements concerning the structure are to be agreed upon when the enquiry and order are placed.

[3] The hardness of wire for needle roller bearings should not exceed 321 HB.

[4] The hardness of cold-finished tubes should not exceed 321 HB.

[5] Under suitable conditions, this grade is shearable in the untreated condition.

[6] Depending on the degree of cold forming, the values may exceed those for the condition +AC by up to approx. 50 HB.

Continuation of table, Hardness, from Page 280.

Steel designation		Hardness in the delivered condition						Previous designation
Short name	Material number	+S max. HB	+A max. HB	+HR HB	+AC[1] max. HB	+AC[1] +C max. HB	+FP HB	
Induction hardening rolling bearing steels								
C56E2	1.1219	255[2]	229	–	–	–	–	Cf 54
–	1.3561	255	–	–	–	–	–	44 Cr 2
43CrMo4	1.3563	255	241	–	–	–	–	43 CrMo 4
–	1.3565	255	–	–	–	–	–	48 CrMo 4
Stainless rolling bearing steels								
X47Cr14	1.3541	[3]	–	–	248	[4]	–	X 45 Cr 13
X108CrMo17	1.3543	[3]	–	–	255	[4]	–	X 102 CrMo 17
X89CrMoV18-1	1.3549	[3]	–	–	255	[4]	–	X 89 CrMoV 18 1
High-temperature rolling bearing steels								
80MoCrV42-16	1.3551	[3]	–	–	248	[4]	–	80 MoCrV 42 16
X82WMoCrV6-5-4	1.3553	[3]	–	–	248	[4]	–	X 82 WMoCrV 6 5 4
X75WCrV18-4-1	1.3558	[3]	–	–	269	[4]	–	X 75 WCrV 18 4 1

[1] For case hardening steels, this condition is applied if cold forming operations are intended. For through hardening, stainless and high-temperature rolling bearing steels, this condition is also used if the steel is processed by machining operations.

[2] Depending on the chemical composition of the cast and the dimensions, condition +A may be necessary.

[3] In general, shearability will only apply in condition +AC.

[4] Depending on the degree of cold forming, the values may exceed those for the condition +AC by up to approx. 50 HB.

Free-cutting steels

According to the standard DIN EN 10087 (January 1999), free-cutting steels are characterised by good cutting properties and good chip brittleness. These are essentially achieved by higher sulphur contents and with further additives (for example, lead) where required.

Bright free-cutting steels differ from the hot formed free-cutting steels by virtue of the fact that they have obtained a smooth, bright surface and a significantly higher dimensional accuracy through cold forming (drawing) or machining (pre-turning, rough grinding).

Mechanical properties

The following table gives the mechanical properties of a number of free-cutting steels as an extract from the corresponding standard.

Steel designation		Diameter d		Untreated		Quenched and tempered		
Short name	Material number	over mm	incl. mm	Hardness[1)] HBW	Tensile strength R_m N/mm^2	Yield strength R_e min. N/mm^2	Tensile strength R_m N/mm^2	Elongation A min. %
Free-cutting steels not intended for heat treatment								
11SMn30 11SMnPb30 11SMn37 11SMnPb37	1.0715 1.0718 1.0736 1.0737	5	10	–	380 ... 570	–	–	–
		10	16	–	380 ... 570	–	–	–
		16	40	112 ... 169	380 ... 570	–	–	–
		40	63	112 ... 169	370 ... 570	–	–	–
		63	100	107 ... 154	360 ... 520	–	–	–
Case hardening steels								
10S20 10SPb20	1.0721 1.0722	5	10	–	360 ... 530	–	–	–
		10	16	–	360 ... 530	–	–	–
		16	40	107 ... 156	360 ... 530	–	–	–
		40	63	107 ... 156	360 ... 530	–	–	–
		63	100	105 ... 146	350 ... 490	–	–	–
15SMn13	1.0725	5	10	–	430 ... 610	–	–	–
		10	16	–	430 ... 600	–	–	–
		16	40	128 ... 178	430 ... 600	–	–	–
		40	63	128 ... 172	430 ... 580	–	–	–
		63	100	125 ... 160	420 ... 540	–	–	–

Continuation of table, see Page 283.

1) In cases of dispute, the tensile strength values are decisive. The hardness values are for information only.

Continuation of table, Mechanical properties (free-cutting steels), from Page 282.

Steel designation		Diameter d		Untreated		Quenched and tempered		
Short name	Material number	over mm	incl. mm	Hardness[1)] HBW	Tensile strength R_m N/mm^2	Yield strength R_e min. N/mm^2	Tensile strength R_m N/mm^2	Elongation A min. %
Quenched and tempered steels								
35S20 35SPb20	1.0726 1.0756	5	10	–	550 ... 720	430	630 ... 780	15
		10	16	–	550 ... 700	430	630 ... 780	15
		16	40	154 ... 201	520 ... 680	380	600 ... 750	16
		40	63	154 ... 198	520 ... 670	320	550 ... 700	17
		63	100	149 ... 193	500 ... 650	320	550 ... 700	17
36SMn14 36SMnPb14	1.0764 1.7065	5	10	–	580 ... 770	480	700 ... 850	14
		10	16	–	580 ... 770	460	700 ... 850	14
		16	40	166 ... 222	560 ... 750	420	670 ... 820	15
		40	63	166 ... 219	560 ... 740	400	640 ... 700	16
		63	100	163 ... 219	550 ... 740	360	570 ... 720	17
38SMn28 38SMnPb28	1.0760 1.0761	5	10	–	580 ... 780	480	700 ... 850	15
		10	16	–	580 ... 750	460	700 ... 850	15
		16	40	166 ... 216	530 ... 730	420	700 ... 850	15
		40	63	166 ... 216	560 ... 730	400	700 ... 850	16
		63	100	163 ... 207	550 ... 700	380	630 ... 800	16

1) In cases of dispute, the tensile strength values are decisive. The hardness values are for information only.

Cast iron and cast steel

Flake graphite cast iron

According to the standard DIN EN 1561 (August 1997), flake graphite cast iron is a casting alloy based on iron-carbon, where the carbon is chiefly present in the form of flake graphite.

The properties of flake graphite cast iron are dependent on the form and distribution of the graphite and on the metallic matrix.

Tensile strength The following table gives the tensile strength of a number of grades of flake graphite cast iron as an extract from the corresponding standard.

Material designation		Decisive wall thickness values		Tensile strength R_m			Previous designation
				Values that must be maintained[1]		Expected values in casting[4]	
Designation	Number	over	incl.	in test piece cast separately[2]	in integrally cast test piece[3]		
		mm	mm	N/mm^2	min. N/mm^2	min. N/mm^2	
EN-GJL-100	EN-JL 1010	5[5]	40	min. 100	–	–	GG-10
EN-GJL-150	EN-JL 1020	2,5[5]	5	150 ... 250	–	180	GG-15
		5	10		–	155	
		10	20		–	130	
		20	40		120	110	
		40	80		110	95	
		80	150		100	80	
		150	300		90[6]	–	
EN-GJL-200	EN-JL 1030	2,5[5]	5	200 ... 300	–	230	GG-20
		5	10		–	205	
		10	20		–	180	
		20	40		170	155	
		40	80		150	130	
		80	150		140	115	
		150	300		130[6]	–	

Continuation of table, see Page 285.

1) If proof of tensile strength has been agreed on ordering, the type of test piece must be specified in the order.
2) The values refer to test pieces with a 30 mm unfinished casting diameter corresponding to a wall thickness of 15 mm.
3) If nothing can be defined for a specific wall thickness range, this is identified by a dash.
4) The values are for information purposes.
5) This dimension is included as the lower limit of the wall thickness range.
6) These are reference values.

Continuation of table, Tensile strength, from Page 284.

Material designation		Decisive wall thickness values		Tensile strength R_m			Previous designation
				Values that must be maintained[1)]		Expected values in casting[4)]	
Designation	Number	over	incl.	in test piece cast separately[2)]	in integrally cast test piece[3)]		
		mm	mm	N/mm^2	min. N/mm^2	min. N/mm^2	
EN-GJL-250	EN-JL 1040	5[5)]	10	250 ... 350	–	250	GG-25
		10	20		–	225	
		20	40		210	195	
		40	80		190	170	
		80	150		170	155	
		150	300		160[6)]	–	
EN-GJL-300	EN-JL 1050	10[5)]	20	300 ... 400	–	230	GG-30
		20	40		250	205	
		40	80		220	180	
		80	150		210	155	
		150	300		190[6)]	130	
EN-GJL-350	EN-JL 1060	10[5)]	20	350 ... 400	–	315	GG-35
		20	40		290	280	
		40	80		260	250	
		80	150		230	225	
		150	300		210[6)]	–	

1) If proof of tensile strength has been agreed on ordering, the type of test piece must be specified in the order.
2) The values refer to test pieces with a 30 mm unfinished casting diameter corresponding to a wall thickness of 15 mm.
3) If nothing can be defined for a specific wall thickness range, this is identified by a dash.
4) The values are for information purposes.
5) This dimension is included as the lower limit of the wall thickness range.
6) These are reference values.

Spheroidal graphite cast iron

According to the standard DIN EN 1563 (October 2005), spheroidal graphite cast iron is a casting material based on iron-carbon, where the carbon is chiefly present in the form of spheroidal graphite particles. Spheroidal graphite cast iron is also known as ductile cast iron.

Mechanical properties

The following table lists the mechanical properties of a number of spheroidal graphite cast irons.

Material designation		Guaranteed properties[1]			Previous designation
Designation	Number	Tensile strength R_m min. N/mm^2	Proof stress $R_{p0,2}$[2] min. N/mm^2	Elongation at fracture A min. %	
EN-GJS-350-22-LT	EN-JS1015	350	220	22	GGG-35.3
EN-GJS-400-18-LT	EN-JS1025	400	250	18	GGG-40,3
EN-GJS-400-15	EN-JS1030	400	250	15	GGG-40
EN-GJS-500-7	EN-JS1050	500	320	7	GGG-50
EN-GJS-600-3	EN-JS1060	600	370	3	GGG-60
EN-GJS-700-2	EN-JS1070	700	420	2	GGG-70
EN-GJS-800-2	EN-JS1080	800	480	2	GGG-80

[1] Guaranteed properties in test pieces cast separately and mechanically processed test pieces. Particularly in the case of wall thickness values of $>$ 50 mm and compact castings, the manufacturer and consumer are advised to reach agreements.

[2] In the case of the ferritic grades, it is permissible to specify the yield strength that is to be determined from the machine diagram instead of the 0,2% proof stress.

Integrally cast test pieces

The properties of an integrally cast test piece cannot accurately reproduce the properties of the actual casting. Improved approximate values can, however, be achieved in this instance compared with a test piece that is cast separately.

The following table gives the properties of integrally cast test pieces.

Material designation		Decisive wall thickness – casting		Thickness[1]	Tensile strength R_m	0,2% proof stress $R_{p0,2}$	Elongation after fracture A	Notched bar impact work KV[2]	
Designation	Number							Average from 3 test pieces	Individual value
		mm		mm	min. N/mm²	min. N/mm²	min. %	min. J	min. J
EN-GJS-400-18U	EN-JS1062	of	30 ... 60	40	390	250	15	14	11
		over	60 ... 200	70	370	240	12	12	9
EN-GJS-400-15U	EN-JS1072	of	30 ... 60	40	390	250	14	–	–
		over	60 ... 200	70	370	240	11	–	–
EN-GJS-500-7U	EN-JS1082	of	30 ... 60	40	450	300	7	–	–
		over	60 ... 200	70	420	290	5	–	–
EN-GJS-600-3U	EN-JS1092	of	30 ... 60	40	600	360	2	–	–
		over	60 ... 200	70	550	340	1	–	–
EN-GJS-700-2U	EN-JS1102	of	30 ... 60	40	700	400	2	–	–
		over	60 ... 200	70	660	380	1	–	–
EN-GJS-800-2U	EN-JS1112	of	30 ... 60	40	800	480	2	–	–
		over	60 ... 200	70	to be agreed between manufacturer and buyer				

[1] Thickness of the integrally cast test piece.

[2] DVM test pieces at –20 °C.

Cast steel for general applications

In accordance with the standard DIN EN 10293 (Juni 2005), the mechanical and magnetic properties of a number of cast steel grades are listed in the following table as an extract.

Cast steel grade		0,2% proof stress $R_{p0,2}$	Tensile strength R_m	Elongation at fracture[1] A	Reduction in area after fracture[2] Z	Notched bar impact work (ISO test pieces) KV		Magnetic induction[4] at a field strength of		
Short name	Material number					≦30 mm Mean value[3]	>30 mm Mean value[3]	25 A/cm	50 A/cm	100 A/cm
		min. N/mm^2	min. N/mm^2	min. %	min. %	min. J	min. J	min. T	min. T	min. T
GE200	1.0420	200	380	25	40	35	27	1,45	1,60	1,75
GE240	1.0446	240	450	22	31	27	27	1,40	1,55	1,70
GE300	1.0558	300	600[5]	15[6]	21	27	31	1,30	1,50	1,65
GS200	1.0449	200	380	25	–	35	35	–	–	–
GS240	1.0455	240	450	22	–	31	31	–	–	–

[1] Elongation at fracture: initial length $L_0 = 5d_0$.
[2] The values are not decisive for acceptance (not a constituent part of the DIN).
[3] Determined on the basis of three single values.
[4] These values apply only by arrangement.
[5] For thickness $t \leqq 30$ only, for $30 < t \leqq 100$, $R_{m\,min} = 520\ N/mm^2$ applies.
[6] For thickness $t \leqq 30$ only, for $30 < t \leqq 100$, $A = 18\%$.

Creep-resistant cast steel

In accordance with the standard DIN EN 10213 (January 2008), the mechanical properties for a number of creep-resistant cast steels are listed in the following table as an extract.

Cast steel grade		Heat treatment symbol[1)]	Tensile strength R_m	0,2% proof stress $R_{p0,2}$ at a temperature of							Elongation at fracture A	Notched bar impact work KV
Short name	Material number			20 °C	200 °C	300 °C	350 °C	400 °C	450 °C	500 °C		
			N/mm^2	N/mm^2							%	J
GP240GH	1.0619	+N	420 ... 600	240	175	145	135	130	125	–	22	27
		+QT	420 ... 600	240	175	145	135	130	125	–	22	40
GP280GH	1.0625	+N	480 ... 640	280	220	190	170	160	150	–	22	27
		+QT	440 ... 590	280	220	190	170	160	150	–	22	35
G20Mo5	1.5419	+QT	440 ... 590	245	190	165	155	150	145	135	22	27
G17CrMo5-5	1.7357	+QT	490 ... 690	315	250	230	215	200	190	175	20	27
G17CrMo9-10	1.7379	+QT	590 ... 740	400	355	345	330	315	305	280	18	27
G17CrMo5-5	1.7357	+QT	490 ... 690	315	250	230	215	200	190	175	20	27
G17CrMo9-10	1.7379	+QT	590 ... 740	400	355	345	330	315	305	280	18	27
G12CrMoV5-2	1.7720	+QT	510 ... 660	295	244	230	–	214	–	194	17	40
G17CrMoV5-10	1.7706	+QT	590 ... 780	440	385	365	350	335	320	300	15	27
GX15CrMo5	1.7365	+QT	630 ... 760	420	390	380	–	370	–	305	16	27
GX8CrNi12	1.4107	+QT	540 ... 690	355	275	265	–	255	–	–	18	45
		+QT1	600 ... 800	500	410	390	–	370	–	–	16	40
GX4CrNi13-4	1.4317	+QT2	760 ... 960	550	485	455	440	–	–	–	15	50
GX23CrMoV12-1	1.4931	+QT	740 ... 880	540	450	430	410	390	370	340	15	27
GX4CrNiMo16-5-1	1.4408	+QT	760 ... 960	540	485	455	–	–	–	–	15	60

1) +N means: normalising.
+QT means: quenching and tempering (hardening in air or liquid + tempering).
If alternative heat treatments are available, the required alternative should be specified in the order, for example GX8CrNi12 +QT1 or 1.4107 +QT.

Malleable cast iron

According to the standard DIN EN 1562 (August 2006), malleable cast iron is an iron-carbon cast material whose castings solidify largely without graphite if the design is appropriate for the material involved.

Depending on the type of heat treatment used for the unfinished casting, we arrive at:

- blackheart malleable cast iron (non-decarburised)
- whiteheart malleable cast iron (decarburised).

After annealing treatment, the iron carbide (cementite) present in the structure disappears completely. With the exception of fully decarburised, whiteheart malleable cast iron, both groups contain free carbon in the form of graphite, known as temper carbon. Both groups have material grades with structures that can range from ferrite to pearlite and/or other transformation products of austenite.
The chemical composition of the unfinished malleable cast iron and the nature of the temperature-dependent and time-dependent annealing process define the structural constitution and consequently the properties of the material.

The materials are designated in terms of tensile strength and elongation. This takes place relative to a 12 mm diameter test piece for whiteheart malleable cast iron and 12 mm or 15 mm diameter test pieces for blackheart malleable cast iron. However, comparative values of tensile strength and elongation after fracture are also given for other test piece diameters.

If welding work is to be carried out during the course of production or when using malleable iron castings, this must be agreed between the purchaser and the manufacturer of the casting. Heat treatment is required after repair welding.

Distortion as the result of heat treatment must be eliminated by straightening. Hot straightening or stress relief annealing can be agreed upon in special cases.

Malleable cast iron is easy to machine. The suitability of individual grades depends on the respective structural constitution.

The following shrinkage dimensions are valid for model production:

- 1% up to 2% for whiteheart malleable cast iron
- 0% to 1,5% for blackheart malleable cast iron.

The average density of the material is 7,4 kg/dm^3.

Mechanical properties The following table gives the mechanical properties of malleable cast iron as an extract from the corresponding standard.

Material designation: Designation	Material designation: Number	Diameter of test piece d mm	Tensile strength R_m min. N/mm^2	0,2% proof stress $R_{p0,2}$ min. N/mm^2	Elongation at fracture $A_{3,4}$ min. %	Brinell hardness max. HB	Previous designation
Whiteheart malleable (decarburised) cast iron							
EN-GJMW-350-4	EN-JM1010	9	340	–	5	230	GTW-35-04
		12	350	–	4	230	
		15	360	–	3	230	
EN-GJMW-360-12	EN-JM1020	9	320	170	15	200	GTW-S-38-12
		12	380	200	12	200	
		15	400	210	8	200	
EN-GJMW-400-5	EN-JM1030	9	360	200	8	220	GTW-40-05
		12	400	220	5	220	
		15	420	230	4	220	
EN-GJMW-450-7	EN-JM1040	9	400	230	10	220	GTW-45-07
		12	450	260	7	220	
		15	480	280	4	220	
Blackheart malleable (non-decarburised) cast iron							
EN-GIMB-350-10	EN-JM1130	12 or 15	350	200	10	150	GTS-35-10
EN-GIMB-450-6	EN-JM1140	12 or 15	450	270	6	200	GTS-45-06
EN-GIMB-550-4	EN-JM1160	12 or 15	550	340	4	230	GTS-55-04
EN-GIMB-650-2	EN-JM1180	12 or 15	650	430	2	260	GTS-65-02
EN-GIMB-700-2	EN-JM1190	12 or 15	700	530	2	290	GTS-70-02

Heat treatment of steel

Heat treatment processes – hardening

In the case of suitable ferrous materials, hardening results in a martensitic structure and consequently an increase in hardness and strength.

The most important heat treatment processes in the field of "hardening" are presented below.

Hardening over the entire cross-section

During the hardening process, the material is heated up to the hardening or austenitising temperature first, in order to produce the austenitic structure and to dissolve carbides. This temperature is then maintained for a defined period, after which the material is cooled or quenched to room temperature at a speed that is sufficient for forming martensite. The appropriate speed can be found in the time-temperature-transformation curve for the respective steel.

Tempering

Tempering is a heat treatment process which is applied in order to make the hardened and relatively brittle material tougher. This involves increasing the temperature in the region of +160 °C to +650 °C with an adequate holding period and subsequent cooling to room temperature.

Tempering reduces the hardness, the strength decreases and the ductility and toughness increase. Depending on the type of steel, any residual austenite present is converted at temperatures in excess of +230 °C.

During the tempering of steels, different temper colours are produced as a function of the tempering temperature, see table, Annealing and temper colours, Page 296.

Quenching and tempering

The combination of hardening and tempering at a temperature in excess of +500 °C is referred to as quenching and tempering. Quenching and tempering is intended to produce an optimum ratio between strength and toughness.

Surface layer hardening

In the case of surface layer hardening, austenitisation and hardening are restricted to the surface layer of the workpiece. In most cases, the material is heated by electric induction (medium or high-frequency alternating current) or by gas burners. The material is quenched by dipping or spraying.

Through the surface layer hardening of components that have already been quenched and tempered, a high basic strength can be combined with a high surface hardness in areas that are subjected to particularly high loads. The thickness of the hardened surface layer results from the hardness profile as the surface hardening depth SHD in mm (synonym: surface layer hardening depth); cf. standard DIN EN 10328:2005.

Case hardening

Case hardening (carburising, carbonitriding) involves carburising or carbonitriding followed by hardening. This hardening either takes place directly afterwards, or after the material concerned has been subjected to intermediate cooling and re-heated to an appropriate hardening temperature.

Depending on the required use characteristics and the subsequent machining requirements, the material is tempered, or deep-cooled and tempered, after hardening.

Case hardening gives the surface layer of workpieces a significantly higher level of hardness and the entire workpiece improved mechanical properties. To this end, the surface layer is enriched with carbon (carburising) or carbon and nitrogen (carbonitriding) before hardening. In comparison with carburisation, additional enrichment with nitrogen results in increased hardenability (by changing the transformation behaviour in the surface layer) and, after hardening, in improved annealing resistance.

Bainitic hardening

During bainitic hardening (isothermal transformation in the bainite stage), the material is first heated to and maintained at the austenitising temperature. The material is cooled to a temperature of between +200 °C and +350 °C, depending on the steel, and kept at this temperature until the steel structure has transformed to bainite. The material is then cooled to room temperature. In this state, the hardness is less than that of martensite, but toughness is increased.

This process is regarded as an alternative to hardening if a high level of toughness is required without a high level of hardness, and if distortion and dimensional changes have to be minimised.

Heat treatment processes – annealing

Annealing involves heat treating a workpiece at a defined annealing temperature. The aim of this is to influence and optimise both the usage and processing characteristics of the material.

Annealing treatment consists of heating the material up to the required annealing temperature, holding the material at this temperature for a sufficiently long period and cooling it to a temperature that is appropriate to the respective objective of the annealing treatment. The annealing temperature of steels can also be estimated on the basis of the colours which occur during annealing, see table Annealing and temper colours, Page 296.

The most important heat treatment processes in the field of "annealing" are presented below.

Stress relief annealing

Intrinsic stresses resulting from structural transformations or cold deformations can occur in workpieces. These are caused by irregular heating or cooling processes. In order to reduce these intrinsic stresses in workpieces, tools or blanks (as a result of plastic deformations), stress relief annealing is carried out at temperatures of between +450 °C and +650 °C. After an annealing time of 0,5 h to 1 h, the material must be cooled as slowly as possible to ensure that no new stresses arise.

Soft annealing

In order to improve the deformability of C steels and facilitate machining, the material is soft annealed at temperatures in the range of Ac_1. This also applies to workpieces that have been strengthened by hardening, precipitation hardening or cold forming. The temperature depends on the material (this is +650 °C to +750 °C for steel or a value lower than this for non-ferrous metals).

If a specific structural state, characterised by spheroidising of the carbides, is to be achieved, then "annealing to spheroidised cementite" (abbreviation: GKZ annealing) is applied. At the same time, a distinction is made between GKZ 2 (initial state martensite or bainite) and GKZ 1 (initial state normal structure). The spheroidal shape of the cementite can also be achieved by austenitisation and controlled cooling.

Recrystallisation annealing

The possibility of cold forming a material is limited by the increase in hardness and the decrease in formability with the strain caused by deformation.

Recrystallisation annealing is applied to formed workpieces in order to eliminate any strain hardening that may have occurred and bring about a new formation of the grains. This re-enables or facilitates subsequent forming.

The temperature depends on the degree of deformation and, in the case of steel, is generally around +550 °C to +730 °C.

Normalising

Normalising is carried out at austenitisation temperature, in other words at a temperature a little above Ac_3 (in the case of hypereutectoid steels above Ac_1). After an adequate holding period, the material is cooled at an appropriate rate so that a structure consisting of ferrite and pearlite is created at room temperature.

Normalising is used to refine a coarse-grained structure (for example in steel castings and welds) and to achieve as homogeneous a ferrite-pearlite distribution as possible. It should be applied instead of recrystallisation annealing if a coarse-grained structure is to be feared in the case of subcritically deformed workpieces.

If an excessive austenitisation temperature is chosen, the γ-mixed crystals grow, which also leads to a coarse-grained structure after transformation. An excessively slow cooling process can also result in a coarse ferrite grain.

Homogenising

Homogenising takes place at temperatures of between +1 030 °C and +1 150 °C above Ac_3. It serves to eliminate segregation zones in ingots and strands.

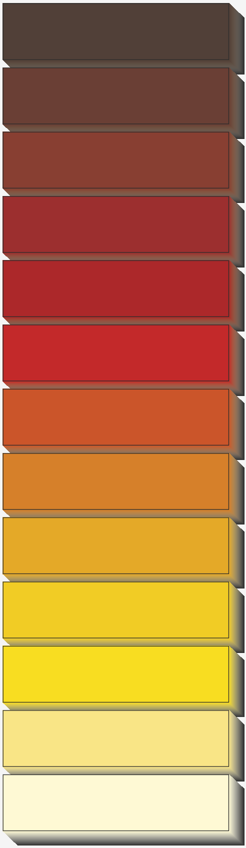
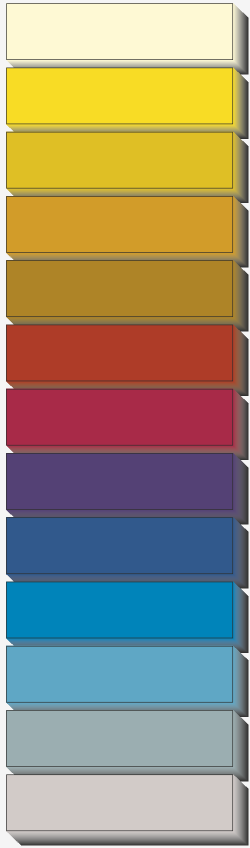
If the material is not subjected to hot forming after homogenising, it must be normalised in order to eliminate the coarse grain.

Precipitation hardening

Precipitation hardening consists of solution annealing, quenching and ageing above room temperature (hot ageing). Ageing results in the segregation and separation of intermetallic compounds made up of specific solution elements dissolved in the base material. This changes material properties, for example hardness, strength, ductility and toughness.

Annealing and temper colours

The following table shows the characteristic annealing and temper colours which occur at specific temperatures during the heat treatment of steel.

Annealing temperature °C	Annealing colours for steels	Tempering temperature °C	Temper colours for unalloyed tool steels[1)]
550		200	
630		220	
680		230	
740		240	
780		250	
810		260	
850		270	
900		280	
950		290	
1000		300	
1100		320	
1200		340	
≧1300		360	

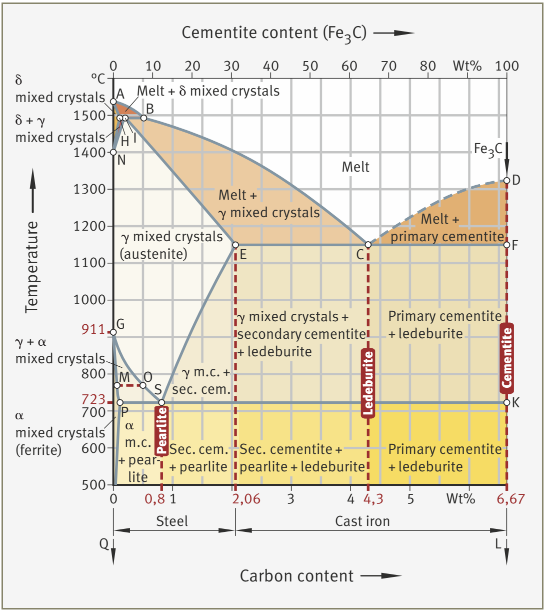
1) In the case of high alloy steels, these temper colours only occur at higher temperatures. The temper colour is also influenced by the tempering time: longer tempering at a lower temperature results in the same temper colour as shorter tempering at a higher temperature.

Iron-carbon phase diagram

If we show the states of different iron grades as a function of their carbon content and the temperature, this gives the iron-carbon phase diagram.

Figure 1

Iron-carbon metastable system



The structure uniformly solidifying at the lowest solidification point of all melts (point C) is known as the eutectic point.

Iron crystal lattice

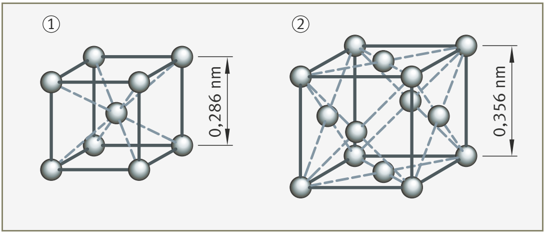
The elementary cell of iron has the following structure in the crystal lattice:

- body-centred cubic: α-iron (ferrite)
- face-centred cubic: γ-iron (austenite).

Figure 2

Crystal lattic structure

① Body-centred cubic crystal lattice
② Face-centred cubic crystal lattice

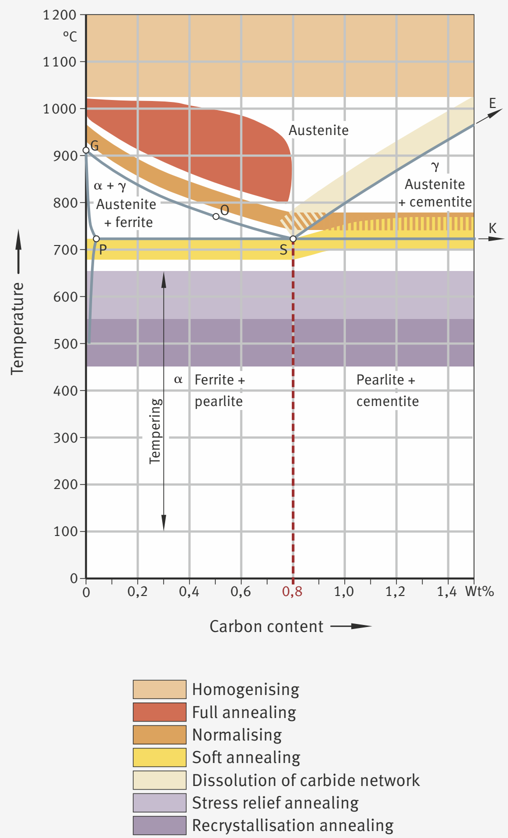


Heat treatment of steel

Heat treatment (hardening, annealing) gives the steel the required properties.

Figure 3

Heat treatment of steel



Case hardening The usual temperatures and heat treatments associated with the case hardening of case hardening steels are listed in the following tables.

Temperatures for case hardening According to the standard DIN EN 10084, the following temperatures are defined for the case hardening of case hardening steels:

Steel designation		Carburising temperature[1)]	Hardness at		Cooling agent	Tempering[3)]
Short name	Material number		Core hardness temperature[2)]	Surface hardness temperature[2)]		
		°C	°C	°C		°C
C10E	1.1121	880 ... 980	880 ... 920	780 ... 820	With a view to the required component properties, the choice of cooling (quenching) agent depends on: ■ the hardenability or case hardenability of the steel used ■ the shape and cross-section of the workpiece that is to be hardened ■ the effect of the cooling agent	150 ... 200
C10R	1.1207					
C15E	1.1141					
C15R	1.1140					
17Cr3	1.7016		860 ... 900			
17CrS3	1.7014					
28Cr4	1.7030					
28CrS4	1.7036					
16MnCr5	1.7131					
16MnCrS5	1.7139					
20MnCr5	1.7147					
20MnCrS5	1.7149					
20MoCr4	1.7321					
20MoCrS4	1.7323					
20NiCrMo2-2	1.6523					
20NiCrMoS2-2	1.6526					
17NiCrMo6-4	1.6566		830 ... 870			
17NiCrMoS6-4	1.6569					
20NiCrMoS6-4	1.6571					

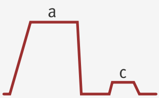
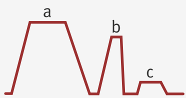
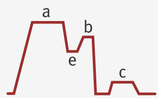
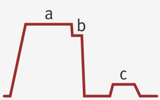
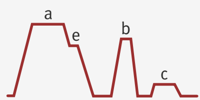
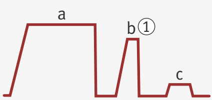
1) The criteria that have a crucial influence on the choice of carburising temperature mainly include the required carburisation time, the chosen carburisation agent, the facility available, the planned process and the required structural state.
Carburisation is normally carried out at less than +950 °C for direct hardening.
In special cases, carburising temperatures in excess of +1 000 °C are used.

2) In direct hardening, the material is either quenched at the carburising temperature or at a lower temperature. Where there is a risk of distortion in particular, the lower hardening temperatures from this range are considered in preference.

3) Tempering duration not less than 1 h (guide value).

Heat treatment during case hardening

The usual heat treatments associated with case hardening correspond to the following processes:

A Direct hardening or double hardening	B Single hardening	C Hardening after isothermal transformation
 Direct hardening from carburising temperature	 Single hardening from core or surface hardness temperature	 Hardening after isothermal transformation in the pearlite stage (e)
 Direct hardening after cooling to hardening temperature	 Single hardening after intermediate annealing (d) (soft annealing)	 Hardening after isothermal transformation in the pearlite stage (e) and cooling to room temperature
 Double hardening 1 Surface zone		

a = carburising temperature.
b = hardening temperature.
c = tempering temperature.
d = intermediate annealing (soft annealing) temperature.
e = transformation temperature in the pearlite stage.

Heat treatment of rolling bearing steels

The heat treatment of rolling bearing steels is described in accordance with the standard DIN EN ISO 683-17.

Steel designation		Hardening temperature for end quench test	Normal-ising	Preheat-ing tem-perature	Harden-ing in oil[1]	Harden-ing in water[1]	Tem-pering	Previous designation
Short name	Material number	°C ± 5 °C	°C	°C	°C	°C	°C	
Through hardening rolling bearing steels								
–	1.3501	–	–	–	820 ... 850	–	150 ... 180	100 Cr 2
100Cr6	1.3505	–	–	–	830 ... 870	–	150 ... 180	100 Cr 6
100CrMnSi6-6	1.3520	–	–	–	830 ... 870	–	150 ... 180	100 CrMn 6
100CrMo7	1.3537	–	–	–	840 ... 880	–	150 ... 180	100 CrMo 7
100CrMo7-3	1.3536	–	–	–	840 ... 880	–	150 ... 180	100 CrMo 7 3
100CrMoSi8-4-6	1.3539	–	–	–	840 ... 880	–	150 ... 180	100 CrMnMo 8
Case hardening rolling bearing steels								
17MnCr5	1.3521	870	–	–	810 ... 840	–	150 ... 180	17 MnCr 5
19MnCr5	1.3523	870	–	–	810 ... 840	–	150 ... 180	19 MnCr 6
–	1.3531	860	–	–	800 ... 830	–	150 ... 180	16 CrNiMo 6
18NiCrMo14-6	1.3533	830	–	–	780 ... 820	–	150 ... 180	17 NiCrMo 14

Continuation of table, see Page 302.

With the exception of the hardening temperatures for the end quench test, these are reference data. From an operational perspective, the temperatures and other conditions should be selected in such a way that the required properties are achieved.

[1] The choice of quenching agent for the tempering steels is dependent on the shape and dimensions of the workpiece.

Continuation of table, Heat treatment of rolling bearing steels, from Page 301.

Steel designation		Hardening temperature for end quench test	Normalising	Preheating temperature	Hardening in oil[1]	Hardening in water[1]	Tempering	Previous designation
Short name	Material number	°C ± 5 °C	°C	°C	°C	°C	°C	
Induction hardening rolling bearing steels								
C56E2	1.1219	840	830 ... 860	–	815 ... 845	805 ... 835	550 ... 660	Cf 54
–	1.3561	850	840 ... 870	–	830 ... 860	820 ... 850	550 ... 660	44 Cr 2
43CrMo4	1.3563	850	840 ... 880	–	830 ... 860	820 ... 850	540 ... 680	43 CrMo 4
–	1.3565	850	840 ... 880	–	830 ... 860	820 ... 850	540 ... 680	48 CrMo 4
Stainless rolling bearing steels								
X47Cr14	1.3541	–	–	–	1020 ... 1070	–	100 ... 200	X 45 Cr 13
X108CrMo17	1.3543	–	–	–	1030 ... 1080	–	100 ... 200	X 102 CrMo 17
X89CrMoV18-1	1.3549	–	–	–	1030 ... 1080	–	100 ... 200	X 89 CrMoV 18 1
High-temperature rolling bearing steels								
80MoCrV42-16	1.3551	–	–	750 ... 875	1070 ... 1120[2]	–	500 ... 580[3]	80 MoCrV 42 16
X82WMoCrV6-5-4	1.3553	–	–	750 ... 875	1180 ... 1230[2]	–	500 ... 580[3]	X 82 WMoCrV 6 5 4
X75WCrV18-4-1	1.3558	–	–	750 ... 875	1220 ... 1270[2]	–	500 ... 580[3]	X 75 WCrV 18 4 1

With the exception of the hardening temperatures for the end quench test, these are reference data. From an operational perspective, the temperatures and other conditions should be selected in such a way that the required properties are achieved.

[1] The choice of quenching agent for the tempering steels is dependent on the shape and dimensions of the workpiece.

[2] This steel is normally quenched in a salt bath at a temperature of +500 °C to +560 °C.

[3] Tempering period 2 h.

Vickers hardness, Brinell hardness, Rockwell hardness, tensile strength

The data for Vickers hardness, Brinell hardness, Rockwell hardness and tensile strength cannot be converted directly into one another.
Therefore comparative values are given in the following (conversion) table:

Tensile strength	Vickers hardness[1)]	Brinell hardness[2)]	Rockwell hardness			Tensile strength	Vickers hardness[1)]	Brinell hardness[2)]	Rockwell hardness		
N/mm^2	HV	HB	HRB	HRC	HRA	N/mm^2	HV	HB	HRB	HRC	HRA
255	80	76,0	–	–	–	640	200	190	91,5	–	–
270	85	80,7	41,0	–	–	660	205	195	92,5	–	–
285	90	85,5	48,0	–	–	675	210	199	93,5	–	–
305	95	90,2	52,0	–	–	690	215	204	94,0	–	–
320	100	95,0	56,2	–	–	705	220	209	95,0	–	–
335	105	99,8	–	–	–	720	225	214	96,0	–	–
350	110	105	62,3	–	–	740	230	219	96,7	–	–
370	115	109	–	–	–	755	235	223	–	–	–
385	120	114	66,7	–	–	770	240	228	98,1	20,3	60,7
400	125	119	–	–	–	785	245	233	–	21,3	61,2
415	130	124	71,2	–	–	800	250	238	99,5	22,2	61,6
430	135	128	–	–	–	820	255	242	–	23,1	62,0
450	140	133	75,0	–	–	835	260	247	(101)	24,0	62,4
465	145	138	–	–	–	850	265	252	–	24,8	62,7
480	150	143	78,7	–	–	865	270	257	(102)	25,6	63,1
495	155	147	–	–	–	880	275	261	–	26,4	63,5
510	160	152	81,7	–	–	900	280	266	(104)	27,1	63,8
530	165	156	–	–	–	915	285	271	–	27,8	64,2
545	170	162	85,0	–	–	930	290	276	(105)	28,5	64,5
560	175	166	–	–	–	950	295	280	–	29,2	64,8
575	180	171	87,1	–	–	965	300	285	–	29,8	65,2
595	185	176	–	–	–	995	310	295	–	31,0	65,8
610	190	181	89,5	–	–	1030	320	304	–	32,2	66,4
625	195	185	–	–	–	1060	330	315	–	33,3	67,0

Continuation of table, see Page 304.

The numbers in parentheses are hardness values that lie outside the range of the standardised hardness testing methods but which, in practical terms, are frequently used as approximated values.

1) Test force: $F \geqq 98$ N.

2) Degree of load: $0{,}102F/D^2 = 30\ N/mm^2$.

Continuation of table, Vickers hardness, Brinell hardness, Rockwell hardness, tensile strength, from Page 303.

Tensile strength	Vickers hardness[1)]	Brinell hardness[2)]	Rockwell hardness			Tensile strength	Vickers hardness[1)]	Brinell hardness[2)]	Rockwell hardness	
N/mm²	HV	HB	HRB	HRC	HRA	N/mm²	HV	HB	HRC	HRA
1095	340	323	–	34,4	67,6	1920	580	(551)	54,1	78,0
1125	350	333	–	35,5	68,1	1955	590	(561)	54,1	78,4
1155	360	342	–	36,6	68,7	1995	600	(570)	55,2	78,6
1190	370	352	–	37,7	69,2	2030	610	(580)	55,7	78,9
1220	380	361	–	38,8	69,8	2070	620	(589)	56,3	79,2
1255	390	371	–	39,8	70,3	2105	630	(599)	56,8	79,5
1290	400	380	–	40,8	70,8	2145	640	(608)	57,3	79,8
1320	410	390	–	41,8	71,4	2180	650	(618)	57,8	80,0
1350	420	399	–	42,7	71,8	–	660	–	58,3	80,3
1385	430	409	–	43,6	72,3	–	670	–	58,8	80,6
1420	440	417	–	44,5	72,8	–	680	–	59,2	80,8
1455	450	428	–	45,3	73,3	–	690	–	59,7	81,1
1485	460	437	–	46,1	73,6	–	700	–	60,1	81,3
1520	470	447	–	46,9	74,1	–	720	–	61,0	81,8
1555	480	(456)	–	47,7	74,5	–	740	–	61,8	82,2
1595	490	(466)	–	48,4	74,9	–	760	–	62,5	82,6
1630	500	(475)	–	49,1	75,3	–	780	–	63,3	83,0
1665	510	(485)	–	49,8	75,7	–	800	–	64,0	83,4
1700	520	(494)	–	50,5	76,1	–	820	–	64,7	83,8
1740	530	(504)	–	51,1	76,4	–	840	–	65,3	84,1
1775	540	(513)	–	51,7	76,7	–	860	–	65,9	84,4
1810	550	(523)	–	52,3	77,0	–	880	–	66,4	84,7
1845	560	(532)	–	53,0	77,4	–	900	–	67,0	85,0
1880	570	(542)	–	53,6	77,8	–	920	–	67,5	85,3
						–	940	–	68,0	85,6

The numbers in parentheses are hardness values that lie outside the range of the standardised hardness testing methods but which, in practical terms, are frequently used as approximated values.

1) Test force: $F \geqq 98$ N.

2) Degree of load: $0{,}102F/D^2 = 30\ N/mm^2$.

Non-ferrous metals

Non-ferrous metal grades

Various grades of non-ferrous metals are defined in DIN standards.

The following tables compile a small selection of non-ferrous metals (NF metals) for general machine building, covering:

- copper alloys
- aluminium alloys
- magnesium alloys.

Copper alloys

The following table gives the mechanical properties of a number of copper alloys as an extract from the corresponding standards.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2]	0,2% proof stress $R_{p0,2N}$[2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. N/mm²	min. N/mm²	
Wrought copper-zinc alloys in accordance with DIN EN 12163							
CuBe2	CW101C	R420 R600 R1150	2 ... 80 25 ... 80 2 ... 80 Round rods	35 10 2	420 600 1150	140 480 1000	E = 122 000 N/mm²; for very high demands in terms of hardness, elasticity and wear, good solderability, optimum precipitation hardening time; springs of all types, membranes, tension bands, non-magnetic design parts, bearing blocks, worm and spur gears, turned clockmaking parts, injection moulding dies, spark-proof tools
CuCr1Zr CuCr1	CW106C CW105C	R200 R400 R470	8 ... 80 50 ... 80 4 ... 25 Round rods	30 12 7	200 400 470	60 310 380	E = 120 000 N/mm²; high electrical conductivity, high softening temperature and creep rupture strength, negligible weldability and solderability, high temperature resistance, precipitation hardenable; continuous casting moulds, springs and contacts for conduction of current, electrodes for resistance welding, continuously cast profiles

Continuation of table, see Page 306.

Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 312.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Continuation of table, Copper alloys, from Page 305.

Material designation		Condition[1)]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m [2)]	0,2% proof stress $R_{p0,2N}$ [2)]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. N/mm^2	min. N/mm^2	
Wrought copper-zinc based multiple element alloys in accordance with DIN EN 12163							
CuZn31Si1	CW708R	R460 R530	5 ... 40 5 ... 14 Round rods	22 12	460 530	250 330	E = 109 000 N/mm^2; good sliding characteristics even under heavy loads, suitable for cold forming, suitable for soldering and welding under certain conditions; bearing bushes, sliding elements, guide systems, drop forged parts
Wrought copper-tin alloys in accordance with DIN EN 12163							
CuSn6	CW452K	R340 R400 R470 R550	2 ... 60 2 ... 40 2 ... 12 2 ... 6 Round rods	45 26 15 8	340 400 470 550	230 250 350 500	E = 118 000 N/mm^2; highly suitable for cold forming, good suitability for welding and soldering, resistant to seawater and industrial atmospheres; springs of all types, hosepipes and coiled pipes, membranes, mesh and sieve wire, gears, bushes, parts for the chemicals industry
Wrought copper-zinc-lead alloys in accordance with DIN EN 12164							
CuZn37Mn3 Al2PbSi	CW713R	R540 R590 R620	6 ... 80 6 ... 50 15 ... 50 Round rods	15 12 8	540 590 620	280 320 350	E = 93 000 N/mm^2; high strength, high wear resistance, good resistance to atmospheric corrosion, resistant to oil corrosion; design parts in machine building, plain bearings, valve guides, gearbox parts, piston rings

Continuation of table, see Page 307.

Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

1) Condition designations and casting methods, see table Condition designations and casting methods, Page 312.

2) The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Continuation of table, Copper alloys, from Page 306.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m [2]	0,2% proof stress $R_{p0,2N}$ [2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. N/mm²	min. N/mm²	
Wrought copper-aluminium alloys in accordance with DIN EN 12163							
CuAl10FE2Mn2	CW306G	R590 R690	10 ... 80 10 ... 50 Round rods	12 6	590 690	330 510	E = 120 000 N/mm²; high fatigue strength under reversed stresses even under corrosive conditions, good corrosion resistance, resistant to seawater, resistant to scaling, erosion and cavitation, creep-resistant; design parts for chemical apparatus construction, scale-resistant parts, screws, shafts, gears, valve seats
Wrought copper-nickel alloys in accordance with DIN EN 12163							
CuNi10Fe1Mn	CW352H	R280 R350	10 ... 80 2 ... 20 Round rods	30 10	280 350	90 150	E = 134 000 N/mm²; excellent resistance to erosion, cavitation and corrosion, insensitive to stress corrosion, tendency to pitting under foreign deposition, good suitable for cold forming and soldering; pipework, brake pipes, plates and bases for heat exchangers, condensers, apparatus construction, freshwater processing plant

Continuation of table, see Page 308.

Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 312.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Continuation of table, Copper alloys, from Page 307.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m [2]	0,2% proof stress $R_{p0,2N}$ [2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. N/mm^2	min. N/mm^2	
Cast copper-tin alloys (cast tin bronze) in accordance with DIN EN 1982							
CuSn12-C	CC483K	GS GM GC GZ	–	7 5 6 5	260 270 300 280	140 150 150 150	E = 94 000 N/mm^2 to 98 000 N/mm^2; standard alloy with good sliding and wear characteristics together with good corrosion resistance, very good emergency running characteristics; bushes, sliding elements, sliding strips, bearing shells
Cast copper-zinc alloys in accordance with DIN EN 1982							
CuZn37Al1-C	CC766S	GM	–	25	450	170	E = 100 000 N/mm^2; moderate strength; design and conducting material in machine building and precision engineering, gravity diecast parts for machine building and electrical engineering
Cast copper-tin-zinc-(lead) alloys (gunmetal) and cast copper-tin-lead alloys (cast tin-lead bronze) in accordance with DIN EN 1982							
CuSn7Zn4Pb7-C	CC483K	GS GM GC, GZ	–	15 12 12	230 230 260	120 120 120	E = 95 000 N/mm^2; standard sliding material with excellent emergency running characteristics, moderate strength and hardness; plain bearings for hardened and unhardened shafts, sliding plates and strips, printing rollers, propeller shaft sleeves

Continuation of table, see Page 309.

Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 312.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Continuation of table, Copper alloys, from Page 308.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2]	0,2% proof stress $R_{p0,2N}$[2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. N/mm²	min. N/mm²	
Cast copper-aluminium alloys (cast aluminium bronze) in accordance with DIN EN 1982							
CuAl10Fe5Ni5-C	CC333G	GS GM GZ GC	–	13 7 13 13	600 650 650 650	250 280 280 280	E = 120 000 N/mm²; very good fatigue strength under reversed stresses even under corrosive conditions (seawater), high resistance to cavitation and erosion, long term loading up to 250 °C, weldable to S235; ships' propellers, stern tubes, impellers, pump housings

Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 312.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Aluminium alloys The following table gives the mechanical properties of a number of aluminium alloys as an extract from the corresponding standards.

Material designation – Short name	Material designation – Number	Condition[1]	Thickness or diameter mm	Elongation at fracture A min. %	Tensile strength R_m[2] min. N/mm^2	0,2% proof stress $R_{p0,2N}$[2] min. N/mm^2	Modulus of elasticity, properties and examples of use
Aluminium and wrought aluminium alloys, not precipitation hardened, in accordance with DIN EN 485-2, DIN EN 754-2, DIN EN 755-2							
ENAW-(Al99,5)	ENAW-1050A	O/H111 H14 H18	≦50 ≦25 ≦3 Sheets	>20 2 ... 6 2	65 105 140	20 85 120	good corrosion resistance, highly suitable for cold and hot forming, good suitability for welding and soldering, surface protection by means of anodising; apparatus, containers, pipes for foodstuffs and beverages, deep drawn, pressed and sheet metal formed parts, busbars, overhead power lines, packaging
Wrought aluminium alloys, precipitation hardenable, in accordance with DIN EN 485-2, DIN EN 754-2, DIN EN 755-2							
ENAW-AlZn4,5Mg1	ENAW-7020	T6	≦40 Profiles	10	350	290	$E = 70\,000\ N/mm^2$; construction alloys of series 7000 with very high strength and low resistance, good cold forming suitability in the soft condition (O), self-hardening by fusion welding process (subsequent hot ageing recommended); profiles, tubes and sheets for welded supporting structures in building construction, vehicle engineering and machine building as well as in rail vehicles
Cast aluminium alloys in accordance with DIN EN 1706							
ENAC-AlSi9Mg	ENAC-43300	S T6 K T6	–	2 4	230 290	190 210	$E = 75\,000\ N/mm^2$; for complicated, thin-walled castings of high strength, good toughness and very good weather resistance, precipitation hardenable, good weldability and solderability, good machinability; engine blocks, gearbox and converter housings

Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 312.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Magnesium alloys The following table gives the mechanical properties of a number of magnesium alloys as an extract from the corresponding standards.

Material designation		Condition[1]	Thickness or diameter	Elongation at fracture A	Tensile strength R_m[2]	0,2% proof stress $R_{p0,2N}$[2]	Modulus of elasticity, properties and examples of use
Short name	Number		mm	min. %	min. N/mm^2	min. N/mm^2	
Wrought magnesium alloys in accordance with DIN 1729 and DIN 9715							
MgAl8Zn	3.5812.08	F27 F29 F31	– ≦10 ≦10	8 10 6	270 290 310	195 205 215	E = 43 000 N/mm^2 to 45 000 N/mm^2; very high strength, vibration-resistant, not weldable; components subjected to vibrations and shocks
Cast aluminium alloys in accordance with DIN EN 1753							
EN-MCMgAl8Zn1	EN-MC 21110	S, K F S, K T4 D F	–	2 8 1 ... 7	160 240 200 ... 250	90 90 140 ... 170	E = 41 000 N/mm^2 to 45 000 N/mm^2; good castability, weldable, good sliding characteristics, suitable for dynamic loading; components subjected to vibrations and shocks, gearbox and engine housings, oil trays

Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

[1] Condition designations and casting methods, see table Condition designations and casting methods, Page 312.

[2] The mechanical and physical properties of the materials are heavily influenced by fluctuations in the alloy composition and the structural state. The strength characteristics stated can only be taken as valid for particular dimension ranges.

Condition designations and casting methods

Some examples of condition designations and casting methods for non-ferrous metals are given in the following two tables.

Strength	Description
Copper alloys	
R600	Minimum tensile strength $R_m = 600\ N/mm^2$

Strength	Description
Wrought magnesium alloys	
F22	Minimum tensile strength $R_m = 10 \cdot 22\ N/mm^2 = 220\ N/mm^2$

Casting methods	Description
Copper alloys	
GS	Sand casting
GM	Gravity diecasting
GZ	Centrifugal casting
GC	Continuous casting
GP	Pressure diecasting
Aluminium and cast aluminium alloys	
S	Sand casting
K	Gravity diecasting
D	Pressure diecasting
L	Investment casting

Condition	Description
Wrought aluminium alloys, precipitation hardenable	
T3	Solution annealed, cold formed and cold aged
T351	Solution annealed, stress relieved by controlled stretching and cold aged
T4	Solution annealed and cold aged
T5	Quenched and hot aged
T6	Solution annealed and completely hot aged
Wrought aluminium alloys, not precipitation hardenable	
O	Soft annealed
F	Cast condition
H111	Annealed with subsequent slight work hardening
H12	Work hardened, 1/4 hard
H14	Work hardened, 1/2 hard
H16	Work hardened, 3/4 hard
H18	Work hardened, 4/4 hard
H22	Work hardened and post-annealed, 1/4 hard
H24	Work hardened and post-annealed, 1/2 hard

Plastic

Structure and properties

Plastics are macromolecular, organic materials that are produced artificially by chemical means, in other words they do not occur in nature.

Depending on the synthesis process, plastics can be subdivided into:

- polycondensation products
- polymerides
- polyaddition compounds.

Due to their different chemical structures, polymers can also be subdivided into:

- thermoplastics
- elastomers
- thermosets.

Thermoplastics

Thermoplastics occur as amorphous and semi-crystalline polymers. They consist of linear or ramified macromolecules. Thermoplastics can soften or melt when heated and solidify when cooled, processes that may be repeated. During the original shaping process, they undergo reversible changes of state. Thermoplastics can be shaped in the free-flowing state by means of various types of processing technology, for example injection moulding, extrusion and calendering, thus arriving at complex components or semi-finished products. Semi-finished products made of hard thermoplastics can largely be shaped whilst hot. Thermoplastics are weldable. Thermoplastic polymers are generally physically soluble in specific, organic solvents.

Elastomers

The structure of elastomers is based on a network of widely meshed macromolecules. In the operating temperature range above the glass temperature, they have a rubber-elastic behaviour. In other words, low stresses result in considerable deformations which recede almost completely once the stress has been removed.
In the event of a temperature increase, they exhibit a rubber-elastic behaviour up to the limit temperature of the irreversible, thermochemical degradation of the network molecules.

Elastomers are obtained through the polymerisation of dienes (for example NBR) or are the result of polycondensation and polyaddition reactions in starting materials (for example PUR).
Elastomers are generally processed in the plastic state before cross-linking with the addition of vulcanisation agents or cross-linking accelerators.

Thermoplastic elastomers (TPE)

Thermoplastic elastomers are polymers that can be processed thermoplastically with elastomer-like properties.
They are not chemically cross-linked.
TPEs are mostly block copolymers with "hard" and "soft" ranges.
The hard segments form aggregated zones. Owing to secondary compounds, these result in physical cross-linking points in the amorphous matrix, which reversibly dissolve at a temperature that is determined by the chemical structure. These polymers become thermoplastically free-flowing above this temperature.

Thermosetting plastics

In the original shaping process, thermosets come into being by virtue of the fact that free-flowing preproducts of a low molecular weight react with one another, forming chemically narrow cross-linked macromolecules. Up to the limit temperature of thermochemical degradation, the physical properties of the irreversibly "cured" thermosets are not very dependent on the temperature. They are not weldable, are not soluble in organic solvents when cured and some of them are capable of swelling.
Thermosetting preproducts can be obtained as "moulding compounds" for processing by means of melting and with subsequent thermal curing. They are also available as liquid "reaction resins" that can be processed and catalytically cured at room temperature.

Recycling

Corresponding to their chemical structure, polymer materials are open to diverse recycling possibilities or disposal concepts. Besides being suitable for more cost-intensive, chemical recycling (for example hydrolysis or hydration), thermoplastics are also suitable for mostly economical, physical recycling. Cross-linked polymers can only be recycled chemically, or used as fillers after grinding.

Application

Depending on the application area, thermoplastics are subdivided into:

- so-called mass plastics (for example PE, PS, PVC, PP)
- engineering plastics (for example PA, PBT, PC, POM)
- high-performance polymers (for example PESU, PPS, PEEK).

Engineering and high-performance plastics are used in automotive engineering, machine construction, the electrical industry and chemicals and plant engineering.
In order to improve the level of the physical and mechanical properties, glass or carbon fibres are frequently added to the base polymers as reinforcements and glass balls or minerals (for example talcum, mica or silica sand etc.) are added as fillers. Thermosets are also often used in the stated areas of engineering.

For further information on plastics for rolling bearings, please see chapter Design elements, section Rolling bearings, Page 575.

Classification of plastics

Plastics can be subdivided into:

- fully synthetic plastics
- modified natural substances.

The following table lists a number of plastics and the corresponding manufacturing method.

Description	Thermoplastics	Hardenable plastics	Manufacturing method
Fully synthetic plastics			
Polycondensation products	Linear polyester resins, polyamides, mixed polyamides	Phenolic plastic, carbamide polymers, melamine plastics, silicones, polyester resins, alkyd resins	**Polycondensation** During polycondensation, various basic, monomeric building blocks combine with each other, accompanied by the separation of water and other volatile substances of low molecular weight (for example ammonia), to form: ■ linear macromolecules, if monomers with two groups that are capable of reacting are present ■ spatially cross-linked macromolecules if two or more groups capable of reacting are present Most thermosets are an important example of this. In such cases, the process often takes place at an increased temperature and under pressure.

Continuation of table, see Page 316.

Continuation of table, Classification of plastics, from Page 315.

Description	Thermoplastics	Hardenable plastics	Manufacturing method
Fully synthetic plastics			
Polymerides	Polyethylene, polyvinyl chlorides, polystyrenes, polyisobutylenes, polymethacrylate, polyacrylonitrile, polyfluorethylenes	Alkyl resins, unsaturated polyester resins	**Polymerisation** When the double bonds split, the liquid or gaseous initial products, which are mostly equally monomeric, deposit themselves on one another, producing filamentary molecules. No cleavage products are produced. The process is started by initiators (radical or ion generators) and then continues to run exothermally. During this process, the polymeride becomes tenacious and ultimately solid as the molecular weight rises. Most thermoplastics are obtained through polymerisation.
Polyaddition products	Linear polyurethanes	Cross-linked polyurethanes, ethyoxyline resins, epoxy resins	**Polyaddition** As a result of the intermolecular repositioning of atoms (for example hydrogen), various basic, monomeric building blocks with reactive groups combine with macromolecules. The process is very similar to that of polycondensation but, in this case, no cleavage products of low molecular weight are produced. Depending on the choice of preproducts, polymers that are either cross-linked or not cross-linked can also be synthesised here. Depending on the starting material, the properties of the products can be intentionally adjusted within wide limits.
Modified natural substances			
Natural substances of high molecular weight	Cellulose ester, cellulose ether	Casein resins	**Chemical transformation** The oldest plastics are modified natural substances such as casein plastics, vulcanised fibre, celluloid, cellophane and artificial silk. Galalith, for example, is a casein plastic that results in a material with properties similar to those of natural horn due to the influence of formaldehyde on rennet casein. Hardening takes place after plastic deformation by introducing the shaped parts to a 5% formaldehyde solution. Depending on the thickness, hardening may take up to a few months. High water absorption and associated dimensional changes prevent such substances from being used for technical parts.

Strength characteristics and dimensional stability

The following tables give the strength characteristics and dimensional stability values for thermoplastics and thermosetting plastics.

Unreinforced thermoplastics

The following values are valid for the strength characteristics and dimensional stability of a number of unreinforced thermoplastics:

Plastic	Desig-nation	Tensile strength N/mm^2	Tensile modulus of elasticity (30 s) N/mm^2	Indentation hardness N/mm^2	Flexural strength N/mm^2	Impact strength kJ/m^2	Notched impact strength kJ/m^2	Dimensional stability (Vicat B) °C
Polyethylene, soft	PE-LD	18 ... 35	700 ... 1 400	40 ... 65	36	■	■	60 ... 70
Polyethylene, hard	PE-HD	8 ... 23	200 ... 500	13 ... 20	–	■	■	$<$40
Polypropylene	PP	21 ... 37	1 100 ... 1 300	36 ... 70	43	■	3 ... 17	85 ... 100
Polyvinyl chloride, hard	PVC-U	50 ... 75	2 500 ... 3 500	75 ... 155	110	■	2 ... 50	75 ... 110
Polyvinyl chloride, soft	PVC-P	10 ... 25	$<$100	A90[1]	–	■	■	40
Polystyrene	PS	45 ... 65	3 200 ... 3 250	140 ... 150	90	15 ... 20	2 ... 2,5	78 ... 99
Styrene/acrylo-nitrile copolymer	SAN	75	3 600	160 ... 170	100	16 ... 20	2 ... 3	100 ... 115
Acrylonitrile-butadiene-styrene copolymer	ABS	32 ... 60	1 900 ... 2 700	80 ... 120	75	70	7 ... 20	95 ... 110
Polymethyl methacrylate	PMMA	50 ... 77	2 700 ... 3 200	180 ... 200	105	18	2	70 ... 100
Polyoxymethylene	POM	62 ... 80	3 200	150 ... 170	110	■	8	160 ... 173
Polytetrafluoro-ethylene	PTFE	25 ... 36	410	27 ... 35	18	■	13 ... 15	–
Polyamide 6[2]	PA6	70 ... 85	1 400	75	50	■	■	180
Polyamide 66[2]	PA66	77 ... 84	2 000	100	50	■	15 ... 20	200
Polyamide 11[2]	PA11	40	1000	75	–	■	30 ... 40	175
Polyamide 12[2]	PA12	40	1 200	75	–	■	10 ... 20	165
Polycarbonate	PC	56 ... 67	2 100 ... 2400	110	100	■	20 ... 30	160 ... 170
Cellulose acetate (432)	CA	40	1 600	50	50	■	15	50 ... 63
Cellulose acetate butyrate (413)	CAB	35	1 600	55	38	■	20	60 ... 75

■ No fracture.

[1] Shore hardness scale A.

[2] Conditioned at +23 °C and 50% relative humidity.

Reinforced thermoplastics The following values are valid for the strength characteristics and dimensional stability of a number of reinforced thermoplastics:

Plastic	Designation	Tensile strength N/mm²	Tensile modulus of elasticity N/mm²	Elongation at tear %	Flexural strength N/mm²	Impact strength kJ/m²	Notched impact strength kJ/m²	Dimensional stability (Vicat B) °C
Polypropylene	PP GF30[1]	50	5 500	5	65	16[5]	6[5]	110
Polybutylene terephthalate	PBT GF30	145	10 500	2,5	210	50[6]	8,5[6]	205
Polyethylene terephthalate	PET GF30	175	11 000	2	228	32[6]	10[6]	260
Polyamide 6[2]	PA6 GF30	180	8 500	3	250	60[6]	12[6]	210
Polyamide 66[2]	PA66 GF30	190	10 000	3	270	45[6]	8,5[6]	250
Polyoxymethylene	POM GF30	130	10 000	3	170	32[6]	5,5[6]	160
Polyphenylene oxide, modified	PPO GF30	105	8 500	2,5	135	20[5]	6[5]	145
Polyphenylene sulphide	PPS GF40[3]	180	14 000	1,6	240	35[6]	6,5[5]	255
Polysulfone	PSU GF30	125	10 000	1,8	160	20[6]	7[6]	190
Polyethersulfone	PESU GF30	150	10 500	2,1	200	30[6]	8[6]	215
Polyetherimide	PEI GF30	160	9 000	3	220	35[5]	8[5]	220
Polyaryl ether ketone[4]	PAEK GF30	190	12 000	3,5	250	42[5]	11[5]	>300
Liquid crystal polymer	LCP GF30	200	23 000	1	–	20[5]	12[5]	170

[1] GF30 = filled with 30% glass fibre.
[2] Values specifically dry.
[3] 30% not commercially available.
[4] PEKEKK.
[5] Charpy test method.
[6] Izod test method.

Thermosetting plastics The following values are valid for the strength characteristics and dimensional stability of a number of thermosetting plastics:

Type of resin	Group	Type	Filler	Flexural strength min. N/mm^2	Impact strength min. kJ/m^2	Notched impact strength min. kJ/m^2	Dimensional stability (Martens) °C
Phenol	I	31	Wood flour	70	6	1,5	125
	II	85	Wood flour	70	5	2,5	125
		51	Cellulose	60	5	3,5	125
		83	Cotton fibres	60	5	3,5	125
		71	Cotton fibres	60	6	6	125
		84	Cotton fabric shreds	60	6	6	125
		74	Cotton fabric shreds	60	12	12	125
		75	Artificial silk skeins	60	14	14	125
	III	12	Asbestos fibres	Hardly any asbestos products are now offered in the marketplace.			
		15	Asbestos fibres				
		16	Asbestos rope				
	IV	11,5	Rock flour	50	3,5	1,3	150
		13	Mica	50	3	2	150
		13,5	Mica	50	3	2	150
		30,5	Wood flour	60	5	1,5	100
		31,5	Wood flour	70	6	1,5	125
		51,5	Cellulose	60	5	3,5	125

Continuation of table, see Page 320.

Group I: types for general use.

Group II: types with increased notched impact strength.

Group III: types with increased dimensional stability under heat.

Group IV: types with increased electrical characteristics.

Continuation of table, Thermosetting plastics, from Page 319.

Type of resin	Group	Type	Filler	Flexural strength min. N/mm^2	Impact strength min. kJ/m^2	Notched impact strength min. kJ/m^2	Dimensional stability (Martens) °C
Aminoplastic and aminoplastic phenol	I	131	Cellulose	80	6,5	1,5	100
		150	Wood flour	70	6	1,5	120
		180	Wood flour	80	6	1,5	120
	II	153	Cotton fibres	60	5	3,5	125
		154	Cotton fabric shreds	60	6	6	125
	III	155	Rock flour	40	2,5	1	130
		156	Asbestos fibres	Hardly any asbestos products are now offered in the marketplace.			
		157	Asbestos fibres + wood flour				
		158	Asbestos fibres				
	IV	131,5	Cellulose	80	6,5	1,5	100
		152	Cellulose	80	7	1,5	120
		181	Cellulose	80	7	1,5	120
		181,5	Cellulose	80	7	1,5	120
		182	Wood and rock flour	70	4	1,2	120
		183	Cellulose + rock flour	70	5	1,5	120
Polyester		801	Glass fibres	60	22	22	125
		802	Glass fibres	55	4,5	3	140
		830	Glass fibre mats	120	50	40	–
		832	Glass fibre mats	160	70	60	–
Epoxy		870	Rock flour	50	5	1,5	110
		871	Glass fibres	80	8	3	120
		872	Glass fibres	90	15	15	125
Phenol		HP 2061	Paper webs	150	20	15	–
		Hgw 2081	Cotton fabric, coarse	100	18	15	–
		Hgw 2082	Cotton fabric, fine	130	30	15	–
		Hgw 2083	Cotton fabric, extremely fine	150	35	15	–

Group I: types for general use.
Group II: types with increased notched impact strength.
Group III: types with increased dimensional stability under heat.
Group IV: types with increased electrical characteristics.

Processing and use A number of processing procedures and special uses of the most important plastics are presented below.

Plastic	Shaping, reshaping and joining behaviour for workpieces							
	Creating shape by						Shape-changing	
	Melt-on, casting and spraying methods	Low-pressure procedures for reinforced plastics	Injection moulding	Blow moulding	Pressing	Extrusion	(Hot) shaping	Welding
Thermoplastics								
Polyolefins	++	–	+++	+++	(+)	+++	+	+++
Styrene polymers	(+)	–	+++	+	–	++	++	+
Vinyl chloride polymers (hard)	(+)	–	+	++	(+)	+++	++	+++
Polyvinyl chloride (soft)	+	–	+	(+)	+++	++	+	++
Fluorine-containing polymers	+	–	+	–	(+)	(+)	–	+
Poly(meth)acrylic plastics	++	(+)	++	–	–	++	++	–
Heteropolymers	+	–	++	+	–	+	(+)	+
Cellulose ester, ether	+	–	++	++	–	++	+	–
Cellulose hydrate (viscose sheet, cellophane)	–	–	–	–	–	–	(+)	–
Casein plastics and similar casein products	–	–	–	–	–	(+)	–	–
Thermosetting plastics								
Phenolic resins, cresylic resins and furan resins	+	(+)	++	–	++	(+)	(+)	–
Urea-formaldehyde resins	–	–	+	–	+	(+)	–	–
Melamine resins	–	–	+	–	++	–	(+)	–
Reaction resins								
Unsaturated polyesters	++	+++	+	–	++	(+)	–	–
Epoxy resins	++	++	+	–	+	(+)	–	–
Special reaction resins	+	(+)	(+)	–	+	–	–	–
Isocyanate resins (PIR)	+++	–	(+)	–	–	–	–	–
High-temperature plastics								
Polyarylenes, polyarylamides (PARA), polyesters, polyoxides, polyimides	+	+	(+)	–	+	–	–	(+)

Continuation of table, see Page 322.

– = not possible or not common.

+, ++, +++ = corresponds to growing importance.

(+) = special case.

Continuation of table, Processing and use, from Page 321.

Plastic	Shaping, reshaping and joining behaviour for workpieces					
	Special uses					
	Film and fabric synthetic leather	Packaging and insulating films	Foam plastics	Adhesives	Paints and coating materials	Fibres and filaments
Thermoplastics						
Polyolefins	–	+++	+	–	–	+
Styrene polymers	–	++	+++	–	+	(+)
Vinyl chloride polymers (hard)	–	++	+	(+)	(+)	(+)
Polyvinyl chloride (soft)	+++	+	++	–	+	–
Fluorine-containing polymers	–	(+)	(+)	–	–	–
Poly(meth)acrylic plastics	–	–	(+)	+	+	–
Heteropolymers	(+)	++	(+)	(+)	(+)	+++
Cellulose ester, ether	–	++	–	+	+	++
Cellulose hydrate (viscose sheet, cellophane)	–	+++	–	–	–	++
Casein plastics and similar casein products	–	–	–	(+)	(+)	(+)
Thermosetting plastics						
Phenolic resins, cresylic resins and furan resins	–	–	+	++	+	–
Urea-formaldehyde resins	–	–	++	+++	+	–
Melamine resins	–	–	(+)	+	+	–
Reaction resins						
Unsaturated polyesters	–	–	(+)	++	++	–
Epoxy resins	–	–	–	++	+	–
Special reaction resins	–	–	–	+	(+)	–
Isocyanate resins (PUR)	++	–	+++	+	+	(+)
High-temperature plastics						
Polyarylenes, polyarylamides (PARA), polyesters, polyoxides, polyimides	–	+	+	+	+	(+)

– = not possible or not common.

+, ++, +++ = corresponds to growing importance.

(+) = special case.

Material selection

In this section, the essential construction materials, their properties and most important characteristic values have been presented. The task of design engineers and product developers is to select the right materials for the components in a technical system, as there is no one material available which can be used universally.

The following criteria play a decisive role in material selection:

- safety (strength characteristics)
- specific gravity (weight expenditure)
- price
- availability
- workability (castable, hot or cold forming properties, machinable, weldable)
- hardness and hardenability
- elongation and toughness
- damping capability
- corrosion resistance
- surface condition and treatment
- behaviour at high and low temperatures
- resistance to ageing.

The optimum material constitutes a compromise between the requirements and possibilities of the various areas:

- Development and Design (use characteristics)
- Production (production characteristics)
- Material and Business Administration (costs)
- Planning and Scheduling (dates).

Technical drawing guideline

Projection methods

Representation of technical objects

There are various possibilities for the clear representation of technical objects, from a freehand drawing through a technical drawing to a data set in a 3-D CAD system. The following descriptions are focussed exclusively on the area of the technical drawing. The technical drawing as an information carrier serves in the case of a production order as a binding means of communication between Design and Production. In addition to the pictorial representation of the object and its dimensioning, it also contains technological and organisational information (cf. VDI 2211).

Orthogonal projection

Orthogonal projection is normally used in technical drawings. The body is depicted and dimensioned in several two-dimensional views (2-D views). These are always perpendicular parallel projections.

As a result of different projection methods, there are different arrangements of views on the drawing. The standards DIN ISO 128-30 (May 2002) and DIN ISO 5456-2 (April 1998) give more detailed explanations of the basic rules governing views and orthogonal projection in technical drawings.

Descriptions are given here of three projection methods:

- arrow method
- first angle projection
- third angle projection.

Attention!

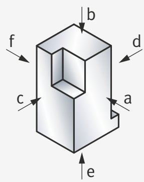
All drawings in this book follow the German standard projection rule: first angle projection, see Page 326.

Basic rules

The basic rules of orthogonal projection are as follows:

- The most informative view is always selected as the front view or main view
- The body is to be depicted in the main view as far as possible in its operating position, manufacturing position or mounting position
- The number of additional views and sections must be restricted to what is necessary.

The following table shows the standardised descriptions of the views:

Views of body	Direction of observation		Description of views	
	View from	View in direction	Letter	Name
	front	a	A	Front view, main view
	above	b	B	Top view
	left	c	C	Lateral view from left
	right	d	D	Lateral view from right
	below	e	E	Bottom view
	rear	f	F	Rear view

Arrow method In the arrow method according to DIN ISO 128-30, the views on the technical drawing in relation to the main view can be in any arrangement but are individually marked.

Rules for the arrow method:

- Arrows with any letters, predominantly in the main view, indicate the direction of observation of the other views
- Each view except the main view must be marked at the top by means of the corresponding upper case letter
- In the drawing, no graphical symbol is necessary for this method (in contrast to first and third angle projection).

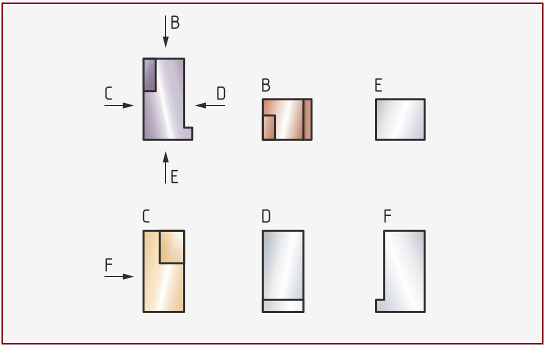


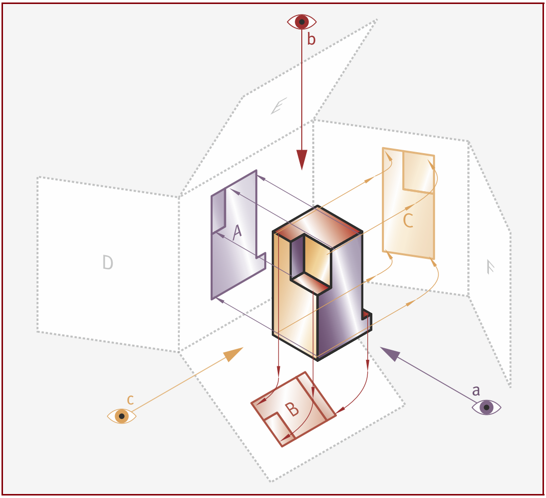
Figure 1
Arrow method, arrangement of views

First angle projection

First angle projection is used in preference in most of Europe.

The body is positioned between the observer and the surface that represents the plane of projection. Each view is projected in a perpendicular manner through the body onto the plane at the rear. The planes of projection of all 6 views form a cube, the inner faces of which bear the projected views.

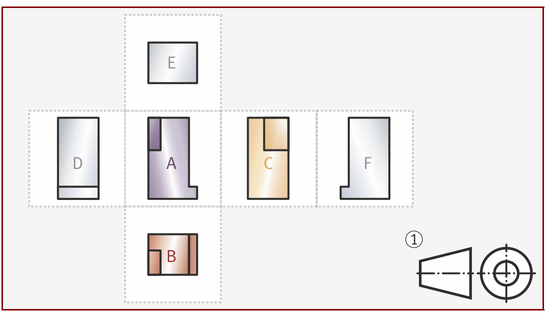
Figure 2
Folding rule in first angle projection



The technical drawing corresponds to the internal surface of this cube arrangement, which means that the views can be placed on the drawing area in this arrangement without any further description. The title block on the drawing contains a symbol indicating first angle projection (DIN 6).

Figure 3
First angle projection arrangement of views

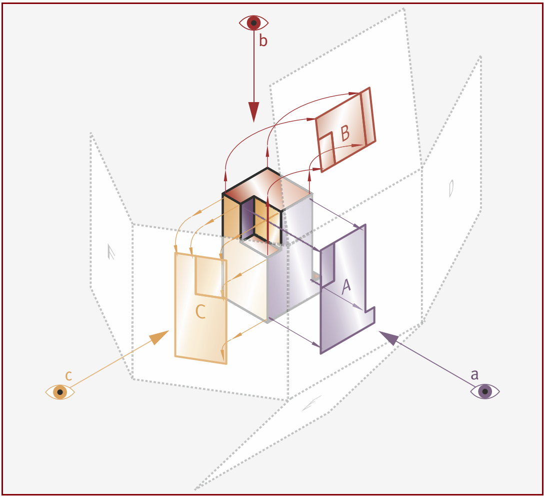
① Graphical symbol (doubled in size)



Third angle projection

The body is positioned, from the observer's perspective, behind the plane of projection. Each of the 6 views is projected perpendicular to the plane in front, which is closer to the observer. All the planes of projection form a cube on which the projected views are shown.

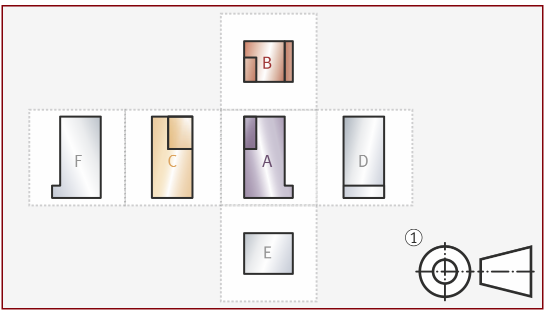
Figure 4
Folding rule
in third angle projection



The technical drawing corresponds to the external surface of the cube arrangement. The views are arranged on the drawing area in this way without any further description. The title block on the drawing contains a symbol indicating third angle projection.

Figure 5
Third angle projection,
arrangement of views

① Graphical symbol
(doubled in size)



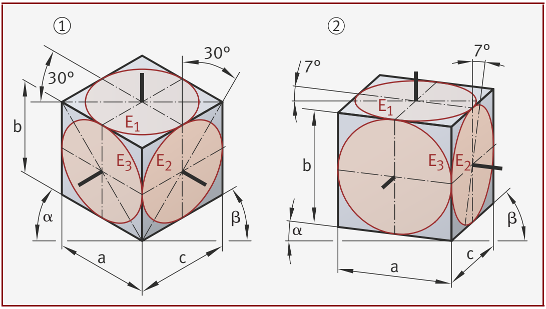
Pictorial representation The pictorial representation of an object gives a three-dimensional view (3-D view). It conveys the impression of a three-dimensional object, is easier to understand than 2-D views and is therefore used in a supporting capacity in technical drawings.

Axonometric representations The standard DIN ISO 5456-3 (April 1998) shows axonometries for technical drawings. Axonometries are parallel projections that create a three-dimensional view of a three-dimensional object. Lines that run parallel in the 3-D object remain parallel in axonometric representations.

Two common methods of axonometric representation, isometric projection and dimetric projection, are explained here.

Figure 6
Axonometric representations

① Isometric projection
② Dimetric projection



Isometric projection In **isometric projection**, see Figure 6 Part ①, the following apply:

- representation of a cube and the circles in three views
- ratio of sides a:b:c = 1:1:1
- $\alpha = \beta = 30°$
- ellipse E_1: large axis horizontal
- ellipses E_2 and E_3: large axes at 30° to perpendicular
- ratio of the axes of the three ellipses 1:1,7.

Dimetric projection In **dimetric projection**, see Figure 6 Part ②, the following apply:

- representation of a cube and the circles in three views
- ratio of sides a:b:c = 1:1:0,5
- $\alpha = 7°$ and $\beta = 42°$
- ellipse E_1: large axis horizontal
- ellipses E_1 and E_2: large axes at 7° to perpendicular
- ellipses E_1 and E_2 have ratio of axes of 1:3
- ellipse E_3 becomes a circle for the purposes of simplicity.

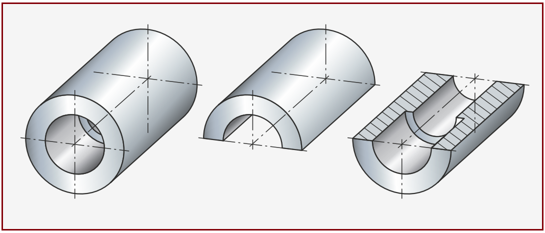
Cross-sections

Representation in accordance with DIN

Cross-sections allow insights into the interior of components or hollow bodies such as housings, workpieces with holes or openings. A virtual cross-section through the relevant body provides clarity. Its representation is implemented in accordance with DIN ISO 128-40 and DIN ISO 128-50.

The cross-sectional surfaces are marked by crosshatching: continuous narrow lines running parallel at an angle of 45°.

Figure 7
Cross-sectional surfaces



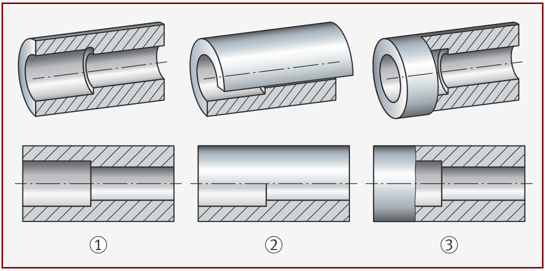
Types of cross-section

The types of cross-section can be classified as follows, see Figure 8:

- Full section:
  complete cross-section of the relevant component
- Half section:
  showing both a cross-section and a view
- Partial section:
  internal contours exposed only in certain selected areas
  - Breakout: all section lines are placed such that the area to be shown is exposed. The crosshatching lines are delimited by a freehand line, see Figure 9
  - Crop section: representation of a detail and associated with a magnification. The crosshatching lines end at a straight virtual edge, see Figure 9.

Figure 8
Types of cross-section

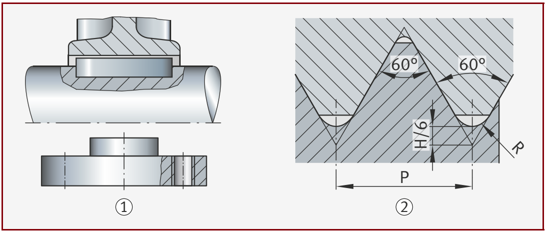
① Full section
② Half section
③ Partial section



A breakout and crop section can be represented as follows:

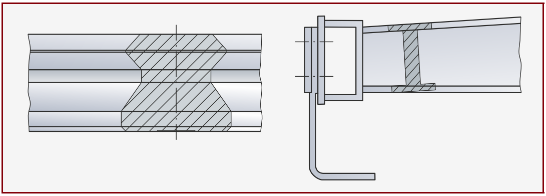
Figure 9
Breakout and crop section

① Breakout
② Crop section



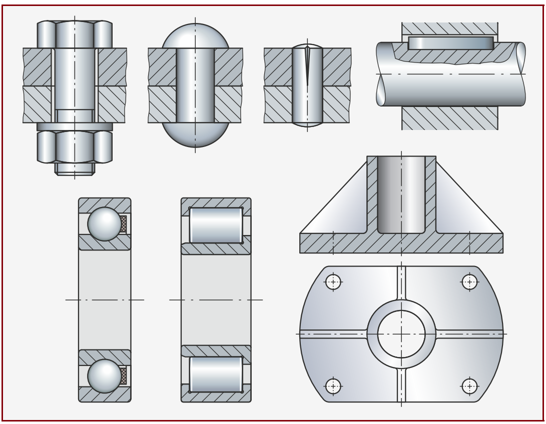
If clarity is still ensured, a cross-section can be rotated in a simplified form into the suitable view, see Figure 10. In this case, it is only bordered by continuous narrow lines; further marking is not necessary.

Figure 10
Rotated view of cross-section



Solid workpieces are not drawn in longitudinal section, see Figure 11. For example, these include shafts, studs, rivets, pins, screws, nuts, washers, feather keys, splines, rolling bearing bodies as well as ribs on castings and knobs on handwheels.

Figure 11
Longitudinal sections



Marking of the section line

In the case of more complex components, the section line can be represented more specifically by means of:

- representation of cross-sectional planes
- several cross-sections in order to represent all features
- allocation within the cross-section by means of upper case letters
- positioning of the cross-section on the projection axis, possibly below the projection axis, see Figure 12
- representation of the section line where it undergoes a change or change in its direction, see Figure 13
- kinking of the cross-section if it is present in several planes
- kinking of the cross-section away from the 90° angle, see Figure 14.
- Outlines and edges behind the cross-sectional plane can be omitted if they do not contribute to illustration of the drawing.

Figure 12
Cross-section below the projection axis

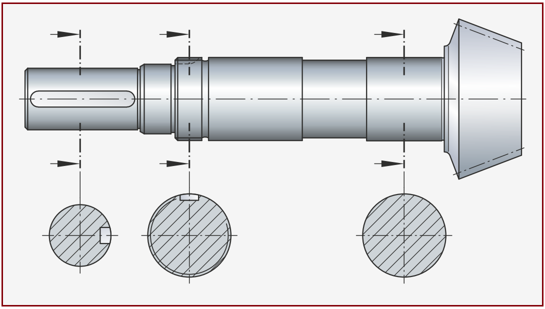


Figure 13
Section line with kinking

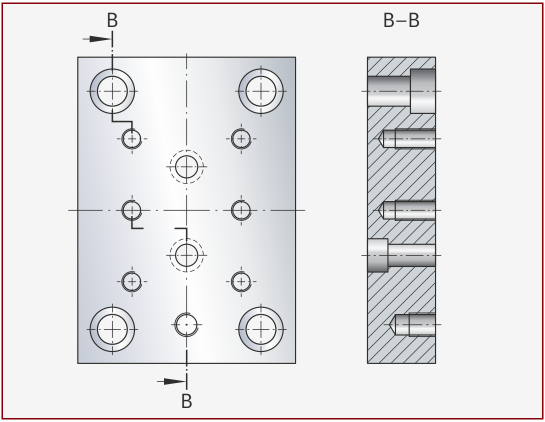
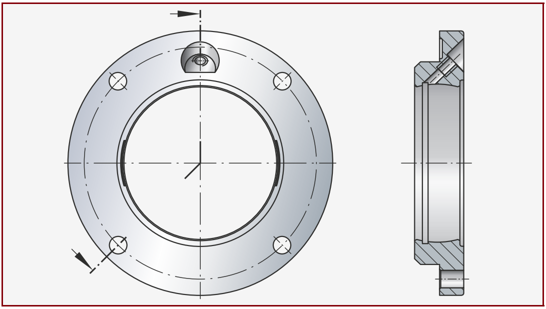


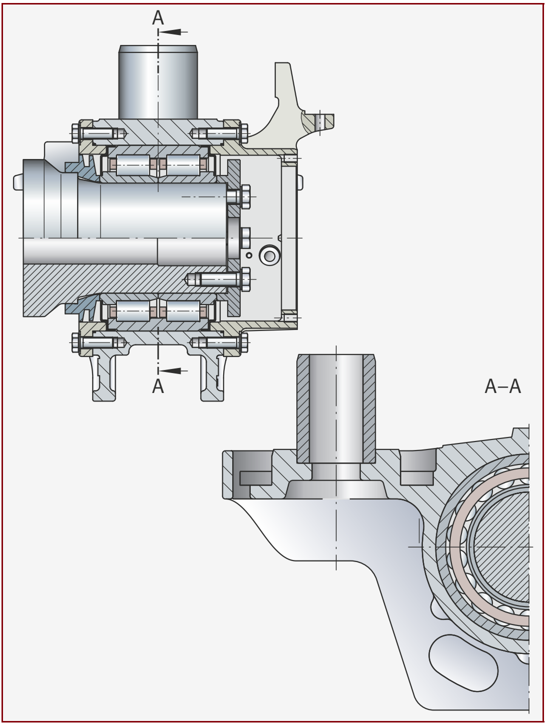
Figure 14
Section line with kinking away from the 90° angle



Example of a cross-section

The following example shows a more complex cross-section.

Figure 15
Example of a cross-section



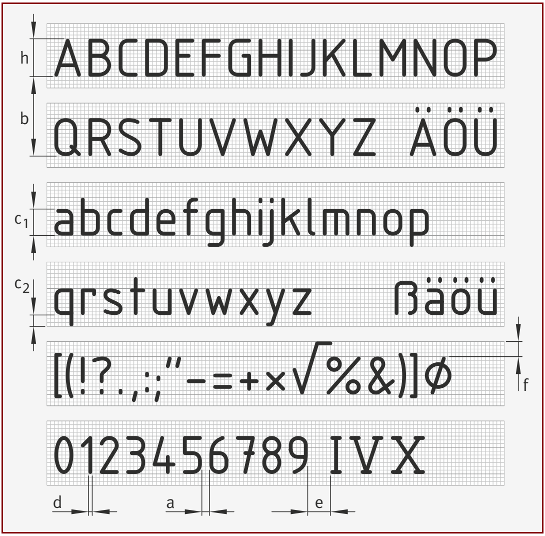
Basic elements in drawings

Standard font

In technical drawings, standard fonts are used for labelling.
The standard DIN EN ISO 3098 (April 1998) defines the fonts A, B, CA and CB (CA and CB only for CAD applications) in either a vertical or inclined form as appropriate.

The following diagram shows the preferred font for manual labelling. In the case of the font size of 5 mm shown here, this font is designated as follows: font ISO 3098 – BVL – 5.

Figure 16
Font B, vertical (V), Roman alphabet (L), with German characters



Font B For font B (d = h/10) in the standard font, the dimensions are as follows:

Labelling feature		Ratio	Dimension mm						
Font size, height of upper case letters	h	(10/10) h	2,5	3,5	5	7	10	14	20
Height of lower case letters (x height)	c_1	(7/10) h	1,75	2,5	3,5	5	7	10	14
Descenders of lower case letters	c_2	(3/10) h	0,75	1,05	1,5	2,1	3	4,2	6
Range of diacritical characters	f	(4/10) h	1	1,4	2	2,8	4	5,6	8
Line thickness	d	(1/10) h	0,25	0,35	0,5	0,7	1	1,4	2
Minimum spacing between:									
Characters	a	(2/10) h	0,5	0,7	1	1,4	2	2,8	4
Base lines 1[1]	b_1	(19/10) h	4,75	6,65	9,5	13,3	19	26,6	38
Base lines 2[2]	b_2	(15/10) h	3,75	5,25	7,5	10,5	15	21	30
Base lines 3[3]	b_3	(13/10) h	3,25	4,55	6,5	9,1	13	18,2	26
Words	e	(6/10) h	1,5	2,1	3	4,2	6	8,4	12

1) In the use of upper case and lower case letters with diacritical characters (Ä, Ö, Ü, ...).

2) In the use of upper case and lower case letters without diacritical characters.

3) Only upper case letters are used, diacritical characters are not used.

Preferred numbers and preferred number series In accordance with the standard DIN 323 (August 1974), the structure of the preferred number series is based on geometric number sequences with 5, 10, 20 or 40 elements in each group of ten.

Starting from the basic series

Equation 1

$$a \cdot q^0, a \cdot q^1, a \cdot q^2, \ldots, a \cdot q^{n-1}$$

the individual preferred number series are determined as having the increment steps q:

Equation 2

$$q = \sqrt[5]{10} = 1{,}6 \quad \text{for R 5} \qquad q = \sqrt[20]{10} = 1{,}12 \quad \text{for R 20}$$

$$q = \sqrt[10]{10} = 1{,}25 \quad \text{for R 10} \qquad q = \sqrt[40]{10} = 1{,}06 \quad \text{for R 40}$$

Basic series The following table gives the basic series (main values and precise values).

Main value Basic series				Sequence number	Mantissa	Precise value	Deviation of main value from precise value
R 5	R 10	R 20	R 40	N			%
1,00	1,00	1,00	1,00	0	000	1,0000	0
			1,06	1	025	1,0593	+0,07
		1,12	1,12	2	050	1,1220	−0,18
			1,18	3	075	1,1885	−0,71
	1,25	1,25	1,25	4	100	1,2589	−0,71
			1,32	5	125	1,3353	−1,01
		1,40	1,40	6	150	1,4125	−0,88
			1,50	7	175	1,4962	+0,25
1,60	1,60	1,60	1,60	8	200	1,5849	+0,95
			1,70	9	225	1,6788	+1,26
		1,80	1,80	10	250	1,7783	+1,22
			1,90	11	275	1,8836	+0,87
	2,00	2,00	2,00	12	300	1,9953	+0,24
			2,12	13	325	2,1135	+0,31
		2,24	2,24	14	350	2,2387	+0,06
			2,36	15	375	2,3714	−0,48
2,50	2,50	2,50	2,50	16	400	2,5119	−0,47
			2,65	17	425	2,6607	−0,40
		2,80	2,80	18	450	2,8184	−0,65
			3,00	19	475	2,9854	+0,49
	3,15	3,15	3,15	20	500	3,1623	−0,39
			3,35	21	525	3,3497	+0,01
		3,55	3,55	22	550	3,5481	+0,05
			3,75	23	575	3,7584	−0,22

Continuation of table, see Page 336.

The method of writing preferred numbers without trailing zeroes is also used internationally.

Continuation of table, Basic series, from Page 335.

Main value Basic series				Sequence number	Mantissa	Precise value	Deviation of main value from precise value
R 5	R 10	R 20	R 40	N			%
4,00	4,00	4,00	4,00	24	600	3,9811	+0,47
			4,25	25	625	4,2170	+0,78
		4,50	4,50	26	650	4,4668	+0,74
			4,75	27	675	4,7315	+0,39
	5,00	5,00	5,00	28	700	5,0119	−0,24
			5,30	29	725	5,3088	−0,17
		5,60	5,60	30	750	5,6234	−0,42
			6,00	31	775	5,9566	+0,73
6,30	6,30	6,30	6,30	32	800	6,3096	−0,15
			6,70	33	825	6,6834	+0,25
		7,10	7,10	34	850	7,0795	+0,29
			7,50	35	875	7,4989	+0,01
	8,00	8,00	8,00	36	900	7,9433	+0,71
			8,50	37	925	8,4140	+1,02
		9,00	9,00	38	950	8,9125	+0,98
			9,50	39	975	9,4406	+0,63
10,00	10,00	10,00	10,00	40	000	10,0000	0

The method of writing preferred numbers without trailing zeroes is also used internationally.

Line types and line groups The standard DIN EN ISO 128-20 (December 2002) defines line types and line groups.

Line types The following table shows some line types from the standard.

DIN ISO 128-20 DIN ISO 128-24[1]	Line type	Application in accordance with DIN ISO 128-24 (December 1999)
01.2	Continuous wide line	■ Visible edges ■ Visible outlines ■ Crests of screw threads ■ Limit of length of usable thread depth ■ Main representations in diagrams, charts, flow charts ■ System lines (steel construction) ■ Parting lines of moulds in views ■ Lines of cuts and section arrows
01.1	Continuous narrow line	■ Imaginary lines of intersection ■ Dimension lines ■ Extension lines ■ Leader lines and reference lines ■ Hatching ■ Outlines of revolved sections ■ Short centre lines ■ Root of screw threads ■ Bending lines ■ Origin and terminations of dimension lines ■ Diagonals for the indication of flat surfaces ■ Framing of details ■ Indication of repetitive details, e.g. root circles of gears ■ Interpretation lines of tapered features ■ Location of laminations ■ Projection lines ■ Grid lines
01.1	Continuous narrow freehand line	■ Preferably manually represented termination of partial or interrupted views, cuts and sections, if the limit is not a line of symmetry or a centre line[2]
	Continuous narrow line with zigzags	■ Termination represented in CAD of partial or interrupted views, cuts and sections, if the limit is not a line of symmetry or a centre line[2]
02.2	Dashed wide line	■ Possible indication of permissible areas of surface treatment
02.1	Dashed narrow line	■ Hidden edges ■ Hidden outlines

Continuation of table, see Page 338.

[1] The first part of the number of the lines indicates the basic type of lines in accordance with DIN ISO 128-20.
[2] It is recommended to use only one type of line on one drawing.

Continuation of table, Line types, from Page 337.

DIN ISO 128-20 DIN ISO 128-24[1]	Line type	Application in accordance with DIN ISO 128-24 (December 1999)
04.2	Long-dashed dotted wide line	■ Position of cross-sectional planes ■ Indication of (limited) required areas of surface treatment (e.g. heat treatment)
04.1	Long-dashed dotted narrow line	■ Centre lines ■ Lines of symmetry ■ Pitch circles of gears ■ Pitch circle of holes
05.1	Long-dashed double-dotted narrow line	■ Outlines (adjacent parts, alternative designs, finished parts into blank parts) ■ Initial outlines prior to forming ■ Extreme positions of movable parts ■ Centroidal lines ■ Parts situated in front of a cross-sectional plane ■ Framing of particular fields/areas ■ Projected tolerance zone

[1] The first part of the number of the lines indicates the basic type of lines in accordance with DIN ISO 128-20.

Line thicknesses and line groups

The following table lists some line groups and the associated line thicknesses.

Line group	Associated line thickness (nominal dimensions in mm) for		
	Line type		Dimension and text notation; graphical symbol
	01.2, 02.2, 04.2	01.1, 02.1, 04.1	DIN EN ISO 81714-1
0,25	0,25	0,13	0,18
0,35	0,35	0,18	0,25
0,5[1]	0,5	0,25	0,35
0,7[1]	0,7	0,35	0,5
1	1	0,5	0,7
1,4	1,4	0,7	1
2	2	1	1,4

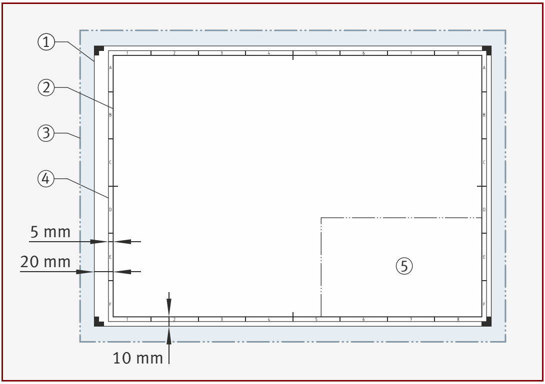
[1] These line groups should be used in preference.

Sheet sizes

The standard DIN EN ISO 5457 (November 2010) defines sheet sizes.

Figure 17
Sheet sizes based on the example of DIN A3

① Trimmed drawing
② Drawing area
③ Untrimmed sheet
④ Field division
⑤ Title block[1)]



[1)] For details about the title block, see DIN EN ISO 7200 (May 2004).

Sheet size[1)]	Trimmed drawing and trimmed blueprint[2)] mm	Drawing area mm	Untrimmed sheet[3)] mm	Usable favourable roller width mm			Base sheet[4)] mm
A0	841×1189	821×1159	880×1230	–	900	–	–
A1	594×841	574×811	625×880	–	900	660	660×900
A2	420×594	400×564	450×25	(2×450)	900	660	450×660
A3	297×420	277×390	330×450	(2×330) (2×450)	660	900	330×450
A4	210×297	180×277	240×330	250	660	–	225×330

[1)] In accordance with DIN EN ISO 216 Series A.
[2)] Finished sheet.
[3)] Base sheet for single printing.
[4)] From 660 mm×900 mm.

Scales The standard DIN ISO 5455 (December 1979) defines notation for scales.

The title block contains the principal scale of the drawing and the other scales at different locations. The latter are presented in each case against the associated representations. As far as possible, all objects (except in standards) must be drawn true to scale.

If parts are depicted in an enlarged representation, it is advisable to add a representation on a 1:1 scale in order to show the natural size. In this case, it is not necessary to reproduce the details.

Enlargement scale	50:1 5:1	20:1 2:1	10:1
Natural scale	1:1		
Reduction scales	1:2 1:20 1:200 1:2 000	1:5 1:50 1:500 1:5 000	1:10 1:100 1:1 000 1:10 000

Dimensioning in accordance with standards

Dimension indications in accordance with DIN Dimension indications in technical drawings apply to the final condition of a part (raw, premachined or finished). Their representation is implemented in accordance with DIN 406-10, DIN 406-11 or respectively ISO 129-1.

Types of dimensioning The type of dimensioning is carried out

- in relation to function:
  selection, entry and tolerancing of dimensions in relation to purely functional considerations
- in relation to production:
  entry of dimensions as a function of the production process
- in relation to inspection:
  entry of dimensions and tolerances relevant to inspection.

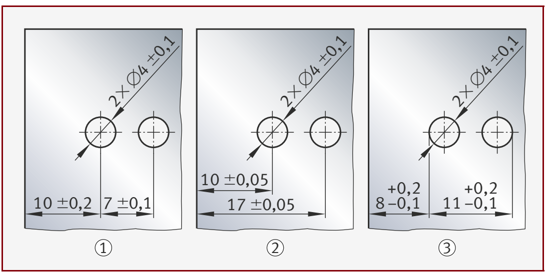


Figure 18
Types of dimensioning

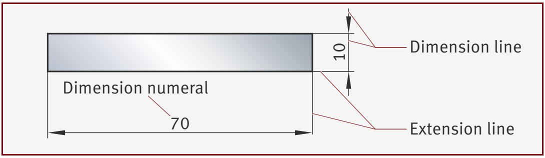
① In relation to function
② In relation to production
③ In relation to inspection

Elements in entry of dimensions

The following elements and specifications must be observed when entering dimensions:

- Dimension lines (continuous narrow lines):
  - are at a distance of approx. 10 mm from the edge of the body
  - other parallel dimension lines have a spacing of at least 7 mm
  - centre lines and edges must not be used as dimension lines
- Extension lines (continuous narrow lines):
  - project approx. 2 mm beyond the dimension line
- Dimension lines and extension lines should not intersect any other lines
- Dimension numerals:
  - are positioned at the approximate centre of the dimension line (although exceptions occur)
  - are positioned approx. 0,5 mm to 1,5 mm above the dimension line.

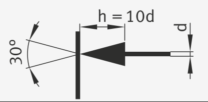
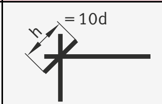
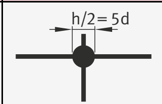
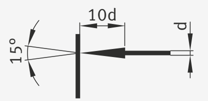
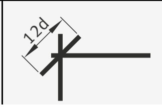
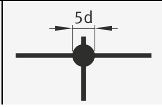
Figure 19
Elements in entry of dimensions



Termination of dimension lines

For the termination of dimension lines, the following applies:

- Within a drawing, only one of the possible terminations of dimension lines may be used, see table Terminations of dimension lines
- In general, a black arrows is used as the dimension arrow, while an open arrow is used in CAD drawings
- In specialised drawings (for example in the case of construction projects), obliques can be used instead of dimension arrows
- If there is a shortage of space, solid points can also be used in conjunction with dimension arrows (in the case of chain dimensions)
- Arrows are always used as terminations of dimension lines on arcs, radii and diameters
- In order to indicate an origin, an open circle is used.

Terminations of dimension lines	Dimension arrows	Obliques	Solid dots
To ISO 129-1			
To DIN 406-10			

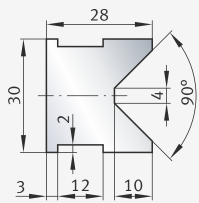
Methods of entry of dimensions

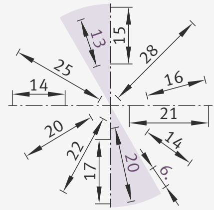
Dimensions can be entered in accordance with the following methods:

- in two main reading directions (standard):
  Dimension numerals should where possible be entered such that, in the reading position of the drawing, they can be read in the main reading directions from the bottom and from the right
- on dimension reference edges:
  Suitable reference planes are selected for the dimensioning.

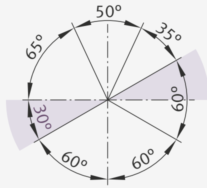
Entry of dimensions in two main reading directions

For length dimensions

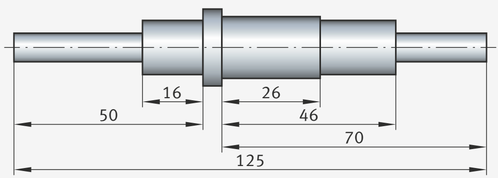




For angle dimensions

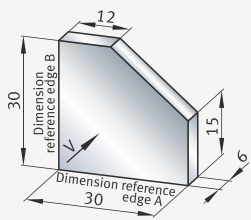


Example

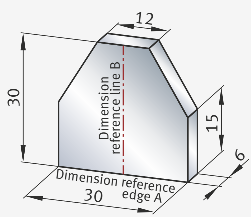


Entry of dimensions on dimension reference edges

As a function of the symmetry of components



For asymmetrical components



For symmetrical components

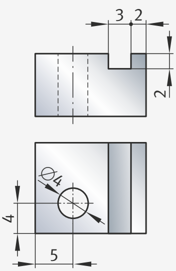
Arrangement of dimensions, dimensioning rules

Each dimension of a component may only be entered once within a drawing.

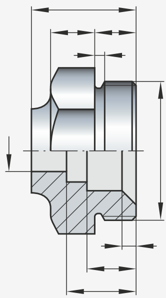
Entries of dimensions are arranged as follows:

- according to views
- according to internal and external dimensions
- according to individual components
- without dimension chains in order to avoid, for example, summation of individual tolerances, but possibly with an auxiliary dimension (in brackets).

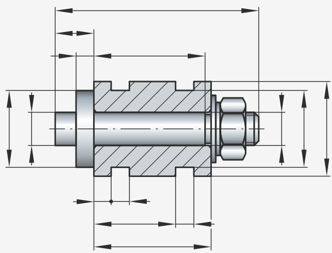
Arrangement of dimensions



According to views

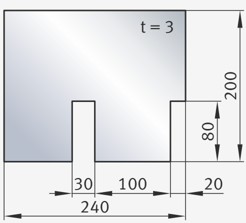


According to internal and external dimensions

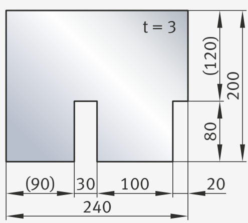


According to individual components

Avoidance of dimension chains



Without dimension chains

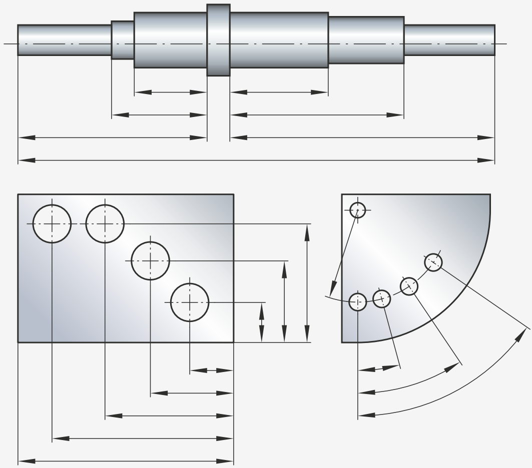


Without dimension chains, with auxiliary dimension (in brackets)

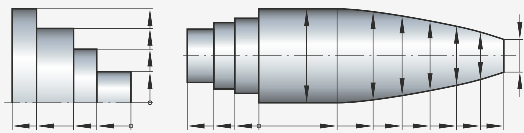
Types of entry of dimensions

Entry of dimensions can be carried out as follows:

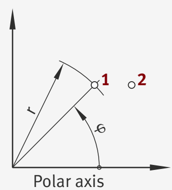
Parallel dimensioning



Stepped dimensioning

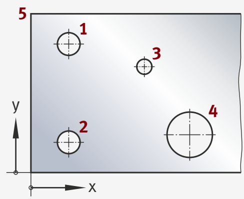


Coordinate dimensioning



Pos.	r	φ
1	15	45°
2	19	35°

With polar coordinates



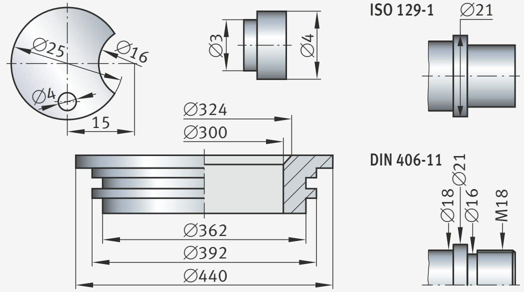
Pos.	x	y	d
1	5	17	∅3
2	5	4	∅3
3	15	14	∅2
4	21	5	∅6
5	0	21	–

With Cartesian coordinates

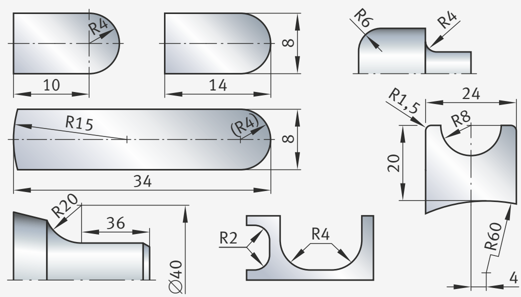
Dimensioning of features

Examples of the dimensioning of geometrical parts are given below:

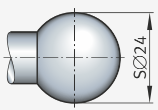
Diameter



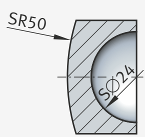
Radius



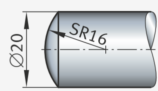
Sphere



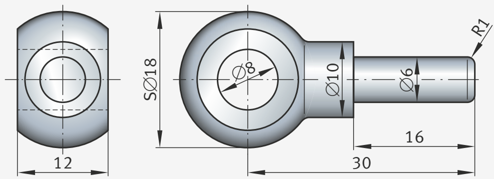
Sphere diameter



Sphere radius



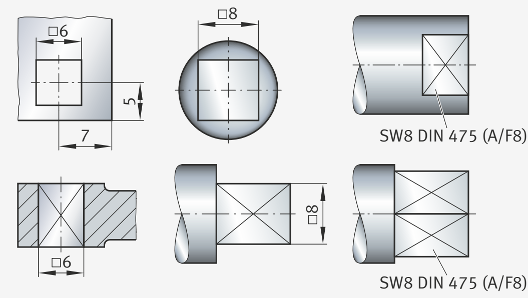
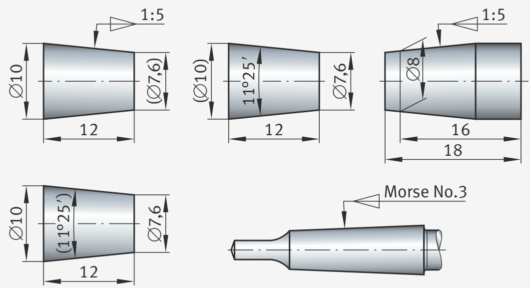
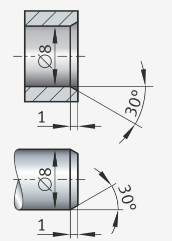
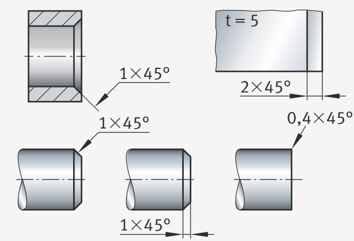
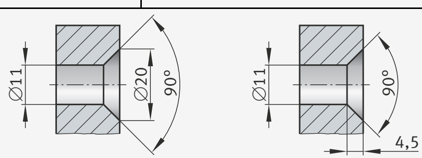
Oval point



Sphere/cylinder transition with light edge

Continuation of table, see Page 346.

Continuation of table, Dimensioning of features, from Page 345.

Square form and width across flats The width across flats is selected in accordance with DIN 475	
Taper (in accordance with DIN EN ISO 3040)	
Chamfer	 Chamfer other than 45°  Chamfer at 45°
Counterbore	

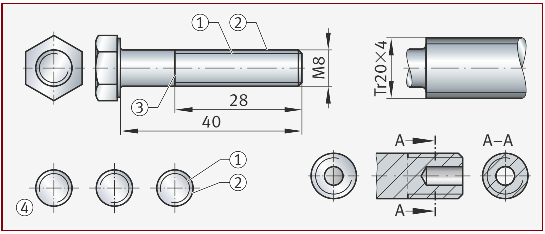
Dimensioning of threads

The simplified representation of threads is defined in DIN ISO 6410-1:1993.

In the case of external threads, the thread root must be drawn as a continuous narrow line while the outside diameter and the thread limit must be drawn as a continuous wide line.

Figure 20
Bolt thread/ external thread

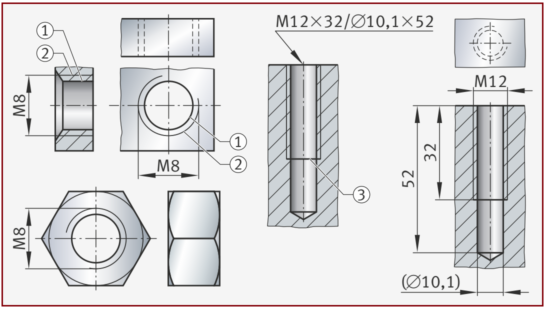
① Thread root
② Outside diameter
③ Thread limit
④ 3/4 circle, variable position and opening



In the case of internal threads, the outside diameter must be drawn as a continuous narrow line while the core diameter must be drawn as a continuous wide line.

Figure 21
Nut thread/ internal thread

① Core diameter
② Outside diameter
③ Thread limit



Short designations for threads are defined in DIN 202. These comprise:

- designation for thread type: M, R, Tr
- nominal diameter (thread size)
- lead or pitch as appropriate
- number of turns
- any additional indications necessary: tolerance, direction of turns.

Dimensions for threads, thread undercuts and thread runouts: see chapter Design elements, section Thread runout and thread undercut, Page 468, and section Metric ISO threads, Page 460.

Dimensioning of undercuts

The representation of undercuts is defined in DIN 509:2006.

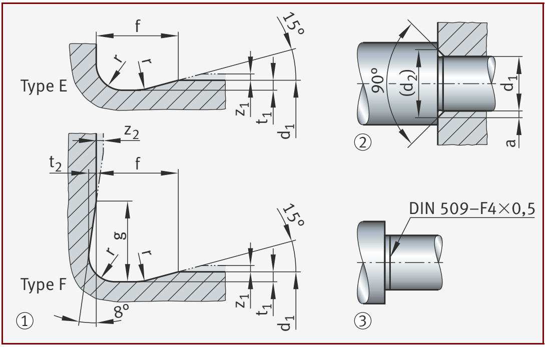
Undercuts are used at the surface transitions of stepped shafts or flat surfaces with steps that are to be ground. As a result, the edge of the grinding wheel can run out cleanly.

The indication of undercuts in drawings is carried out as follows:

- complete with all dimensions:
  - large representation of the workpiece in a breakout
  - small representation of a detail drawn in magnification
- simplified as a designation with extension line.

Figure 22
Dimensioning of undercuts

① Undercuts, type E and F, complete representation
② Counterbore on mating part
③ Simplified indication of undercut F 4 × 0,5; dimensions r = 4 mm, t_1 = 0,5 mm



A distinction is made between the following types of undercut:

- **type E** for workpieces with one machining surface
- **type F** for workpieces with two machining surfaces perpendicular to each other.

The necessary counterbore depth in the mating part is dependent on the undercut type.

The following table shows the dimensions of undercuts of type E and F and the counterbore in the mating part in accordance with DIN 509:2006, see also Figure 22.

Dimensions mm					Recommended allocation[2] to diameter d_1 for workpieces mm		Minimum dimension a for counterbore mm	
r ±0,1	t_1 +0,1	t_2 +0,05	f +0,2	g	under normal loading	under increased loading	Type E	Type F
R0,2	0,1	0,1	1	(0,9)	> ∅ 1,6 to ∅ 3	–	0,2	0
R0,4	0,2	0,1	2	(1,1)	> ∅ 3 to ∅ 18	–	0,3	0
R0,6	0,2	0,1	2	(1,4)	> ∅ 10 to ∅ 18	–	0,5	0,15
R0,6	0,3	0,2	2,5	(2,1)	> ∅ 18 to ∅ 80	–	0,4	0
R0,8	0,3	0,2	2,5	(2,3)	> ∅ 18 to ∅ 80	–	0,6	0,05
R1	0,2	0,1	2,5	(1,8)	–	> ∅ 18 to ∅ 50	0,9	0,45
R1	0,4	0,3	4	(3,2)	> ∅ 80	–	0,7	0
R1,2	0,2	0,1	2,5	(2)	–	> ∅ 18 to ∅ 50	1,1	0,6
R1,2	0,4	0,3	4	(3,4)	> ∅ 80	–	0,9	0,1
R1,6	0,3	0,2	4	(3,1)	–	> ∅ 50 to ∅ 80	1,4	0,6
R2,5	0,4	0,3	5	(4,8)	–	> ∅ 80 to ∅ 125	2,2	1
R4	0,5	0,3	7	(6,4)	–	> ∅ 125	3,6	2,1

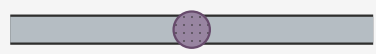
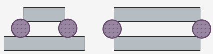
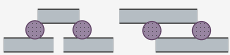
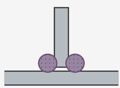
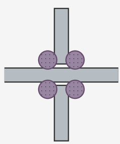
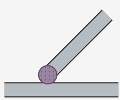
[1] Numerals printed **bold** correspond to undercuts of series 1. Series 1 should be used in preference.

[2] The allocation to the diameter range is a guideline only; it does not apply in the case of short steps and thin-walled parts.

Symbols for weld connections

Weld connections Weld connections are material contact connections between materials of the same type (for example steel, aluminium, certain plastics). Depending on the intended purpose and type of loading, use is made of various joint types, weld seam types and seam configurations.

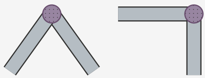
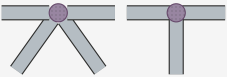
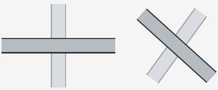
Joint types in accordance with DIN The following table shows joint types in accordance with DIN 1912.

Joint type	Arrangement of parts	Explanation of joint type	Suitable seam configurations (symbols), guidelines
Butt joint		The parts lie in a single plane and are butted against each other	 Favourable in relation to flow of forces and material usage
Parallel joint		The parts lie parallel on top of each other	 Frequently used in top flange plates of bending beams
Overlap joint		The parts lie parallel on top of and overlap each other	 Frequently used as member connections in steelwork
T joint		The parts are butted perpendicular against each other	 Measures are necessary in the case of transverse tensile loading[1]
Double T joint (cross joint)		(derived from T joint) Two parts lying in a single plane are butted perpendicular against an interjacent third part – unfavourable!	 Measures are necessary in the case of transverse tensile loading[1]
Angular joint		(derived from T joint) One part is butted obliquely against another	 Fillet angle $\geqq 60°$ Measures are necessary in the case of transverse tensile loading[1]

Continuation of table, see Page 351.

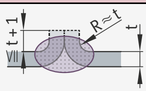
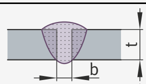
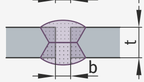
[1] Due to the risk of fractures (lamellar fractures): examples include ultrasonic inspection, increase in weld connection area.

Continuation of table, Joint types in accordance with DIN, from Page 350.

Joint type	Arrangement of parts	Explanation of joint type	Suitable seam configurations (symbols), guidelines
Corner joint		Two parts are butted against each other at a corner of any angle	 Lower load capacity than T joint
Multiple joint		Two or more parts are butted against each other at any angle	Difficult to assess all parts. Unsuitable for higher loading.
Crossing joint		Two parts lie crosswise on top of each other	 Isolated cases in steelwork

Seam types and seam configurations

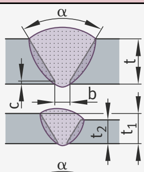
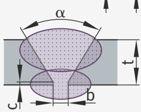
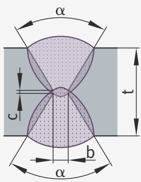
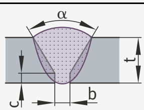
The following table shows a selection of butt seam configurations and their preparation in accordance with DIN EN ISO 9692-1.

Seam type	Seam configuration (joining configuration)	Work-piece thick-ness t mm	Execu-tion	Sym-bol	Dimensions Gap b mm	Angle α, β °	Welding process[1]	Production costs[2]	Comments Application
Flanged seam		up to 2	One side		–	–	G, E, WIG, MIG, MAG	–	Welding of thin sheet metal without filler material
I seam		up to 4	One side		≈ t	–	G, E, WIG	0,5	No seam preparation, little filler material. With welding on one side, the possibility of root defects and fusion defects cannot be ruled out.
		up to 8	Both sides		≈ t/2	–	E, WIG (MIG, MAG)		

Continuation of table, see Page 352.

[1] Recommended welding process.
[2] Relative production costs.

Continuation of table, Seam types and seam configurations, from Page 351.

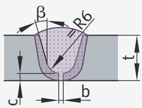
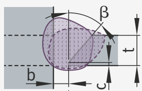
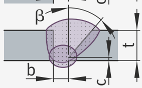
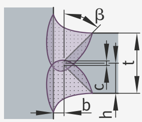
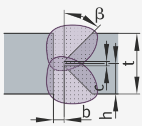
Seam type	Seam configuration (joining configuration)	Workpiece thickness t mm	Execution	Symbol	Dimensions Gap b mm	 Angle α, β °	Welding process[1)]	Production costs[2)]	Comments Application
V seam		3 to 10	One side	V	≦ 4	40 to 60	G	1	In case of dynamic loading: ■ work out and back weld the root ■ for $t_1 - t_2 > 3$ mm, bevel the thicker part to an inclination of 1:4 (flow of forces!)
		3 to 40	Both sides		≦ 3	≈ 60	E, WIG		
						40 to 60	MIG, MAG		
DV seam or X seam		over 10	Both sides	X	1 to 4	≈ 60	E, WIG	2	More favourable for larger sheet thicknesses than V seam since, for an identical angle α, only half the weld deposit quantity is required. Almost no angle shrinkage with welding on alternate sides. If necessary, work out root before welding of the opposing position.
						40 to 60	MIG, MAG		
Y seam		5 to 40	One side	Y	1 to 4	≈ 60	E, WIG, MIG, MAG	1,5	Crosspiece height c = 2 ... 4 mm

Continuation of table, see Page 353.

1) Recommended welding process.

2) Relative production costs.

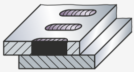
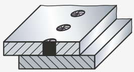
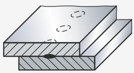
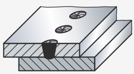
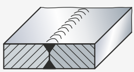
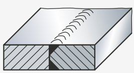
Continuation of table, Seam types and seam configurations, from Page 352.

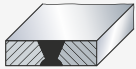
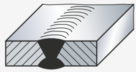
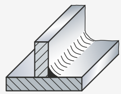
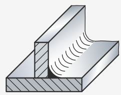
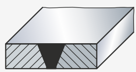
Seam type	Seam configuration (joining configuration)	Work-piece thick-ness t mm	Execu-tion	Sym-bol	Dimensions Gap b mm	Dimensions Angle α, β °	Welding process[1]	Production costs[2]	Comments Application
U seam		over 12	One side		1 to 4	8 to 12	E, WIG, MIG, MAG	4	Crosspiece height c = 3 mm advantageous with inaccessible opposing side. Expensive preparation (planing).
HV seam		3 to 10	One side		2 to 4	35 to 60	E, WIG, MIG, MAG	0,7	Frequently used in conjunction with a fillet seam in a T joint. Execution with unwelded web (HV web seam) reduces production costs. Web height c ≦ 2 mm
		3 to 10	Both sides		1 to 4				
DHV seam (double HV seam, K seam)	 	over 10	Both sides		1 to 4	35 to 60	E, WIG, MIG, MAG	1	Frequently used in conjunction with fillet seams in a T joint. Execution with unwelded web (K web seam) reduces production costs. Flank height h = t/2 or t/3

[1] Recommended welding process.

[2] Relative production costs.

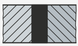
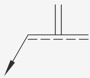
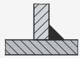
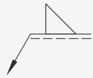
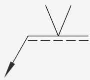
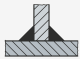
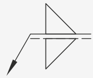
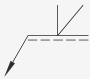
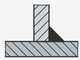
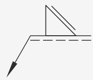
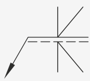
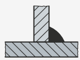
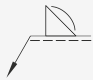
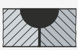
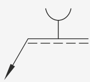
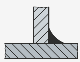
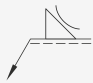
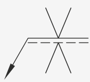
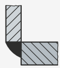
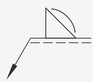
The following table shows further examples of seam types taken from DIN EN ISO 22553:1997.

Seam type	Seam configuration (joining configuration)	Symbol
Plug seam		
		
Spot seam		
		
Composite symbols		
DY seam (double Y seam)		
DHY seam (double HY seam, K web seam)		

Seam type	Seam configuration (joining configuration)	Symbol
Application examples for additional symbols		
Flat V seam with flat opposing seam		
Y seam with worked out root and opposing seam		
Fillet seam with concave surface		
Fillet seam with notch-free seam transition (machined as necessary)		
Flat V seam levelled on upper workpiece surface by additional machining		

Seam symbols for butt and corner seams

The following table shows weld seam symbols for butt seams and corner seams in accordance with DIN EN 22553:1997, in the form in which they can be used in drawings.

Symbol for seam type	Seam configuration (cross-section)	Symbolic representation in drawings	Name	Symbol for seam type	Seam configuration (cross-section)	Symbolic representation in drawings	Name
Butt seams				**Corner seams and fillet seams**			
			I seam				Fillet seam
			V seam				Double fillet seam
			HV seam				Flat seam
			DHV seam or K seam				Camber seam
			U seam				Concave seam
			DV seam or X seam				Corner seam (outer fillet seam, executed here as a camber seam)

Indication of surface quality and roughness parameters in drawings

Surface quality

The insertion of surface roughness data in drawings is carried out with the aid of graphical symbols and is described in the standard DIN EN ISO 1302 (June 2002).

Surfaces on workpieces that are to remain unfinished (unmachined), i.e. surfaces which are the result of the manufacturing process, such as rolling, forging, casting or flame cutting etc., are not assigned a surface symbol.

Indications of surface quality are necessary where there are higher requirements for the quality of the surface. The required surface quality can be achieved by cutting or non-cutting production processes.

For recording of the surface quality, various surface (quality) parameters are defined, see section Roughness profile parameters, Page 364. The values for the surface parameter Ra that can be achieved using various production processes are compiled in the section Achievable mean roughness values, Page 368.

Graphical symbols without indications

Requirements for surface quality are represented in drawings by means of the following graphical symbols:

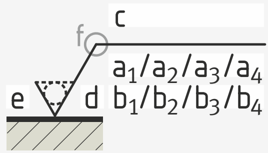
Graphical symbol	Explanation
	**Basic symbol** Meaning must be explained by means of additional indications
	**Expanded symbol** Surface must be machined with removal of material
	**Expanded symbol** Surface may not be subjected to machining by removal of material or must remain in its delivered condition

Indications with the symbol

If the roughness of the surface is specified or further indications relating to the coating, production process or surface structure (direction of grooves) are necessary, a horizontal line is added to the graphical symbols.

The possible additional requirements a-f are positioned in the symbol as shown in Figure 23 and are explained in the following table:

Figure 23
Position of indications with the symbol



Indication	Explanation
a	Individual requirement for surface quality a. The indication a comprises the elements a_1, a_2, a_3 and a_4, the indication a_4 forms the basic information, a_1 – a_3 are optional.
a_1	Tolerance types: upper limit (U) or lower limit (L) for surface parameters.
a_2	As appropriate, the type of filter (for example X = digital Gaussian filter, standardised in accordance with ISO 11562), and the transfer characteristic that is determined from the values for the limit wavelengths of the short wave and/or long wave filter. In the case of 0,0025-0,8 the indication 0,0025 is the wavelength of the short wave filter and the indication 0,8 is the wavelength of the long wave filter in mm. Indications only in the case of deviations from the standard transfer characteristic taken from ISO 4288 and ISO 3274.
a_3	Length of the measurement distance (only for subject parameters). Indications only in the case of deviations from the standard measurement distance in accordance with ISO 1 2085.
a_4	Surface parameter (Rz, Rzmax, Ra, Ramax, Rt, Pt, Wz) with the required limit value in µm. If there is a deviation from the standard measurement distance in accordance with ISO 4288, the surface parameter is increased by one digit. In ISO 4288, the roughness parameters are determined as follows: standard measurement distance ln = 5 × individual measurement distance lr. Rp becomes Rp3 if ln = 3 × lr.

Continuation of table, see Page 358.

Continuation of table, Indications with the symbol, from Page 357.

Indication	Explanation
b	As appropriate, the second requirement for surface quality (composition on the same basis as a).
b' b'' ...	The third, fourth and subsequent requirements for surface quality are inserted on additional lines below b and the symbol is then increased by the height of the additional lines.
c	Indications of production process, surface treatment, coating.
d	Symbol for surface structure, surface grooves, direction of grooves.
e	Machining allowance in mm.
f	An additional circle with the symbol circle indicates all around application. The complete symbol is then valid for the complete contour of the workpiece, the enclosed outer outline of the view, but not for the front and rear sides.

Preferred parameters

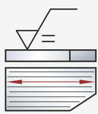
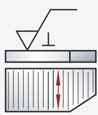
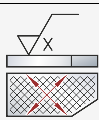
When indicating roughness dimensions in drawings, it must be ensured that only the preferred parameters are used for mean roughness values.

The preferred parameters for Ra and Rz are as follows:

Ra	0,025	0,05	0,1	0,2	0,4	0,8	1,6	3,2	6,3	12,5	25	50
Rz	0,2	0,4	0,8	1,6	3,2	6,3	12,5	25	50	100	200	200

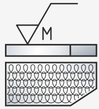
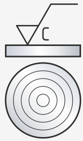
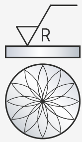
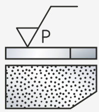
Symbols for the direction of grooves

The direction of grooves is indicated by means of the following symbols:

Symbol	Example	Explanation
=		■ Parallel to the plane of projection of the view in which the symbol is used
⊥		■ Perpendicular to the plane of projection of the view in which the symbol is used
X		■ Crossed in two oblique directions relative to the plane of projection of the view in which the symbol is used

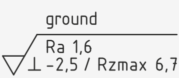
Continuation of table, see Page 359.

Continuation of table, Symbols for the direction of grooves, from Page 358.

Symbol	Example	Explanation
M		■ Multi-directional
C		■ Approximately concentric to the centre of the surface to which the symbol applies
R		■ Approximately radial to the centre of the surface to which the symbol applies
P		■ Surface without grooves, non-directional or with troughs

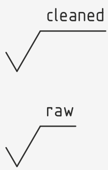
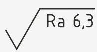
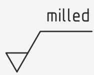
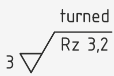
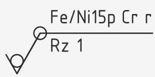
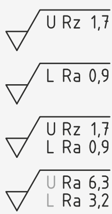
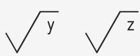
Surface symbols

The following table shows some examples of surface symbols and their meaning or application:

Symbol	Explanation
	Complete example: ■ Production process: grinding ■ Direction of grooves perpendicular to the plane of projection of the view ■ Ra = 1,6 μm, application of 16% rule, standard measurement distance, standard transfer characteristic ■ Rz = Rz max = 6,7 μm (with application of the max rule), standard measurement distance, transfer characteristic –2,5 mm (long wave filter in accordance with ISO 16610-21, deviating from the transfer characteristic in DIN EN ISO 4288).

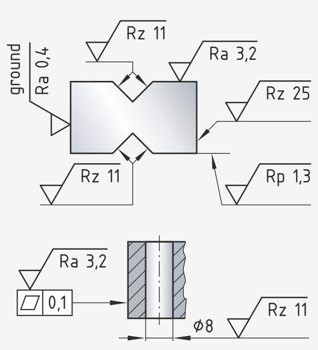
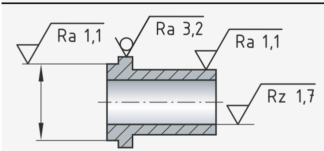
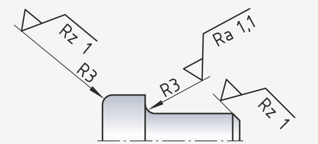
Continuation of table, see Page 360.

Continuation of table, Surface symbols, from Page 359.

Symbol	Explanation
	Surface not subject to requirements for smoothness and uniformity, but certain operations are carried out as appropriate: ■ cleaned: surface devoid of coarse uneven areas, smoothed if necessary (e.g. by grinding, filing) ■ raw: raw condition or cleaned, machining by cutting only permissible if the dimension has not been observed.
	Any machining method, cutting or non-cutting, at upper limit of Ra = 6,3 μm.
	Final condition of the surface is to be achieved by means of the stated production process (milled, turned, ground etc.).
	Machining allowance 3 mm. Application for example in blank part drawings that already show the finished part.
	Surface treatment by nickel/chromium coating, on all surfaces along the outer outline in the view.
	■ Upper limit (U) or lower limit (L) for surface roughness ■ Indication of U and L is optional if they refer to the same parameter (see the lowest image in this line). They are then indicated by their position
	■ Simplified symbol representation by means of letters ■ The meaning of the symbol is explained further at another point on the drawing, preferably in the vicinity of the title block or in the indications section

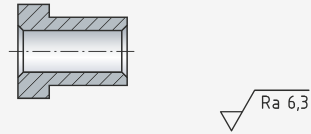
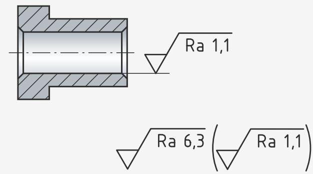
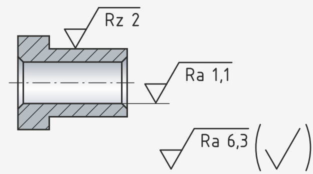
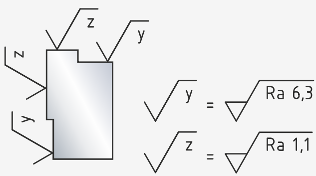
Arrangement of symbols

In the drawing, the symbols are arranged as follows:

Drawing	Application
	Arrangement, general rules: ■ Symbols must be legible from the right or from the bottom ■ Symbols are located directly on the surface or connected with it by means of reference or extension lines and a dimension arrow ■ Symbols may also be placed on the tolerance framework for geometrical tolerances ■ The surface indication can be made together with a dimension indication if there is no possibility of misinterpretation
	■ Cylindrical and prismatic surfaces
	■ Radii and chamfers

Continuation of table, see Page 362.

Continuation of table, Arrangement of symbols, from Page 361.

Drawing	Application
	■ Simplified drawing entry for identical surface quality
	■ Simplified drawing entry for predominantly identical surface quality
	
	■ Simplified drawing entry for identical surface quality on several individual surfaces

Surface parameters

Geometrical parameters are defined for the description of surface quality (DIN EN ISO 4287, July 2010):

- R parameters (relating to the roughness profile)
- W parameters (relating to the waviness profile)
- P parameters (relating to the primary profile).

The quality of a surface is measured in accordance with the standardised profile method (DIN EN ISO 3274, April 1998).

The terms used in conjunction with measurement and recording of surface parameters are explained in the following table.

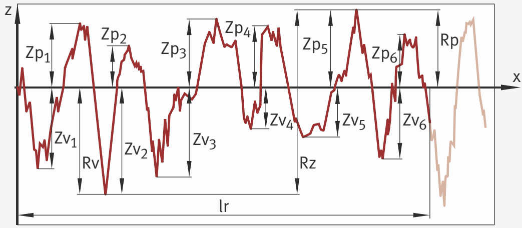
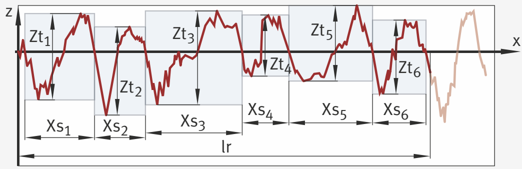
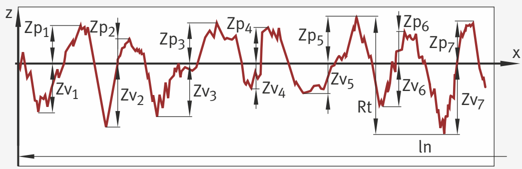
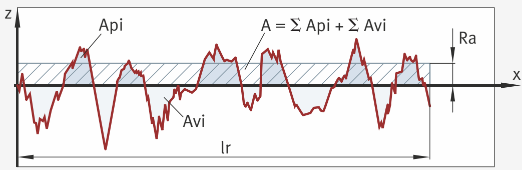
Term	Definition
Individual measurement distance lr, lw, lp	Length in the direction of the x axis that is used for identifying the geometrical deviation of the profile under evaluation: ■ lr = individual measurement distance for roughness ■ lw = individual measurement distance for waviness ■ lp = individual measurement distance for primary profile.
Traced profile	Line of the centre point of a sensor tip of geometrically ideal shape (taper with spherical rounding, nominal dimensions, nominal measurement force) that traces the surface in a cross-sectional plane. The following defined profiles are derived from this profile.
Total profile	Digital form of the traced profile relative to the reference profile, derived from vertical and horizontal coordinates relative to the reference profile.
Measurement distance In	Length in the direction of the x axis that is used for evaluation of the profile. The measurement distance may encompass one or more individual measurement distances.
Surface profile	Profile that results from the intersection of a workpiece surface with a specified plane.
Ordinate value Z(x)	Height of the measured profile at any specified position x. The heights are considered as negative if ordinates lie below the x axis (centre line).
Primary profile	Total profile after application of the filter for short wavelengths. The primary profile serves as the starting point for digital processing by the profile filter and for calculation of the parameters of the primary profile.
Profile element	A profile peak and the adjacent profile trough. Dimensions: ■ Zp = height of the largest profile peak ■ Zv = depth of the largest profile trough ■ Zt = height of a profile element ■ Xs = width of a profile element.
Profile filter λs, λc, λf	Filter that separates a profile into long wave and short wave components.
Roughness profile	Primary profile following separation of the long wave profile components using the profile filter λc. The roughness profile is the basis for calculation of the parameters of the roughness profile.
Reference profile	Line onto which the sensor in the cross-sectional plane is moved along the guide.
Traced section	Measurement of the surface in accordance with the profile method.
Waviness profile	Primary profile following separation of the very long wave profile components using the profile filter λf and subsequent separation of the short wave components using the profile filter λc.

Roughness profile parameters Roughness profile parameters (R parameters) are calculated from the roughness profile, W parameters from the waviness profile and P parameters from the primary profile.

R parameters The following table show a selection of R parameters in accordance with DIN EN ISO 4287.

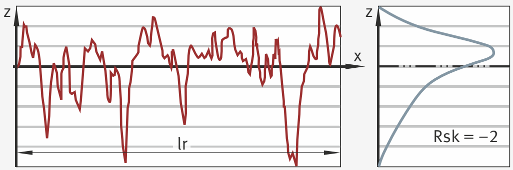
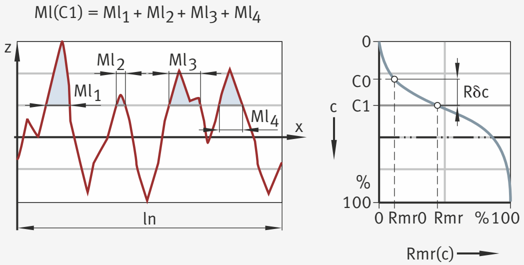
Parameter		Definition	Mathematical definition
Rp	Height of the largest profile peak	Height of the largest profile peak Zp within the individual measurement distance.	–
Rv	Depth of the largest profile trough	Depth of the largest profile trough Zv within the individual measurement distance.	
Rz	Largest height of the profile	Sum of the height of the largest profile peak Zp and the depth of the largest profile trough Zv within an individual measurement distance.	
Rc	Mean height of the profile elements	Mean value of the height of the profile elements Zt within an individual measurement distance.	$Rc = \frac{1}{m} \cdot \sum_{i=1}^{m} Zt_i$
Rt	Total height of the profile	Sum of the height of the largest profile peak Zp and the depth of the largest profile trough Zv within the measurement distance.	$Rt \geq Rz$
Ra	Arithmetic mean value of the profile ordinates	Arithmetic mean value of the magnitude of the ordinate values Z(x) within an individual measurement distance.	$Ra = \frac{1}{lr} \cdot \int_{0}^{lr} \lvert Z(x) \rvert dx$

Continuation of table, see Page 366.

Statement	Geometrical representation
■ Defined by outliers ■ No statement on profile shape	
–	
■ Rt is defined by means of the measurement distance, which is greater than the individual measurement distance. As a result, $Rt \geq Rz$	
■ Surfaces of the same character can be compared ■ Good-natured response ■ No statement on profile shape	

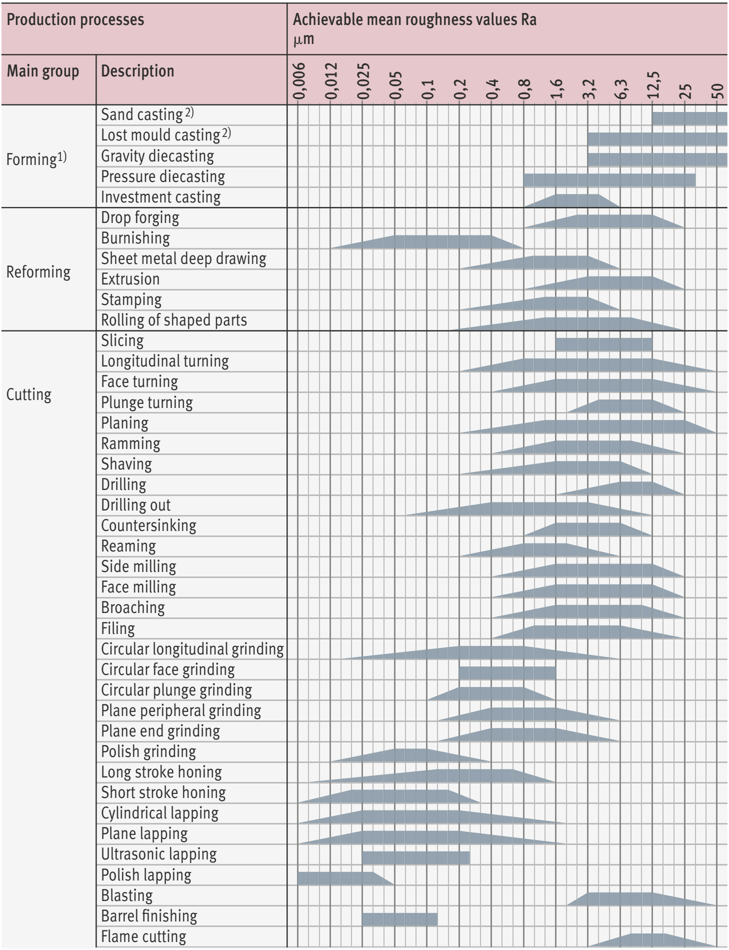
Continuation of table, R parameters, from Page 364.

Parameter		Definition	Mathematical definition
Rq	Root mean square of the profile ordinates	Root mean square of the ordinate values Z(x) within the individual measurement distance.	$Rq = \sqrt{\frac{1}{lr} \cdot \int_0^{lr} Z^2(x)\,dx}$
Rsk	Skewness of the profile	Quotient of the mean cube value of the ordinate values Z(x) and the cube of Rq within an individual measurement distance.	$Rsk = \frac{1}{Rq^3} \cdot \left[\frac{1}{lr} \cdot \int_0^{lr} Z^3(x)\,dx\right]$
Rmr(c)	Material component of the profile	Quotient of the sum of the material lengths of the profile elements Ml(c) at the specified section height c and the measurement distance. This gives the Abbott-Firestone curve, which represents the material component of the profile as a function of the section height.	$Rmr(c) = \frac{Ml(c)}{ln}$
Rmr	Relative material component of the profile	Material component at the section line height Rδc relative to a reference section line height C0.	Rmr = Rmr(C1) C1 = C0 – Rδc C0 = C(Rmr0)
Rδc	Material component at the section line height	Difference in height between two section lines with specified material component values.	Rδc = C(Rmr1) – C(Rmr2) Rmr1 < Rmr2

Statement	Geometrical representation
■ Parameter with greater statistical security than Ra ■ No statement on profile shape	–
■ Good description of the profile shape ■ A negative Rsk value indicates a plateau-like surface	
■ Integral value, so a good description of the profile shape is possible	

Achievable mean roughness values

The standard DIN 4766-2 (March 1981, which is now invalid but nevertheless of considerable practical relevance) shows the achievable mean roughness values Ra for various production processes.



[1] For further details, see VDG Instruction Sheet K 100, obtainable from Verein Deutscher Giessereifachleute (VDG), Sohnstrasse 70, 40237 Düsseldorf.

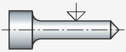
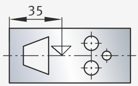
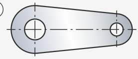
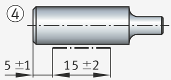
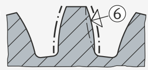
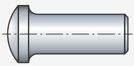
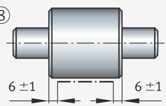
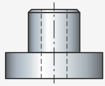
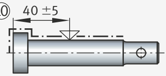
[2] In this casting method, Ra values of up to 125 µm must be expected in the case of castings up to 250 kg.

Hardness indications in drawings in accordance with ISO 15787:2016

The drawing must, in addition to the indications of the material, describe the required final condition such as “quench-hardened”, “surface-hardened and tempered”, “case-hardened” or “nitrided” and include the necessary indications of the surface hardness[1], core hardness[1] and the hardening depth (SHD, CHD, NHD).

Examples of hardness indications

The following table shows some examples of hardness indications in drawings:

Explanation	Example
Quench-hardening, quench-hardening and tempering	
Indications of heat treatment of the entire part by quench-hardening, quench-hardening and tempering are shown in Figure ①, ② and ③: In addition to the text indication such as “quench-hardened”, the hardness indication must always be provided together with with an upper limit deviation and lower limit deviation. The indication of Brinell hardness in Figure ③ must, following the abbreviation HBW, state the ball diameter and the associated testing force in accordance with DIN EN ISO 6506.	①  quench-hardened $(60\ ^{+4}_{0})$ HRC ②  quench-hardened and tempered (60 +4/0) HRC ③  quench-hardened and tempered (350 ±25) HBW 2,5/187,5
Surface-hardening	
The surface-hardened areas are indicated by wide long-dashed dotted lines outside the edges of the body, see Figure ④ and ⑤: Figure ⑤ shows the pattern of the surface-hardening in the tooth by means of narrow long-dashed dotted lines and a measurement point ⑥.	④  –·– surface-hardened (700 ±80) HV30 SHD 500 = 1,2 ±0,4 ⑤  –·– surface-hardened and entire part tempered ⑥ (58 ±3) HRC SHD 475 = 1,5 ±0,5
Case-hardening	
In case-hardening, a distinction is made as follows: ■ case-hardening on all sides, see Figure ⑦ ■ case-hardening on all sides with differing surface hardness or case-hardening depth ■ case-hardening at specific points, see Figure ⑧ When case-hardening is carried out at specific points, the case-hardened area is indicated by wide long-dashed dotted lines outside the edges of the body.	⑦  case-hardened and tempered (62 ±2) HRC CHD = 1 ±0,2 ⑧  –·– case-hardened and entire part tempered (750 ±50) HV10 CHD = 0,4 ±0,1
Nitriding	
In nitriding, a distinction is made as follows: ■ nitriding of the entire part, see Figure ⑨ ■ localised nitriding, see Figure ⑩ In the case of localised nitriding, the areas of the workpiece to be nitrided are indicated by wide long-dashed dotted lines outside the edges of the body.	⑨  nitrided NHD HV0,3 = 0,12 ±0,2 ≧800 HV3 ⑩  –·– nitrided NHD400 = 0,4 ±0,1 ≧900 HV10

Source: ISO 15787:2016

[1] See DIN EN ISO 6508-1 (March 2006) Rockwell hardness; DIN EN ISO 6507-1 (March 2006) Vickers hardness; DIN EN ISO 6506-1 (March 2006) Brinell hardness.

Tolerances and fits

Tolerances – general definition

Allocation of tolerances

The absolutely precise production of components to the size indicated in the technical drawing, the nominal size, is not possible for reasons of manufacturing or not appropriate for reasons of cost. It is possible to determine, however, the permissible variations from these nominal sizes. These deviations are described as deviations or limit deviations. In addition, the variation from the geometrical shapes represented or their position relative to each other can be permitted or delimited in such a way that the function of the component is still ensured.

In order to give an adequate definition of all the required sizes and geometrical characteristics of a workpiece in relation to its production, tolerances are allocated that must then be indicated in the technical drawing.

Tolerances in drawings

There are different types of geometrical variations: dimensional, geometrical, positional and surface variations. These must be delimited by means of tolerance indications.

Tolerances can be entered in a technical drawing by means of:

- Deviations: Entry of tolerances for linear and angular sizes in the form of values after the nominal size
- Symbols for tolerance classes:
  The use of fit systems in accordance with the ISO tolerance system for linear sizes, starting Page 387
- Symbols for geometrical and positional tolerances:
  Identification of the permissible variation of geometry, direction, location or run-out by means of symbols, starting Page 413
- Tolerancing principle starting Page 426
- General tolerances:
  Global indication of tolerances for the simplification of drawings, starting Page 429.

Dimensional tolerances in drawings

The following image shows the nominal size, limit deviations and limit sizes.

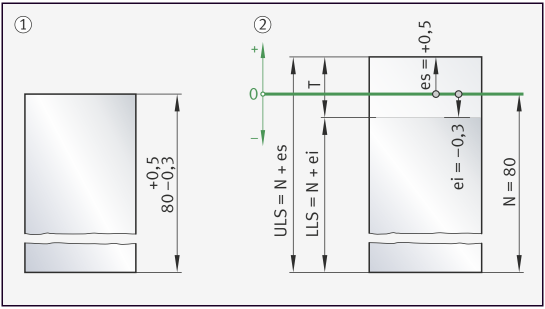


Figure 1
Nominal size, limit deviations and limit sizes

1 Drawing with linear size and tolerance
2 Appropriate representation of nominal size, limit sizes and deviations

Definition of terms

For the indication of sizes, deviations and tolerances, the standard DIN EN ISO 286-1 (November 2010) defines the following fixed terms:

Name		Symbol	Comments
Nominal size		N[1)]	Dimension of a feature of perfect form, as defined by the specification in the drawing. This size is used, with the aid of of the upper and lower limit deviation, to derive the limit sizes
Actual size		I[1)]	Dimension of the allocated complete dimensional feature, determined by measurement
Local actual size		–	Any individual distance in any cross-section of a form element
Limit size		–	Extreme permitted sizes of a dimensional feature. The actual size of a workpiece may lie between the two limit sizes "maximum size" and "minimum size" (including the limit sizes themselves)
Upper limit of size		ULS	The larger permitted size of the two limit sizes
Lower limit of size		LLS	The smaller permitted size of the two limit sizes
Limit deviation	upper	ES, es	Maximum size minus nominal size (designation ES for holes, es for shafts)
	lower	EI, ei	Minimum size minus nominal size (designation EI for holes, ei for shafts)
Tolerance		T[1)]	Upper limit of size minus lower limit of size
Fundamental tolerance		IT	International tolerance; any tolerance in the ISO system for tolerances of linear sizes

1) Still included in the standard DIN 7182-1; no longer included in the standard DIN EN ISO 286-1 (November 2010) as a symbol.

Geometrical Product Specification (GPS)

Concept

The Geometrical Product Specification (GPS) is an internationally recognised concept that defines all the different requirements relating to geometry (especially the sizes, geometry and surface of a workpiece or a component) in their specification and applies to all the associated principles in relation to inspection, the measuring equipment used and its calibration. The result ensures optimum function.

The Masterplan from 1995 (ISO 14638) gave the first specifications for GPS, while the GPS symbols were defined from 2010 in ISO 14405 and now serve worldwide as a common "language". They facilitate precise and detailed indication of the technical requirements in technical drawings or in a 3D model. The standard ISO 8015 revised in 2011 defines the fundamental concepts, principles and rules that apply in the creation, interpretation and application of all other international standards, technical specifications and technical reports (where these concern the geometrical product specification and verification). In industry, technical drawings worldwide have since been successively created in accordance with the GPS standard.

Aspects of GPS

Significant aspects of GPS are:

- Objective: functionally appropriate specification – specification has a considerable influence on all phases of the product life and the quality of products
- The term "Specification" encompasses the complete package of documentation for specification of the workpiece, for example the technical drawing, 3D model, standards etc.
- Specification and verification: The specification describes the product, while the verification checks the implementation of the specification on the workpiece with the aid of measurements
- Specification in accordance with the GPS principles can be recognised from the entry "Linear Size ISO 14405" in or above the title block of a technical drawing
- In order to exactly describe linear sizes, specification modifiers in accordance with ISO 14405-1 can be used. They define features of size such as the type "cylinder", "sphere", "torus, "circle", "two parallel opposite planes" or "two parallel opposite straight lines". (see also section Sizes, Page 381, and tables Specification modifiers for linear sizes, Page 384, as well as Complementary specification modifiers, Page 385).

- Important standards in the GPS Masterplan that specify products in geometrical terms and regulate the verification of component characteristics are:
  - ISO 14405-1: Dimensional tolerancing – Linear sizes
  - ISO 14405-2: Dimensional tolerancing – Dimensions other than linear sizes
  - ISO 14405-3: Dimensional tolerancing – Angular sizes
  - ISO 8015: Fundamentals – Concepts, principles and rules
  - ISO 1101: Geometrical tolerancing – Tolerances of form, orientation, location and run-out
  - ISO 5459: Geometrical tolerancing – Datums and datum systems
  - ISO 14638: GPS matrix model.

Advantages of GPS

The advantages of GPS compared to the previous standards are:

- Clarity of workpieces and measurement methods – by means of the GPS standard, the designer can clearly describe the nominal geometry, sizes with tolerances and characteristics of the surface of a workpiece. The designer can thus influence the measurement method
- Specification is free from contradictions or additions and can therefore no longer be interpreted in different ways, thus achieving the prerequisite for legal compliance
- Uniform standards apply worldwide for machine building, automotive and the rolling bearing industry (contradictions and redundancies are no longer present)
- The uniform technical “language” increases quality and facilitates more efficient cooperation within the company and globally
- GPS creates the preconditions for machine-readable specifications and thus for Industry 4.0
- The specification and verification are contained in a single specification (for example in the technical drawing or the 3D model). The GPS specification is independent of any measuring device or measurement method. It does not prescribe how measurement or inspection is to be carried out (measurement strategy).

Clarity with GPS

The greatest advantage of the GPS symbolic language is the clarity of workpieces and measurement methods. The finished workpiece in the mounting and functional respect is clearly represented in the specification. This was not always the case in the past. A complex production drawing could be understood in different ways by different parties, see Figure 2. The use of the GPS symbolic language reduces errors and simplifies not only internal communication but also regional and international exchange with suppliers and customers. Companies now speak a uniform technical language globally that defines the requirements for the workpiece so clearly that misunderstandings and misinterpretations are eliminated. With GPS, a specification is obtained that is clear worldwide.

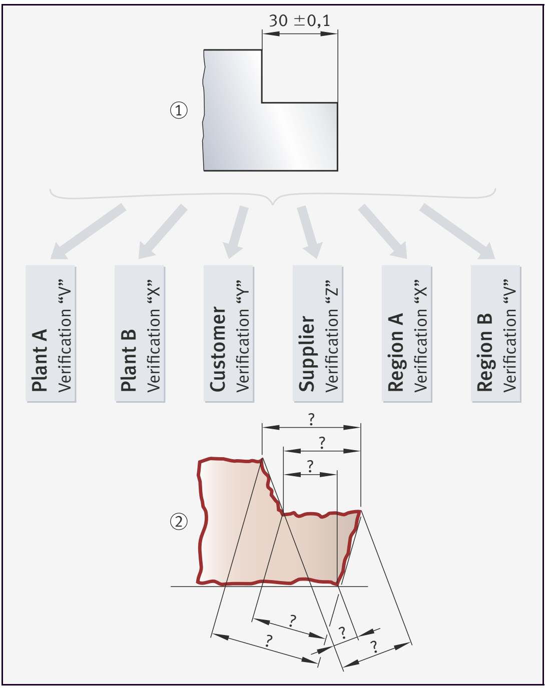
Linear spacing as an example of GPS

Figure 2 shows an example of ISO 14405-2 for the use of plus/minus tolerances in the case of a spacing dimension, Part ①. This type of indication was previously general practice. This specification indication can result in significant ambiguity, see Figure 2 Part ②.

Figure 2
Without GPS, ambiguous solution

Source: ISO 14405-2

① Spacing dimension with symmetrical tolerance indication
② Differing verifications possible



Starting from the functional requirements of the product, Figure 3, Part ① to ③ describes datums in accordance with ISO 5459, geometrical tolerances and a modifier for the combination of tolerance zones in accordance with ISO 1101. The datums represent the mounting situation at the customer.

In Part ①, the position of the integral toleranced feature, in this case a straight line or flat surface, is checked at the theoretically exact spacing 30.

In Part ②, the surface profile of the flat surface is checked as a function of datum A at the theoretically exact spacing 30. The extracted surface must be contained between two parallel datum planes A at a distance of 0,2.

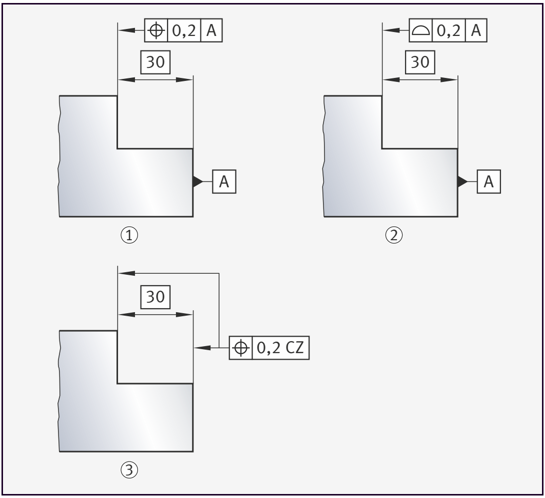
In the case of Part ③, and in contrast to Part ①, the two tolerance zones of the positional tolerance must be checked in relation to each other in location and direction at the theoretically exact spacing 30. The modifier CZ joins the two individual tolerance zones in a combined zone.

Each of these indications conforms to GPS. It represents an unambiguous specification that can be clearly verified.

Figure 3
Conformity with GPS, unambiguous dimensioning concept

Source: ISO 14405-2

① With positional tolerance
② With surface profile tolerance
③ With positional tolerance and combined zone



GPS in tolerance specifications for rolling bearings – examples

The special tolerance specifications for rolling bearings in accordance with GPS are defined, starting from the basis of ISO 14405-1 and ISO 8015, in the rolling bearing standards ISO 492 and ISO 199. Based on examples from the field of rolling bearings, the following section shows changes to and the advantages of tolerance specification using GPS in comparison with previous practice and its significance is explained.

Bearing outer ring, deviation of diameter

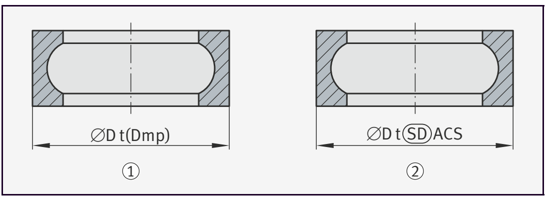
Figure 4 shows, taking the example of the deviation in diameter of a bearing outer ring, the differences in representation with and without GPS.

Part ① shows an outer ring that was specified using the function symbolic language in accordance with the rolling bearing standard ISO 1132-1. The average outside diameter Dmp in a single plane is stated. Evaluation is carried out on the arithmetic mean derived from the maximum and minimum single outside surface diameter ascertainable in a radial plane, thus (Dtmax + Dtmin)/2.

Part ② shows the same ring using the new GPS symbolic language in accordance with ISO 492. The modifier Ⓢⓓ applied to the outside diameter means that this is a statistically determined value. The centre value of the spread is determined, which is the arithmetic mean value derived from the maximum and minimum measured outside diameter, (Dmax + Dmin)/2. This applies in ACS (Any Cross Section), in other words in any plane of the diameter. In this representation, there is no explicit indication of the default modifier (LP) for the two-point size in accordance with ISO 14405-1.

Figure 4
Bearing outer ring, deviation of diameter

① Without GPS (prior to 2011)
② With GPS



While the measurement method and evaluation strategy are identical, the GPS indications facilitate understanding of the measurement method and evaluation strategy without requiring special knowledge of rolling bearing standards.

Bearing inner ring, straightness of end face

Figure 5, Page 377 shows, taking the example of the straightness of the end face of a bearing inner ring, the differences in representation with and without GPS.

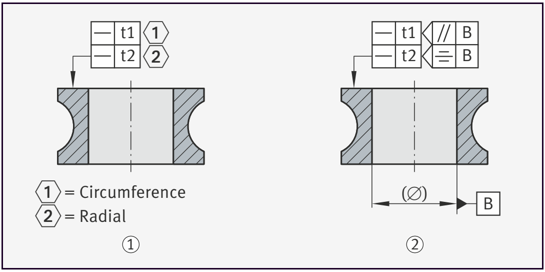
Part ① shows an inner ring whose end face is specified by the indication of two straightness values. One value is to be determined in a circumferential direction, while the other is to be determined in a radial direction. For precise interpretation of the measurement direction, indications in the form of words are necessary, which are stated with the aid of reference highlight symbols.

Part ② shows the same inner ring with straightness indications on the end face, where the specification indications can be given in a non-verbal form. The straightness t1 is supplemented by a intersection plane indicator, which states: "Intersection plane parallel to axis B", which forms the datum as the axis of the bore diameter. Measurement is thus carried out in a circumferential direction on the end face.

The straightness t2 is measured taking account of axis B, symbolised by the intersection plane indicator "Axis B included".

Figure 5
Bearing inner ring, straightness of end face

① Without GPS (prior to 2011)
② With GPS modifier



Due to the additional indications of the two intersection plane indicators, the measurement direction can be specified exactly and no indications in the form of words are used. In this way, the scope for interpretation in measurement (including towards the customer) is reduced.

Tolerances for linear and angular sizes

Dimensional tolerances in technical drawings

In DIN 406-12:1992 (as well as ISO 14405), the methods are defined for entering sizes in technical drawings and specifically the tolerances for linear and angular sizes.

Dimensional tolerances are entered in the form of deviations after the nominal size. In each case, the sum of the nominal size and the specific deviation gives the two permissible limit sizes between which the actual size of the finished component may lie.

Rules for the entry of dimensional tolerances in drawings

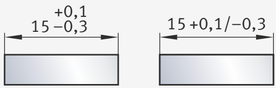
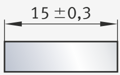
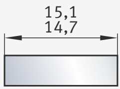
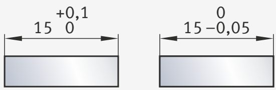
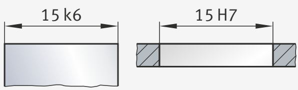
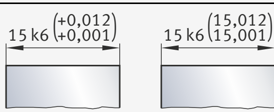
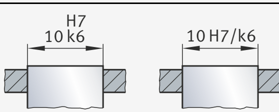
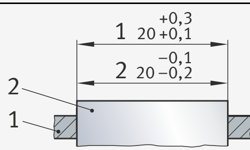
The values for the deviations are always indicated in the same unit as the nominal size after which they are entered. Exception: If an individual size is indicated in a unit different from the rest of the drawing, the units are added for the nominal size and the associated deviations.

Both deviations must have the same number of decimal places. Exception: In the case of the deviation 0, no decimal places are given. In the case of linear sizes, the symbol for the tolerance class can be placed instead of the deviations after the nominal size.

The deviations and symbols for the tolerance class are preferably entered in the same font size as the nominal size or in a font size one step smaller.

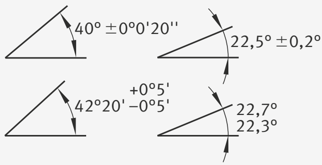
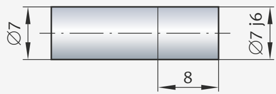
Entry in drawings of deviations, limit sizes and tolerance classes

The following table shows further rules for the entry of tolerances for linear and angular sizes.

Drawing entry	Rule
	The upper deviation is placed above or before the lower deviation.
	If the upper and lower deviation are of the same magnitude, they are combined.
	By agreement, limit sizes calculated from the nominal size and deviations may be entered instead of the nominal size.
	The deviation zero is also given as a limit deviation in accordance with ISO 14405-1 and must not be omitted (this is permitted in DIN 406-12:1992).
	Entry of symbol for tolerance class.
	Deviations or limit sizes may be placed in parentheses after the symbol for the tolerance class.
	Where parts are joined together, the symbol for the tolerance class of the internal size is placed before or above that of the external size.
	Where parts are joined together, the internal size and its deviations is placed above the external size and the components are additionally allocated by means of numbers.

Continuation of table, see Page 379.

Continuation of table Entry in drawings of deviations, limit sizes and tolerance classes from Page 378.

Drawing entry	Rule
	In the case of angular sizes, the units must always be indicated for the nominal size and deviations. In other respects, the rules are identical to those for linear sizes.
	If tolerances apply to a delimited area, this is shown and dimensioned by means of a continuous narrow line.

Tolerance zones, dimensional and positional tolerances

The following example shows that dimensional tolerances alone are not sufficient. Drawings that contain dimensional tolerances only may be ambiguous. The task is to define the position of a hole axis in a plate in relation to the lateral surfaces and delimit possible variations of the axis by means of tolerancing.

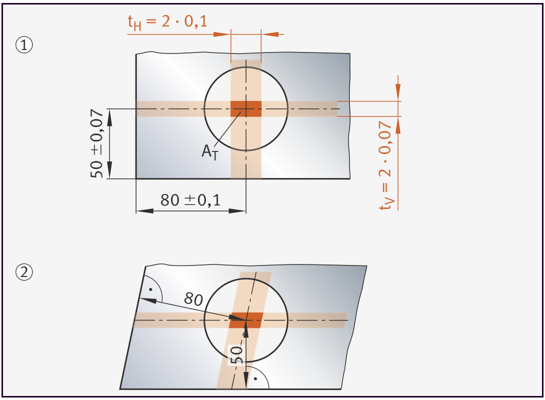
1. Approach: Dimensional tolerances

The axis of the hole is toleranced by means of deviations. This generates a rectangular tolerance zone, see Figure 6. The ideal situation is shown in section ①. Section ② shows the actual situation, namely a workpiece with deviations.

Figure 6
Dimensional tolerances

t_H = horizontal tolerance range
t_V = vertical tolerance range
A_T = tolerance zone for hole axis

① Dimensional tolerancing
② Workpiece subject to variations (exaggerated representation)



It is clear that the use of tolerances alone is not sufficient since this may lead to ambiguity. Datums are important for production and inspection. A complete datum system will therefore be necessary in the future for each workpiece.

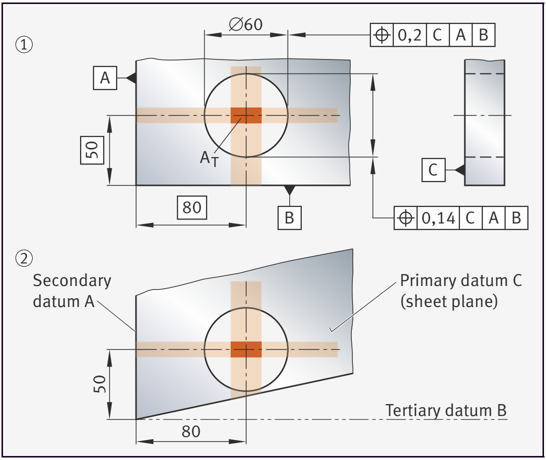
2. Approach: Positional tolerances with datum system

The allocation of positional tolerances with three datum planes gives the following situation, see Figure 7. The ideal situation is shown in section ①. Section ② shows the actual situation, namely a workpiece with deviations.

Figure 7
Positional tolerances

A_T = tolerance zone for hole axis

① Positional tolerancing
② Workpiece subject to variations (exaggerated representation)

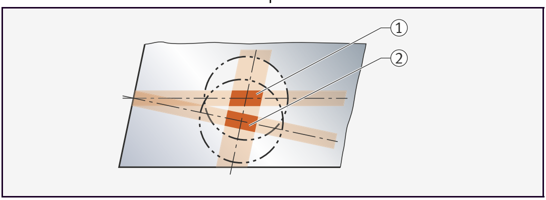


3. Comparison

The following representation shows a comparison of the position of the hole axis for dimensional and positional tolerances:

Figure 8
Comparison: Dimensional and positional tolerances

① Dimensional tolerancing
② Positional tolerancing



Conclusion

The different tolerancing systems lead to different positions for the hole axis. The positional tolerance is significantly "stronger" than the dimensional tolerance, since the datums are defined. With particular regard to foreign manufacturing of components, the formation of a suitable datum system is the most important task in the positional tolerancing of a component.
It is therefore advisable to indicate a complete datum system for each workpiece. The datums are important for production and inspection.

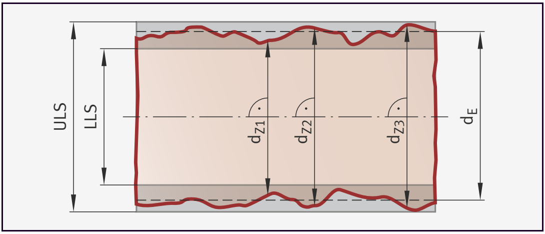
Measurement of components

During measurement of the finished component, the actual size, namely the size of the feature, is determined. The actual size is determined from various local actual sizes. If further specification modification symbols (see DIN EN ISO 14405-1 for example for spherical sizes, cross-sectional sizes or portion sizes) are not indicated, local actual sizes are two-point sizes. They are formed by the Gaussian median plane of the feature of size and are obtained by means of two-point measurement, see Figure 9.

The actual size and the local actual sizes may not be more or less than the upper and lower limit of size (maximum and minimum size) respectively.

Figure 9
Local actual sizes

d_E = size of the substitute element
d_Z = local actual size, two-point size
ULS = upper limit of size
LLS = lower limit of size



Sizes

Manufactured workpieces always deviate from the ideal geometric form. The actual value of the sizes of a feature of size depend on the geometrical deviations and the specific type of the size used. The function of the workpiece determines the type of size that is used for a feature of size.

In DIN EN ISO 14405-1, a differentiation is made between the following types of linear sizes:

- local size
- global size
- calculated size
- rank-order size (statistical value)

Local sizes are explained in further detail below.

Local sizes The local size is defined in accordance with ISO 14405-1 as:

- Two-point size (default or marked with the modifier LP)
- Spherical size (with modifier LS)
- Cross-section size (global size for a given cross section of the measured feature)
- Portion size (global size for a given portion of the measured feature)

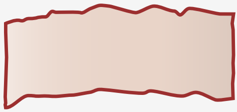
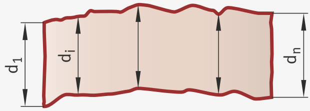
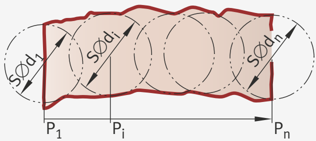
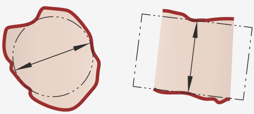
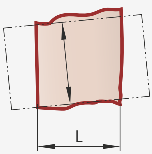
The local size of a measured cylinder and the local diameter of a measured cylinder are defined in accordance with DIN EN ISO 14660-2 as the distance between two opposing points on the feature, where:

- The connecting line between the points encloses the allocated circle centre
- The cross sections are perpendicular to the axis of the allocated cylinder resulting from the measured face.

The local size of two parallel, measured planes is defined, in accordance with DIN EN ISO 14660-2, as the distance between two points on opposing, measured faces, where:

- The connecting lines of pairs of opposing points are perpendicular to the allocated median plane
- The allocated median plane is the median plane of two allocated parallel planes that are derived from the measured faces (this means that the distance can deviate from the nominal value).

The table describes the types of local sizes in accordance with DIN EN ISO 14405-1.

Representation	Definition in accordance with DIN EN ISO 14405-1
	Measured feature under consideration that corresponds to the inner or outer feature and a cylinder or two parallel opposing planes
	Two-point size
	Spherical size
	Cross-section size obtained from a direct global size with the criterion “maximum inscribed” (other criteria are possible)
	Portion size obtained from a direct global size with the criterion “maximum inscribed” Only a portion of the recorded feature of length L is under consideration, other criteria are possible.

Legend

d	mm	Size
L	mm	Length of portion under consideration of cylinder
P		Position
$S\varnothing d$	mm	Diameter of the maximum inscribed sphere.

Specification modifiers

The type of size is indicated on the drawing by a specification modifier in accordance with DIN EN ISO 14405-1, see tables Specification modifiers for linear sizes, Page 384, and Complementary specification modifiers, Page 385.

The specification modifiers comprise the type of size and the required type of evaluation of the size, see Figure 10, Page 386.

Specification modifiers for linear sizes	
Description	**Symbol**
Two-point size	(LP)
Local size defined by a sphere	(LS)
Least-squares association criterion	(GG)
Maximum inscribed feature association criterion	(GX)
Minimum circumscribed feature association criterion	(GN)
Minimax (Chebyshev) association criterion	(GC)
Circumference diameter (calculated size)	(CC)
Area diameter (calculated size)	(CA)
Volume diameter (calculated size)	(CV)
Maximum size	(SX)
Minimum size	(SN)
Average size	(SA)
Median size	(SM)
Mid-range size	(SD)
Range of sizes	(SR)
Standard deviation of sizes	(SQ)

Source: DIN EN ISO 14405-1:2017.

The following table shows complementary specification modifiers.

Complementary specification modifiers		
Description	Symbol	Example of drawing indication
United feature of size	UF	UF 2× ∅30 ±0,2 (GN)
Envelope requirement	(E)	30 ±0,2 (E)
Any restricted portion of feature	/Length	∅30 ±0,2 (GG) / 5
Any cross section	ACS	∅30 ±0,2 (GX) ACS
Specific fixed cross section	SCS	30 ±0,2 (GX) SCS
Any longitudinal section	ALS	30 ±0,2 (GX) ALS
More than one feature	Quantity×	4× 30 ±0,2 (E)
Common toleranced feature of size	CT	4× 30 ±0,2 (E) CT
Free-state condition	(F)	∅30 ±0,2 (LP) (SA) (F)
Between	◄─►	∅30 ±0,2 C◄─►D
Intersection plane[1)]	◁ // | B	8 ±0,01 ALS ◁ ⊥ | A
Direction feature[1)]	◄─ // | B	8 ±0,01 ALS ◄─ ⊥ | A
Flagnote	⟨1⟩	30 ±0,2 ⟨1⟩

Source: DIN EN ISO 14405-1:2017.

[1)] Further information: see ISO 1101.

The following diagram shows the types of sizes and the system of specification modifiers.

Figure 10
Types of sizes – system of specification modifiers

Type of Size	XY	Type for Evaluation of Size
local	L	P point
		S spherical
global	G	G least-squares (Gauß)
		X maximum inscribed
		N minimum circumscribed
		C minimax (Chebyshev)
calculated	C	C circumference diameter
		A area diameter
		V volume diameter
statistical (rank-order)	S	X maximum
		N minimum
		A average
		M median
		D mid-range
		R range
		Q standard deviation (quadratic sum)

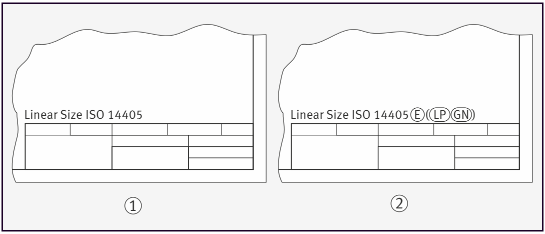
GPS in technical drawings
In technical drawings, the new possibilities for tolerancing of sizes in accordance with GPS unites the requirements in relation to function, production and inspection of a component.

Drawing-specific specification operator
The entry "Linear Size ISO 14405" above or in the vicinity of the drawing text block shows the application of the GPS concept to the technical drawing, especially after revision of a drawing. If no further specification modifier is indicated after this, the drawing is based by default on the two-point size and the independence principle, see Figure 11.

The entry of a specification modifier changes the basis of the entire drawing. Other modifiers used on the drawing can, where necessary, be added in brackets for information.

Figure 11
Specification operator above the drawing title block

1 Default: Two-point size (with principle of independence)
2 Default: Envelope requirement



ISO tolerances and fits

Functions and areas of application

Precise manufacturing and **precise measurement** of machine parts are the basis for interchangeable manufacture and the prerequisite for economical series and mass production in the entire field of technology. Systematic production and the easy repair of technical devices is only possible if the sizes of mutually interchangeable machine parts lie within certain limits and these parts can be assembled or replaced (without special fitting or adaptation work).

In order to achieve a particular joining characteristic or a particular fit tolerance zone of two machine parts (interference fit, transition fit or clearance fit; formerly: press seat, transition seat or clearance seat), the sizes and deviations of the components at the joint must conform to a particular tolerance and it must be possible to check these by means of appropriate measuring equipment.

ISO tolerance system for linear sizes

In order to fulfil all technical requirements to a substantial degree, the standard ISO 286-1 (April 2010) was developed to include the "ISO tolerance system for linear sizes – Part 1: Basis of tolerances, deviations and fits" in accordance with GPS. It is valid for nominal sizes up to 3150 mm and is graduated as far as appropriate in technical terms. ISO 286-2 (June 2010) contains the associated tables of fundamental tolerance grades and limit deviations for holes and shafts.

The ISO system contains the following:

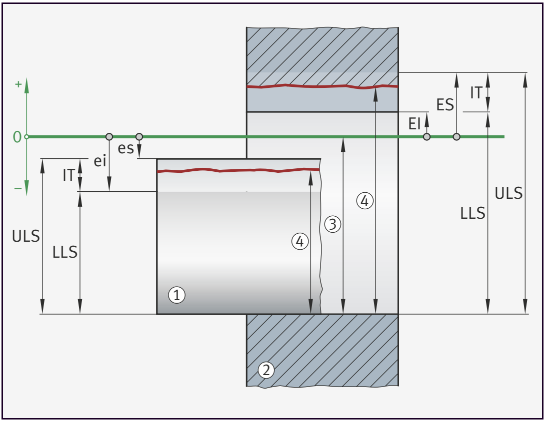
- ISO tolerance system for linear sizes:
  It contains principles and the associated terminology for the system and also provides a standardised selection of tolerance classes for general usage from the extensive possibilities of the system; see also DIN EN ISO 286-1 (November 2010)
- ISO fundamental deviation system:
  The letter defines the position of the tolerance interval in relation to the zero line; it is identified as a fundamental deviation symbol or fundamental deviation identifier (table ISO fundamental deviations for external sizes and ISO fundamental deviations for internal sizes, Page 392 and Page 394; see also DIN EN ISO 286-1 (November 2010)
- ISO fundamental tolerance system:
  Definition of the dimensional tolerances that are indicated by means of ISO tolerance grades (table ISO fundamental tolerances, Page 398); see also DIN EN ISO 286-2 (November 2010)
- System of ISO tolerance classes and ISO tolerance intervals:
  The short designation of the tolerance class comprises the letters for the fundamental deviation (fundamental deviation symbol: lower case letter for external sizes, upper case letter for internal sizes) and the tolerance grade (the number of the fundamental tolerance grade). When the magnitude of the tolerance class using the appropriate limit deviations is represented in graphical form, this is described as a tolerance interval (formerly: "tolerance zone") (graphical representation and calculation starting Page 389).
  The values for limit deviations for generally applied tolerance classes for holes and shafts are part of DIN EN ISO 286-2 (Table ISO tolerances for shafts, Page 400, and Table ISO tolerances for holes, Page 404).

The terms in the ISO tolerance system for linear sizes (see also section Definition of terms, Page 371) are shown below by means of a clearance fit.

Figure 12
Dimensions, limit deviations and tolerances for ISO fits

es, ES = upper limit deviations
ei, EI = lower limit deviations
IT = fundamental tolerance
LLS = lower limit of size
ULS = upper limit of size

① Inner part (outer fit surface)
② Outer part (inner fit surface)
③ Nominal size
④ Local actual size



The terms "hole " and shaft" refer in the ISO tolerance system of linear sizes not only to cylindrical fit surfaces but also to parallel fit surfaces of workpieces, for example the width of a slot or the thickness of a feather key.

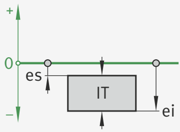
Derivation of ISO tolerance classes or ISO tolerance intervals

The ISO tolerance classes or the ISO tolerance intervals (in diagrammatic form) are derived from ISO fundamental deviations and ISO fundamental tolerances. For this purpose, the ISO fundamental deviations for external sizes are taken from the table ISO fundamental deviations for external sizes, Page 392. In the case of internal sizes, the table ISO fundamental deviations for internal sizes, Page 394, is used. The ISO fundamental deviations are the limit deviations closest to the zero line (smallest distances) taking account of the mathematical signs. The other limit deviation is produced by adding or subtracting the ISO fundamental tolerance IT, see table ISO fundamental tolerances, Page 398.

The following table shows the calculations of limit deviations for external and internal sizes in comparison with each other.

Limit deviations for external sizes (shafts)

Position of tolerance interval a to h
Beneath the zero line

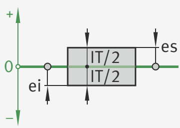


Lower limit deviation ei
= fundamental deviation – fundamental tolerance IT

Example for fit size **25 d15**:

Upper limit deviation es Table Page 392	= Fundamental deviation = −65 μm
Fundamental tolerance for tolerance grade 15 Table Page 398	= 840 μm
Lower limit deviation ei = −65 μm − 840 μm	= −905 μm
Therefore:	25 d15 = $25^{-0{,}065}_{-0{,}905}$

Position of tolerance interval js
Symmetrical on both sides of the zero line



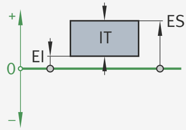
Example for fit size **25 js8**:

Fundamental tolerance for tolerance grade 8 Table Page 398	= 33 μm
Upper limit deviation es = +IT/2 = +33 μm/2	= +16,5 μm
Lower limit deviation ei = −IT/2 = −33 μm/2	= −16,5 μm
Therefore:	25 js8 = $25 \pm 0{,}0165$

From the position of the tolerance interval j, the fundamental deviations can also change with the tolerance grade. When determining the fundamental deviation, it is therefore necessary to observe not only the position of the tolerance interval but also the tolerance grade (table Page 392).

Limit deviations for internal sizes (holes)

Position of tolerance interval A to H
Above the zero line

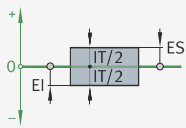


Upper limit deviation es
= fundamental deviation + fundamental tolerance IT

Example for fit size **420 C10**:

Lower limit deviation EI Table Page 394	= Fundamental deviation = +440 μm
Fundamental tolerance for tolerance grade 10 Table Page 398	= 250 μm
Upper limit deviation ES = +440 μm + 250 μm	= +690 μm
Therefore:	420 C10 = $420^{+0{,}690}_{+0{,}440}$

Position of tolerance interval JS
Symmetrical on both sides of the zero line



Example for fit size **200 JS9**:

Fundamental tolerance for tolerance grade 9 Table Page 398	= 115 μm
Upper limit deviation ES = +IT/2 = +115 μm/2	= +57,5 μm
Lower limit deviation EI = −IT/2 = −115 μm/2	= −57,5 μm
Therefore:	200 JS9 = $200 \pm 0{,}0572$

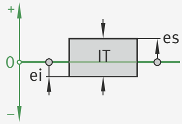
From the position of the tolerance interval J, the fundamental deviations can also change with the tolerance grade. When determining the fundamental deviation, it is therefore necessary to observe not only the position of the tolerance interval but also the tolerance grade (table Page 394).

Continuation of table, see Page 391.

Continuation of table, Calculations of limit deviations for external sizes and internal sizes, from Page 390.

Limit deviations for external sizes (shafts)

Position of tolerance interval j
Approximately symmetrical on both sides of the zero line



Upper limit deviation es
= fundamental deviation + fundamental tolerance IT

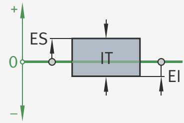
Example for fit size **25 j6**:

Lower limit deviation ei Table Page 392	= Fundamental deviation = −4 μm
Fundamental tolerance for tolerance grade 6 Table Page 398	= 13 μm
Upper limit deviation es = −4 μm + 13 μm	= +9 μm
Therefore:	25 j6 = $25^{+0,009}_{-0,004}$

In the position of the tolerance interval j, the fundamental deviation table always indicates the lower limit deviation ei.

Limit deviations for internal sizes (holes)

Position of tolerance interval J
Approximately symmetrical on both sides of the zero line



Lower limit deviation EI
= fundamental deviation − fundamental tolerance IT

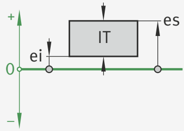
Example for fit size **125 J7**:

Upper limit deviation ES Table Page 394	= Fundamental deviation = +26 μm
Fundamental tolerance for tolerance grade 7 Table Page 398	= 40 μm
Lower limit deviation EI = +26 μm − 40 μm	= −14 μm
Therefore:	125 J7 = $125^{+0,026}_{-0,014}$

In the position of tolerance interval J, the fundamental deviation table always indicates the upper limit deviation ES.

Limit deviations for external sizes (shafts)

Position of tolerance interval k to zc
Above the zero line



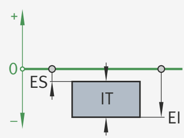
Upper limit deviation es
= fundamental deviation + fundamental tolerance IT

Example for fit size **25 p6**:

Lower limit deviation ei Table Page 392	= Fundamental deviation = +22 μm
Fundamental tolerance for tolerance grade 6 Table Page 398	= 13 μm
Upper limit deviation es = 22 μm + 13 μm	= +35 μm
Therefore:	25 p6 = $25^{+0,035}_{+0,022}$

Limit deviations for internal sizes (holes)

Position of tolerance interval K to ZC
Predominantly beneath the zero line



Lower limit deviation EI
= fundamental deviation − fundamental tolerance IT

Example for fit size **125 T10**:

Upper limit deviation ES Table Page 394	= Fundamental deviation = −122 μm
Fundamental tolerance for tolerance grade 10 Table Page 398	= 160 μm
Lower limit deviation EI = −122 μm −160 μm	= −282 μm
Therefore:	125 T10 = $125^{-0,122}_{-0,282}$

ISO fundamental deviations for external sizes

The following table shows the values for ISO fundamental deviations (minimum distances) for external sizes (shafts):

Tolerance classes[2)]		Nominal size range mm											
Fundamental deviation symbol	Tolerance grade IT	over	1	3	6	10	14	18	24	30	40	50	65
		incl.	3	6	10	14	18	24	30	40	50	65	80
		Sign	Values for fundamental deviations (minimum distances) to DIN EN ISO 286-1 µm										
a	All tolerance grades	–	270	270	280	290	290	300	300	310	320	340	360
b		–	140	140	150	150	150	160	160	170	180	190	200
c		–	60	70	80	95	95	110	110	120	130	140	150
d		–	20	30	40	50	50	65	65	80	80	100	100
e		–	14	20	25	32	32	40	40	50	50	60	60
f		–	6	10	13	16	16	20	20	25	25	30	30
g		–	2	4	5	6	6	7	7	9	9	10	10
h		–	0	0	0	0	0	0	0	0	0	0	0
j[1)]	5 + 6	–	2	2	2	3	3	4	4	5	5	7	7
j[1)]	7	–	4	4	5	6	6	8	8	10	10	12	12
js	All grades	The limit deviations amount to ±1/2 IT in the relevant tolerance grade											
k	4 – 7	+	0	1	1	1	1	2	2	2	2	2	2
k	up to 3, from 8	+	0	0	0	0	0	0	0	0	0	0	0
m	All tolerance grades	+	2	4	6	7	7	8	8	9	9	11	11
n		+	4	8	10	12	12	15	15	17	17	20	20
p		+	6	12	15	18	18	22	22	26	26	32	32
r		+	10	15	19	23	23	28	28	34	34	41	43
s		+	14	19	23	28	28	35	35	43	43	53	59
t		+	–	–	–	–	–	–	41	48	54	66	75
u		+	18	23	28	33	33	41	48	60	70	87	102
v		+	–	–	–	–	39	47	55	68	81	102	120
x		+	20	28	34	40	45	54	64	80	97	122	146
y		+	–	–	–	–	–	63	75	94	114	144	174
z		+	26	35	42	50	60	73	88	112	136	172	210
za		+	32	42	52	64	77	98	118	148	180	226	274
zb		+	40	50	67	90	108	136	160	200	242	300	360
zc		+	60	80	97	130	150	188	218	274	325	405	480

1) In the case of the fundamental deviation symbol j, the table always indicates the lower limit deviation as the fundamental deviation.

2) The special tolerance classes with the fundamental deviation symbols cd, ef and fg for clockmaking and precision engineering are not given here.

80	100	120	140	160	180	200	225	250	280	315	355	400	450
100	120	140	160	180	200	225	250	280	315	355	400	450	500
380	410	460	520	580	660	740	820	920	1050	1200	1350	1500	1650
220	240	260	280	310	340	380	420	480	540	600	680	760	840
170	180	200	210	230	240	260	280	300	330	360	400	440	460
120	120	145	145	145	170	170	170	190	190	210	210	230	230
72	72	85	85	85	100	100	100	110	110	125	125	135	135
36	36	43	43	43	50	50	50	56	56	62	62	68	68
12	12	14	14	14	15	15	15	17	17	18	18	20	20
0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	9	11	11	11	13	13	13	16	16	18	18	20	20
15	15	18	18	18	21	21	21	26	26	28	28	32	32
3	3	3	3	3	4	4	4	4	4	4	4	5	5
0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	13	15	15	15	17	17	17	20	20	21	21	23	23
23	23	27	27	27	31	31	31	34	34	37	37	40	40
37	37	43	43	43	50	50	50	56	56	62	62	68	68
51	54	63	65	68	77	80	84	94	98	108	114	126	132
71	79	92	100	108	122	130	140	158	170	190	208	232	252
91	104	122	134	146	166	180	196	218	240	268	294	330	360
124	144	170	190	210	236	258	284	315	350	390	435	490	540
146	172	202	228	252	284	310	340	385	425	475	530	595	660
178	210	248	280	310	350	385	425	475	525	590	660	740	820
214	254	300	340	380	425	470	520	580	650	730	820	920	1000
258	310	365	415	465	520	575	640	710	790	900	1000	1100	1250
335	400	470	535	600	670	740	820	920	1000	1150	1300	1450	1600
445	525	620	700	780	880	960	1050	1200	1300	1500	1650	1850	2100
585	690	800	900	1000	1150	1250	1350	1550	1700	1900	2100	2400	2600

ISO fundamental deviations for internal sizes

The following table shows the values for ISO fundamental deviations (minimum distances) for internal sizes (holes):

Tolerance classes[2)] Fundamental deviation symbol	Tolerance grade IT	Nominal size range mm											
		over	1	3	6	10	14	18	24	30	40	50	65
		incl.	3	6	10	14	18	24	30	40	50	65	80
		Sign	Values for fundamental deviations (minimum distances) to DIN EN ISO 286-1 µm										
A	All tolerance grades	+	270	270	280	290	290	300	300	310	320	340	360
B		+	140	140	150	150	150	160	160	170	180	190	200
C		+	60	70	80	95	95	110	110	120	130	140	150
D		+	20	30	40	50	50	65	65	80	80	100	100
E		+	14	20	25	32	32	40	40	50	50	60	60
F		+	6	10	13	16	16	20	20	25	25	30	30
G		+	2	4	5	6	6	7	7	9	9	10	10
H			0	0	0	0	0	0	0	0	0	0	0
J[1)]	6	+	2	5	5	6	6	8	8	10	10	13	13
J[1)]	7	+	4	6	8	10	10	12	12	14	14	18	18
J[1)]	8	+	6	10	12	15	15	20	20	24	24	28	28
JS	All grades	The limit deviations amount to ±1/2 IT in the relevant tolerance grade											
K	5	+	0	0	1	2	2	1	1	2	2	3	3
K	6	+	0	2	2	2	2	2	2	3	3	4	4
K	7	+	0	3	5	6	6	6	6	7	7	9	9
K	8	+	0	5	6	8	8	10	10	12	12	14	14
M	6	–	2	1	3	4	4	4	4	4	4	5	5
M	7	–	2	0	0	0	0	0	0	0	0	0	0
M	8		–2	+2	+1	+2	+2	+4	+4	+5	+5	+5	+5
M	9	–	2	4	6	7	7	8	8	9	9	11	11
N	6	–	4	5	7	9	9	11	11	12	12	14	14
N	7	–	4	4	4	5	5	7	7	8	8	9	9
N	8	–	4	2	3	3	3	3	3	3	3	4	4
N	9	–	4	0	0	0	0	0	0	0	0	0	0
P	6	–	6	9	12	15	15	18	18	21	21	26	26
R		–	10	12	16	20	20	24	24	29	29	35	37
S		–	14	16	20	25	25	31	31	38	38	47	53
T		–	–	–	–	–	–	–	37	43	49	60	69

Continuation of table, see Page 396.

1) In the case of the fundamental deviation identifier J, the table always indicates the lower limit deviation as the fundamental deviation.

2) The special tolerance classes with the fundamental deviation symbols CD, EF and FG for clockmaking and precision engineering are not given here.

80	100	120	140	160	180	200	225	250	280	315	355	400	450
100	120	140	160	180	200	225	250	280	315	355	400	450	500
380	410	460	520	580	660	740	820	920	1050	1200	1350	1500	1650
220	240	260	280	310	340	380	420	480	540	600	680	760	840
170	180	200	210	230	240	260	280	300	330	360	400	440	480
120	120	145	145	145	170	170	170	190	190	210	210	230	230
72	72	85	85	85	100	100	100	110	110	125	125	135	135
36	36	43	43	43	50	50	50	56	56	62	62	68	68
12	12	14	14	14	15	15	15	17	17	18	18	20	20
0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	16	18	18	18	22	22	22	25	25	29	29	33	33
22	22	26	26	26	30	30	30	36	36	39	39	43	43
34	34	41	41	41	47	47	47	55	55	60	60	66	66
2	2	3	3	3	2	2	2	3	3	3	3	2	2
4	4	4	4	4	5	5	5	5	5	7	7	8	8
10	10	12	12	12	13	13	13	16	16	17	17	18	18
16	16	20	20	20	22	22	22	25	25	28	28	29	29
6	6	8	8	8	8	8	8	9	9	10	10	10	10
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+6	+6	+8	+8	+8	+9	+9	+9	+9	+9	+11	+11	+11	+11
13	13	15	15	15	17	17	17	20	20	21	21	23	23
16	16	20	20	20	22	22	22	25	25	26	26	27	27
10	10	12	12	12	14	14	14	14	14	16	16	17	17
4	4	4	4	4	5	5	5	5	5	5	5	6	6
0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	30	36	36	36	41	41	41	47	47	51	51	55	55
44	47	56	58	61	68	71	75	85	89	97	103	113	119
64	72	85	93	101	113	121	131	149	161	179	197	219	239
84	97	115	127	139	157	171	187	209	231	257	283	317	347

Continuation of table ISO fundamental deviations for internal sizes from Page 394.

Tolerance classes		Nominal size range mm											
Fundamental deviation symbol	Tolerance grade IT	over	1	3	6	10	14	18	24	30	40	50	65
		incl.	3	6	10	14	18	24	30	40	50	65	80
		Sign	Values for fundamental deviations (minimum distances) to DIN EN ISO 286-1 μm										
U	6	–	18	20	25	30	30	37	44	55	65	81	96
V		–	–	–	–	–	36	43	51	63	76	96	114
X		–	20	25	31	37	42	50	60	75	92	116	140
Y		–	–	–	–	–	–	59	71	89	109	138	168
Z		–	26	32	39	47	57	69	84	107	131	166	204
ZA		–	32	39	49	61	74	94	114	143	175	220	268
ZB		–	40	47	64	87	105	132	156	195	237	294	354
ZC		–	60	77	94	127	147	184	214	269	320	399	474
P	7	–	6	8	9	11	11	14	14	17	17	21	21
R		–	10	11	13	16	16	20	20	25	25	30	32
S		–	14	15	17	21	21	27	27	34	34	42	48
T		–	–	–	–	–	–	–	33	39	45	55	64
U		–	18	19	22	26	26	33	40	51	61	76	91
V		–	–	–	–	–	32	39	47	59	72	91	109
X		–	20	24	28	33	38	46	56	71	88	111	135
Y		–	–	–	–	–	–	55	67	85	105	133	163
Z		–	26	31	36	43	53	65	80	103	127	161	199
ZA		–	32	38	46	57	70	90	110	139	171	215	263
ZB		–	40	46	61	83	101	128	152	191	233	289	349
ZC		–	60	76	91	123	143	180	210	265	316	394	469
P	from 8	–	6	12	15	18	18	22	22	26	26	32	32
R		–	10	15	19	23	23	28	28	34	34	41	43
S		–	14	19	23	28	28	35	35	43	43	53	59
T		–	–	–	–	–	–	–	41	48	54	66	75
U		–	18	23	28	33	33	41	48	60	70	87	102
V		–	–	–	–	–	39	47	55	68	81	102	120
X		–	20	28	34	40	45	54	64	80	97	122	146
Y		–	–	–	–	–	–	63	75	94	114	144	174
Z		–	26	35	42	50	60	73	88	112	136	172	210
ZA		–	32	42	52	64	77	98	118	148	180	226	274
ZB		–	40	50	67	90	108	136	160	200	242	300	360
ZC		–	60	80	97	130	150	188	218	274	325	405	480

80 100	100 120	120 140	140 160	160 180	180 200	200 225	225 250	250 280	280 315	315 355	355 400	400 450	450 500
117	137	163	183	203	227	249	275	306	341	379	424	477	527
139	165	195	221	245	275	301	331	376	416	464	519	582	647
171	203	241	273	303	341	376	416	466	516	579	649	727	807
207	247	293	333	373	416	461	511	571	641	719	809	907	987
251	303	358	406	458	511	566	631	701	781	889	989	087	1237
328	393	463	528	593	661	731	811	911	991	1139	1289	1437	1587
38	518	613	693	773	871	951	1041	1191	1291	1489	1639	1837	2087
578	683	793	893	993	1141	1241	1341	1541	1691	1889	2089	2387	2587
24	24	28	28	28	33	33	33	36	36	41	41	45	45
38	41	48	50	53	60	63	67	74	78	87	93	103	109
58	66	77	85	93	105	113	123	138	150	169	187	209	229
78	91	107	119	131	149	163	179	198	220	247	273	307	337
111	131	155	175	195	219	241	267	295	330	369	414	467	517
133	159	187	213	237	267	293	323	365	405	454	509	572	637
165	197	233	265	295	333	368	408	455	505	569	639	717	797
201	241	285	325	365	408	453	503	560	630	709	799	897	977
245	297	350	400	450	503	558	623	690	770	879	979	1077	1227
322	387	455	520	585	653	723	803	900	980	1129	1279	1427	1577
432	512	605	685	765	863	943	1033	1180	1280	1479	1629	1827	2077
572	677	785	885	985	1133	1233	1333	1530	1680	1879	2079	2377	2577
37	37	43	43	43	50	50	50	56	56	62	62	68	68
51	54	63	65	68	77	80	84	94	98	108	114	126	132
71	79	92	100	108	122	130	140	158	170	190	208	232	252
91	104	122	134	146	166	180	196	218	240	268	294	330	360
124	144	170	190	210	236	258	284	315	350	390	435	490	540
146	172	202	228	252	284	310	340	385	425	475	530	595	660
178	210	248	280	310	350	385	425	475	525	590	660	740	820
214	254	300	340	380	425	470	520	580	650	730	820	920	1000
258	310	365	415	465	520	575	640	710	790	900	1000	1100	1250
335	400	470	535	600	670	740	820	920	1000	1150	1300	1450	1600
445	525	620	700	780	880	960	1050	1200	1300	1500	1650	1850	2100
585	690	800	900	1000	1150	1250	1350	1550	1700	1900	2100	2400	2600

ISO fundamental tolerances

The following table shows the values for ISO fundamental tolerances:

Fundamental tolerance grade	Nominal size range mm						
	over –	3	6	10	18	30	50
	incl. 3	6	10	18	30	50	80
	ISO fundamental tolerances IT to DIN EN ISO 286-1 µm						
IT01	0,3	0,4	0,4	0,5	0,6	0,6	0,8
IT0	0,5	0,6	0,6	0,8	1	1	1,2
IT1	0,8	1	1	1,2	1,5	1,5	2
IT2	1,2	1,5	1,5	2	2,5	2,5	3
IT3	2	2,5	2,5	3	4	4	5
IT4	3	4	4	5	6	7	8
IT5	4	5	6	8	9	11	13
IT6	6	8	9	11	13	16	19
IT7	10	12	15	18	21	25	30
IT8	14	18	22	27	33	39	46
IT9	25	30	36	43	52	62	74
IT10	40	48	58	70	84	100	120
IT11	60	75	90	110	130	160	190
IT12	100	120	150	180	210	250	300
IT13	140	180	220	270	330	390	460
IT14	250	300	360	430	520	620	740
IT15	400	480	580	700	840	1000	1200
IT16	600	750	900	1100	1300	1600	1900
IT17	1000	1200	1500	1800	2100	2500	3000
IT18	1400	1800	2200	2700	3300	3900	4600

Continuation of table, see Page 399.

Continuation of table ISO fundamental tolerances from Page 398.

Fundamental tolerance grade	Nominal size range mm					
	over 80	120	180	250	315	400
	incl. 120	180	250	315	400	500
	ISO fundamental tolerances IT to DIN EN ISO 286-1 μm					
IT01	1	1,2	2	2,5	3	4
IT0	1,5	2	3	4	5	6
IT1	2,5	3,5	4,5	6	7	8
IT2	4	5	7	8	9	10
IT3	6	8	10	12	13	15
IT4	10	12	14	16	18	20
IT5	15	18	20	23	25	27
IT6	22	25	29	32	36	40
IT7	35	40	46	52	57	63
IT8	54	63	72	81	89	97
IT9	87	100	115	130	140	155
IT10	140	160	185	210	230	250
IT11	220	250	290	320	360	400
IT12	350	400	460	520	570	630
IT13	540	630	720	810	890	970
IT14	870	1000	1150	1300	1400	1550
IT15	1400	1600	1850	2100	2300	2500
IT16	2200	2500	2900	3200	3600	4000
IT17	3500	4000	4600	5200	5700	6300
IT18	5400	6300	7200	8100	8900	9700

ISO tolerances for shafts The following table shows a selection of ISO tolerances for shafts and their corresponding limit deviations:

Tolerance classes	Nominal size range mm											
	over 1	3	6	10	18	30	40	50	65	80	100	120
	incl. 3	6	10	18	30	40	50	65	80	100	120	140
	Limit deviations (1 μm = 0,001 mm) μm						Upper limit deviation = es Lower limit deviation = ei					
a12	–270	–270	–280	–290	–300	–310	–320	–340	–360	–380	–410	–460
	–370	–390	–430	–470	–510	–560	–570	–640	–660	–730	–760	–860
a13	–270	–270	–280	–290	–300	–310	–320	–340	–360	–380	–410	–460
	–410	–450	–500	–560	–630	–700	–710	–800	–820	–920	–950	–1090
c12	–60	–70	–80	–95	–110	–120	–130	–140	–150	–170	–180	–200
	–160	–190	–230	–275	–320	–370	–380	–440	–450	–520	–530	–600
d6	–20	–30	–40	–50	–65	–80	–80	–100	–100	–120	–120	–145
	–26	–38	–49	–61	–78	–96	–96	–119	–119	–142	–142	–170
e6	–14	–20	–25	–32	–40	–50	–50	–60	–60	–72	–72	–85
	–20	–28	–34	–43	–53	–66	–66	–79	–79	–94	–94	–110
e7	–14	–20	–25	–32	–40	–50	–50	–60	–60	–72	–72	–85
	–24	–32	–40	–50	–61	–75	–75	–90	–90	–107	–107	–125
e8	–14	–20	–25	–32	–40	–50	–50	–60	–60	–72	–72	–85
	–28	–38	–47	–59	–73	–89	–89	–106	–106	–126	–126	–148
f5	–6	–10	–13	–16	–20	–25	–25	–30	–30	–36	–36	–43
	–10	–15	–19	–24	–29	–36	–36	–43	–43	–51	–51	–61
f6	–6	–10	–13	–16	–20	–25	–25	–30	–30	–36	–36	–43
	–12	–18	–22	–27	–33	–41	–41	–49	–49	–58	–58	–68
f7	–6	–10	–13	–16	–20	–25	–25	–30	–30	–36	–36	–43
	–16	–22	–28	–34	–41	–50	–50	–60	–60	–71	–71	–83
g5	–2	–4	–5	–6	–7	–9	–9	–10	–10	–12	–12	–14
	–6	–9	–11	–14	–16	–20	–20	–23	–23	–27	–27	–32
g6	–2	–4	–5	–6	–7	– 9	– 9	–10	–10	–12	–12	–14
	–8	–12	–14	–17	–20	–25	–25	–29	–29	–34	–34	–39
g7	–2	–4	–5	–6	–7	–9	–9	–10	–10	–12	–12	–14
	–12	–16	–20	–24	–28	–34	–34	–40	–40	–47	–47	–54
h5	0	0	0	0	0	0	0	0	0	0	0	0
	–4	–5	–6	–8	–9	–11	–11	–13	–13	–15	–15	–18
h6	0	0	0	0	0	0	0	0	0	0	0	0
	–6	–8	–9	–11	–13	–16	–16	–19	–19	–22	–22	–25
h7	0	0	0	0	0	0	0	0	0	0	0	0
	–10	–12	–15	–18	–21	–25	–25	–30	–30	–35	–35	–40
h8	0	0	0	0	0	0	0	0	0	0	0	0
	–14	–18	–22	–27	–33	–39	–39	–46	–46	–54	–54	–63
h9	0	0	0	0	0	0	0	0	0	0	0	0
	–25	–30	–36	–43	–52	–62	–62	–74	–74	–87	–87	–100
h10	0	0	0	0	0	0	0	0	0	0	0	0
	–40	–48	–58	–70	–84	–100	–100	–120	–120	–140	–140	–160

Continuation of table, see Page 402.

140 160	160 180	180 200	200 225	225 250	250 280	280 315	315 355	355 400	400 450	450 500
−520 −920	−580 −980	−660 −1120	−740 −1 200	−820 −1 280	−920 −1 440	−1 050 1 570	−1 200 −1 770	−1 350 −1 920	−1 500 −2 130	−1 650 −2 280
−520 −1 150	−580 −1 210	−660 −1380	−740 −1460	−820 −1 540	−920 −1730	−1050 −1 860	−1200 −2 090	−1350 −2 240	−1500 −2 470	−1650 −2 620
−210 −610	−230 −630	−240 −700	−260 −720	−280 −740	−300 −820	−330 −850	−360 −930	−400 −970	−440 −1 070	−480 −1 110
−145 −170	−145 −170	−170 −199	−170 −199	−170 −199	−190 −222	−190 −222	−210 −246	−210 −246	−230 −270	−230 −270
−85 −110	−85 −110	−100 −129	−100 −129	−100 −129	−110 −142	−110 −142	−125 −161	−125 −161	−135 −175	−135 −175
−85 −125	−85 −125	−100 −146	−100 −146	−100 −146	−110 −162	−110 −162	−125 −182	−125 −182	−135 −198	−135 −198
−85 −148	−85 −148	−100 −172	−100 −172	−100 −172	−110 −191	−110 −191	−125 −214	−125 −214	−135 −232	−135 −232
−43 −61	−43 −61	−50 −70	−50 −70	−50 −70	−56 −79	−56 −79	−62 −87	−62 −87	−68 −95	−68 −95
−43 −68	−43 −68	−50 −79	−50 −79	−50 −79	−56 −88	−56 −88	−62 −98	−62 −98	−68 −108	−68 −108
−43 −83	−43 −83	−50 −96	−50 −96	−50 −96	−56 −108	−56 −108	−62 −119	−62 −119	−68 −131	−68 −131
−14 −32	−14 −32	−15 −35	−15 −35	−15 −35	−17 −40	−17 −40	−18 −43	−18 −43	−20 −47	−20 −47
−14 −39	−14 −39	−15 −44	−15 −44	−15 −44	−17 −49	−17 −49	−18 −54	−18 −54	−20 −60	−20 −60
−14 −54	−14 −54	−15 −61	−15 −61	−15 −61	−17 −69	−17 −69	−18 −75	−18 −75	−20 −83	−20 −83
0 −18	0 −18	0 −20	0 −20	0 −20	0 −23	0 −23	0 −25	0 −25	0 −27	0 −27
0 −25	0 −25	0 −29	0 −29	0 −29	0 −32	0 −32	0 −36	0 −36	0 −40	0 −40
0 −40	0 −40	0 −46	0 −46	0 −46	0 −52	0 −52	0 −57	0 −57	0 −63	0 −63
0 −63	0 −63	0 −72	0 −72	0 −72	0 −81	0 −81	0 −89	0 −89	0 −97	0 −97
0 −100	0 −100	0 −115	0 −115	0 −115	0 −130	0 −130	0 −140	0 −140	0 −155	0 −155
0 −160	0 −160	0 −185	0 −185	0 −185	0 −210	0 −210	0 −230	0 −230	0 −250	0 −250

Continuation of table ISO tolerances for shafts from Page 400.

Tolerance classes	Nominal size range mm											
	over 1	3	6	10	18	30	40	50	65	80	100	120
	incl. 3	6	10	18	30	40	50	65	80	100	120	140
	Limit deviations (1 μm = 0,001 mm) μm						Upper limit deviation = es Lower limit deviation = ei					
h11	0	0	0	0	0	0	0	0	0	0	0	0
	−60	−75	−90	−110	−130	−160	−160	−190	−190	−220	−220	−250
h13	−0	0	0	0	0	0	0	0	0	0	0	0
	−140	−180	−220	−270	−330	−390	−390	−460	−460	−540	−540	−630
j5	+2	+3	+4	+5	+5	+6	+6	+6	+6	+6	+6	+7
	−2	−2	−2	−3	−4	−5	−5	−7	−7	−9	−9	−11
j6	+4	+6	+7	+8	+9	+11	+11	+12	+12	+13	+13	+14
	−2	−2	−2	−3	−4	−5	−5	−7	−7	−9	−9	−11
j7	+6	+8	+10	+12	+13	+15	+15	+18	+18	+20	+20	+22
	−4	−4	−5	−6	−8	−10	−10	−12	−12	−15	−15	−18
js5	+2	+2,5	+3	+4	+4,5	+5,5	+5,5	+6,5	+6,5	+7,5	+7,5	+9
	−2	−2,5	−3	−4	−4,5	−5,5	−5,5	−6,5	−6,5	−7,5	−7,5	−9
js6	+3	+4	+4,5	+5,5	+6,5	+8	+8	+9,5	+9,5	+11	+11	+12,5
	−3	−4	−4,5	−5,5	−6,5	−8	−8	−9,5	−9,5	−11	−11	−12,5
js7	+5	+6	+7,5	+9	+10,5	+12,5	+12,5	+15	+15	+17,5	+17,5	+20
	−5	−6	−7,5	−9	−10,5	−12,5	−12,5	−15	−15	−17,5	−17,5	−20
k5	+4	+6	+7	+9	+11	+13	+13	+15	+15	+18	+18	+21
	0	+1	+1	+1	+2	+2	+2	+2	+2	+3	+3	+3
k6	+6	+9	+10	+12	+15	+18	+18	+21	+21	+25	+25	+28
	0	+1	+1	+1	+2	+2	+2	+2	+2	+3	+3	+3
k7	+10	+13	+16	+19	+23	+27	+27	+32	+32	+38	+38	+43
	0	+1	+1	+1	+2	+2	+2	+2	+2	+3	+3	+3
m5	+6	+9	+12	+15	+17	+20	+20	+24	+24	+28	+28	+33
	+2	+4	+6	+7	+8	+9	+9	+11	+11	+13	+13	+15
m6	+8	+12	+15	+18	+21	+25	+25	+30	+30	+35	+35	+40
	+2	+4	+6	+7	+8	+9	+9	+11	+11	+13	+13	+15
m7	+12	+16	+21	+25	+29	+34	+34	+41	+41	+48	+48	+55
	+2	+4	+6	+7	+8	+9	+9	+11	+11	+13	+13	+15
n5	+8	+13	+16	+20	+24	+28	+28	+33	+33	+38	+38	+45
	+4	+8	+10	+12	+15	+17	+17	+20	+20	+23	+23	+27
n6	+10	+16	+19	+23	+28	+33	+33	+39	+39	+45	+45	+52
	+4	+8	+10	+12	+15	+17	+17	+20	+20	+23	+23	+27
n7	+14	+20	+25	+30	+36	+42	+42	+50	+50	+58	+58	+67
	+4	+8	+10	+12	+15	+17	+17	+20	+20	+23	+23	+27
p5	+10	+17	+21	+26	+31	+37	+37	+45	+45	+52	+52	+61
	+6	+12	+15	+18	+22	+26	+26	+32	+32	+37	+37	+43
p6	+12	+20	+24	+29	+35	+42	+42	+51	+51	+59	+59	+68
	+6	+12	+15	+18	+22	+26	+26	+32	+32	+37	+37	+43
p7	+16	+24	+30	+36	+43	+51	+51	+62	+62	+72	+72	+83
	+6	+12	+15	+18	+22	+26	+26	+32	+32	+37	+37	+43

140 160	160 180	180 200	200 225	225 250	250 280	280 315	315 355	355 400	400 450	450 500
0 −250	0 −250	0 −290	0 −290	0 −290	0 −320	0 −320	0 −360	0 −360	0 −400	0 −400
0 −630	0 −630	0 −720	0 −720	0 −720	0 −810	0 −810	0 −890	0 −890	0 −970	0 −970
+7 −11	+7 −11	+7 −13	+7 −13	+7 −13	+7 −16	+7 −16	+7 −18	+7 −18	+7 −20	+7 −20
+14 −11	+14 −11	+16 −13	+16 −13	+16 −13	+16 −16	+16 −16	+18 −18	+18 −18	+20 −20	+20 −20
+22 −18	+22 −18	+25 −21	+25 −21	+25 −21	+26 −26	+26 −26	+29 −28	+29 −28	+31 −32	+31 −32
+9 −9	+9 −9	+10 −10	+10 −10	+10 −10	+11,5 −11,5	+11,5 −11,5	+12,5 −12,5	+12,5 −12,5	+13,5 −13,5	+13,5 −13,5
+12,5 −12,5	+12,5 −12,5	+14,5 −14,5	+14,5 −14,5	+14,5 −14,5	+16 −16	+16 −16	+18 −18	+18 −18	+20 −20	+20 −20
+20 −20	+20 −20	+23 −23	+23 −23	+23 −23	+26 −26	+26 −26	+28,5 −28,5	+28,5 −28,5	+31,5 −31,5	+31,5 −31,5
+21 +3	+21 +3	+24 +4	+24 +4	+24 +4	+27 +4	+27 +4	+29 +4	+29 +4	+32 +5	+32 +5
+28 +3	+28 +3	+33 +4	+33 +4	+33 +4	+36 +4	+36 +4	+40 +4	+40 +4	+45 +5	+45 +5
+43 +3	+43 +3	+50 +4	+50 +4	+50 +4	+56 +4	+56 +4	+61 +4	+61 +4	+68 +5	+68 +5
+33 +15	+33 +15	+37 +17	+37 +17	+37 +17	+43 +20	+43 +20	+46 +21	+46 +21	+50 +23	+50 +23
+40 +15	+40 +15	+46 +17	+46 +17	+46 +17	+52 +20	+52 +20	+57 +21	+57 +21	+63 +23	+63 +23
+55 +15	+55 +15	+63 +17	+63 +17	+63 +17	+72 +20	+72 +20	+78 +21	+78 +21	+86 +23	+86 +23
+45 +27	+45 +27	+51 +31	+51 +31	+51 +31	+57 +34	+57 +34	+62 +37	+62 +37	+67 +40	+67 +40
+52 +27	+52 +27	+60 +31	+60 +31	+60 +31	+66 +34	+66 +34	+73 +37	+73 +37	+80 +40	+80 +40
+67 +27	+67 +27	+77 +31	+77 +31	+77 +31	+86 +34	+86 +34	+94 +37	+94 +37	+103 +40	+103 +40
+61 +43	+61 +43	+70 +50	+70 +50	+70 +50	+79 +56	+79 +56	+87 +62	+87 +62	+95 +68	+95 +68
+68 +43	+68 +43	+79 +50	+79 +50	+79 +50	+88 +56	+ 88 +56	+98 +62	+98 +62	+108 +68	+108 +68
+83 +43	+83 +43	+96 +50	+96 +50	+96 +50	+108 +56	+108 +56	+119 +62	+119 +62	+131 +68	+131 +68

ISO tolerances for holes The following table shows a selection of ISO tolerances for holes and their corresponding limit deviations:

Tolerance classes	Nominal size range mm													
	over 3 incl. 6	6 10	10 18	18 30	30 40	40 50	50 65	65 80	80 100	100 120	120 140	140 160	160 180	180 200
	Limit deviations (1 μm = 0,001 mm) μm						Upper limit deviation = ES Lower limit deviation = EI							
A11	+345	+370	+400	+430	+470	+480	+530	+550	+600	+630	+710	+770	+830	+950
	+270	+280	+290	+300	+310	+320	+340	+360	+380	+410	+460	+520	+580	+660
C11	+145	+170	+205	+240	+280	+290	+330	+340	+390	+400	+450	+460	+480	+530
	+70	+80	+95	+110	+120	+130	+140	+150	+170	+180	+200	+210	+230	+240
D10	+78	+98	+120	+149	+180	+180	+220	+220	+260	+260	+305	+305	+305	+355
	+30	+40	+50	+65	+80	+80	+100	+100	+120	+120	+145	+145	+145	+170
E6	+28	+34	+43	+53	+66	+66	+79	+79	+94	+ 94	+110	+110	+110	+129
	+20	+25	+32	+40	+50	+50	+60	+60	+72	+72	+85	+85	+85	+100
E7	+32	+40	+50	+61	+75	+75	+90	+90	+107	+107	+125	+125	+125	+146
	+20	+25	+32	+40	+50	+ 50	+60	+60	+72	+72	+85	+85	+85	+100
E9	+50	+61	+75	+92	+112	+112	+134	+134	+159	+159	+185	+185	+185	+215
	+20	+25	+32	+40	+50	+50	+60	+60	+72	+72	+85	+85	+85	+100
E10	+68	+83	+102	+124	+150	+150	+180	+180	+212	+212	+245	+245	+245	+285
	+20	+25	+32	+40	+50	+50	+60	+60	+72	+72	+85	+85	+85	+100
F6	+18	+22	+27	+33	+41	+41	+49	+49	+58	+58	+68	+68	+68	+79
	+10	+13	+16	+20	+25	+25	+30	+30	+36	+36	+43	+43	+43	+50
F7	+22	+28	+34	+41	+50	+50	+60	+60	+71	+71	+83	+83	+83	+96
	+10	+13	+16	+20	+25	+25	+30	+30	+36	+36	+43	+43	+43	+50
F8	+28	+35	+43	+53	+64	+64	+76	+76	+90	+90	+106	+106	+106	+122
	+10	+13	+16	+20	+25	+25	+30	+30	+36	+36	+43	+43	+43	+50
G6	+12	+14	+17	+20	+25	+25	+29	+29	+34	+34	+39	+39	+39	+44
	+4	+5	+6	+7	+9	+9	+10	+10	+12	+12	+14	+14	+14	+15
G7	+16	+20	+24	+28	+34	+34	+40	+40	+47	+47	+54	+54	+54	+61
	+4	+5	+6	+7	+9	+9	+10	+10	+12	+12	+14	+14	+14	+15
G8	+22	+27	+33	+40	+48	+48	+56	+56	+66	+66	+77	+77	+77	+87
	+4	+5	+6	+7	+9	+9	+10	+10	+12	+12	+14	+14	+14	+15
H6	+8	+9	+11	+13	+16	+16	+19	+19	+22	+22	+25	+25	+25	+29
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H7	+12	+15	+18	+21	+25	+25	+30	+30	+35	+35	+40	+40	+40	+46
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H8	+18	+22	+27	+33	+39	+39	+46	+46	+54	+54	+63	+63	+63	+72
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H9	+30	+36	+43	+52	+62	+62	+74	+74	+87	+87	+100	+100	+100	+115
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H10	+48	+58	+70	+84	+100	+100	+120	+120	+140	+140	+160	+160	+160	+185
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H11	+75	+90	+110	+130	+160	+160	+190	+190	+220	+220	+250	+250	+250	+290
	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Continuation of table, see Page 406.

200	225	250	280	315	355	400	450	500	560	630	710	800	900
225	250	280	315	355	400	450	500	560	630	710	800	900	1000
+1030	+1110	+1240	+1370	+1560	+1710	+1900	+2050	–	–	–	–	–	–
+740	+820	+920	+1050	+1200	+1350	+1500	+1650	–	–	–	–	–	–
+550	+570	+620	+650	+720	+760	+840	+880	–	–	–	–	–	–
+260	+280	+300	+330	+360	+400	+440	+480	–	–	–	–	–	–
+355	+355	+400	+400	+440	+440	+480	+480	+540	+540	+610	+610	+680	+680
+170	+170	+190	+190	+210	+210	+230	+230	+260	+260	+290	+290	+320	+320
+129	+129	+142	+142	+161	+161	+175	+175	+189	+189	+210	+210	+226	+226
+100	+100	+110	+110	+125	+125	+135	+135	+145	+145	+160	+160	+170	+170
+146	+146	+162	+162	+182	+182	+198	+198	+215	+215	+240	+240	+260	+260
+100	+100	+110	+110	+125	+125	+135	+135	+145	+145	+160	+160	+170	+170
+215	+215	+240	+240	+265	+265	+290	+290	+320	+320	+360	+360	+400	+400
+100	+100	+110	+110	+125	+125	+135	+135	+145	+145	+160	+160	+170	+170
+285	+285	+320	+320	+355	+355	+385	+385	+425	+425	+480	+480	+530	+530
+100	+100	+110	+110	+125	+125	+135	+135	+145	+145	+160	+160	+170	+170
+79	+79	+88	+88	+98	+98	+108	+108	+120	+120	+130	+130	+142	+142
+50	+50	+56	+56	+62	+62	+68	+68	+76	+76	+80	+80	+86	+86
+96	+96	+108	+108	+119	+119	+131	+131	+146	+146	+160	+160	+176	+176
+50	+50	+56	+56	+62	+62	+68	+68	+76	+76	+80	+80	+86	+86
+122	+122	+137	+137	+151	+151	+165	+165	+186	+186	+205	+205	+226	+226
+50	+50	+56	+56	+62	+62	+68	+68	+76	+76	+80	+80	+86	+86
+44	+44	+49	+49	+54	+54	+60	+60	+66	+66	+74	+74	+82	+82
+15	+15	+17	+17	+18	+18	+20	+20	+22	+22	+24	+24	+26	+26
+61	+61	+69	+69	+75	+75	+83	+83	+92	+92	+104	+104	+116	+116
+15	+15	+17	+17	+18	+18	+20	+20	+22	+22	+24	+24	+26	+26
+87	+87	+98	+98	+107	+107	+117	+117	+132	+132	+149	+149	+166	+166
+15	+15	+17	+17	+18	+18	+20	+20	+22	+22	+24	+24	+26	+26
+29	+29	+32	+32	+36	+36	+40	+40	+44	+44	+50	+50	+56	+56
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+46	+46	+52	+52	+57	+57	+63	+63	+70	+70	+80	+80	+90	+90
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+72	+72	+81	+81	+89	+89	+97	+97	+110	+110	+125	+125	+140	+140
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+115	+115	+130	+130	+140	+140	+155	+155	+175	+175	+200	+ 200	+230	+230
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+185	+185	+210	+210	+230	+230	+250	+250	+280	+280	+320	+320	+360	+360
0	0	0	0	0	0	0	0	0	0	0	0	0	0
+290	+290	+320	+320	+360	+360	+400	+400	+440	+440	+500	+500	+560	+560
0	0	0	0	0	0	0	0	0	0	0	0	0	0

Continuation of table ISO tolerances for holes from Page 404.

Tolerance classes	Nominal size range mm												
	over 3 incl. 6	6 10	10 18	18 30	30 40	40 50	50 65	65 80	80 100	100 120	120 140	140 160	160 180
	Limit deviations (1 μm = 0,001 mm) μm					Upper limit deviation = ES Lower limit deviation = EI							
J6	+5	+5	+6	+8	+10	+10	+13	+13	+16	+16	+18	+18	+18
	−3	−4	−5	−5	−6	−6	−6	−6	−6	−6	−7	−7	−7
J7	+6	+8	+10	+12	+14	+14	+18	+18	+22	+22	+26	+26	+26
	−6	−7	−8	−9	−11	−11	−12	−12	−13	−13	−14	−14	−14
J8	+10	+12	+15	+20	+24	+24	+28	+28	+34	+34	+41	+41	+41
	−8	−10	−12	−13	−15	−15	−18	−18	−20	−20	−22	−22	−22
JS6	+4	+4,5	+5,5	+6,5	+8	+8	+9,5	+9,5	+11	+11	+12,5	+12,5	+12,5
	−4	−4,5	−5,5	−6,5	−8	−8	−9,5	−9,5	−11	−11	−12,5	−12,5	−12,5
JS7	+6	+7,5	+9	+10,5	+12,5	+12,5	+15	+15	+17,5	+17,5	+20	+20	+20
	−6	−7,5	−9	−10,5	−12,5	−12,5	−15	−15	−17,5	−17,5	−20	−20	−20
JS8	+9	+11	+13,5	+16,5	+19,5	+19,5	+23	+23	+27	+27	+31,5	+31,5	+31,5
	−9	−11	−13,5	−16,5	−19,5	−19,5	−23	−23	−27	−27	−31,5	−31,5	−31,5
K6	+2	+2	+2	+2	+3	+3	+4	+4	+4	+4	+4	+4	+4
	−6	−7	−9	−11	−13	−13	−15	−15	−18	−18	−21	−21	−21
K7	+3	+5	+6	+6	+7	+7	+9	+9	+10	+10	+12	+12	+12
	−9	−10	−12	−15	−18	−18	−21	−21	−25	−25	−28	−28	−28
K8	+5	+6	+8	+10	+12	+12	+14	+14	+16	+16	+20	+20	+20
	−13	−16	−19	−23	−27	−27	−32	−32	−38	−38	−43	−43	−43
M6	−1	−3	−4	−4	−4	−4	−5	−5	−6	−6	−8	−8	−8
	−9	−12	−15	−17	−20	−20	−24	−24	−28	−28	−33	−33	−33
M7	0	0	0	0	0	0	0	0	0	0	0	0	0
	−12	−15	−18	−21	−25	−25	−30	−30	−35	−35	−40	−40	−40
M8	+2	+1	+2	+4	+5	+5	+5	+5	+6	+6	+8	+8	+8
	−16	−21	−25	−29	−34	−34	−41	−41	−48	−48	−55	−55	−55
N6	−5	−7	−9	−11	−12	−12	−14	−14	−16	−16	−20	−20	−20
	−13	−16	−20	−24	−28	−28	−33	−33	−38	−38	−45	−45	−45
N7	−4	−4	−5	−7	−8	−8	−9	−9	−10	−10	−12	−12	−12
	−16	−19	−23	−28	−33	−33	−39	−39	−45	−45	−52	−52	−52
N8	−2	−3	−3	−3	−3	−3	−4	−4	−4	−4	−4	−4	−4
	−20	−25	−30	−36	−42	−42	−50	−50	−58	−58	−67	−67	−67
P6	−9	−12	−15	−18	−21	−21	−26	−26	−30	−30	−36	−36	−36
	−17	−21	−26	−31	−37	−37	−45	−45	−52	−52	−61	−61	−61
P7	−8	−9	−11	−14	−17	−17	−21	−21	−24	−24	−28	−28	−28
	−20	−24	−29	−35	−42	−42	−51	−51	−59	−59	−68	−68	−68
P8	−12	−15	−18	−22	−26	−26	−32	−32	−37	−37	−43	−43	−43
	−30	−37	−45	−55	−65	−65	−78	−78	−91	−91	−106	−106	−106
R6	−12	−16	−20	−24	−29	−29	−35	−37	−44	−47	−56	−58	−61
	−20	−25	−31	−37	−45	−45	−54	−56	−66	−69	−81	−83	−86
R7	−11	−13	−16	−20	−25	−25	−30	−32	−38	−41	−48	−50	−53
	−23	−28	−34	−41	−50	−50	−60	−62	−73	−76	−88	−90	−93

180	200	225	250	280	315	355	400	450	500	560	630	710	800	900
200	225	250	280	315	355	400	450	500	560	630	710	800	900	1000
+22	+22	+22	+25	+25	+29	+29	+33	+33	–	–	–	–	–	–
−7	−7	−7	−7	−7	−7	−7	−7	−7	–	–	–	–	–	–
+30	+30	+30	+36	+36	+39	+39	+43	+43	–	–	–	–	–	–
−16	−16	−16	−16	−16	−18	−18	−20	−20	–	–	–	–	–	–
+47	+47	+47	+55	+55	+60	+60	+66	+66	–	–	–	–	–	–
−25	−25	−25	−26	−26	−29	−29	−31	−31	–	–	–	–	–	–
+14,5	+14,5	+14,5	+16	+16	+18	+18	+20	+20	+22	+22	+25	+25	+28	+28
−14,5	−14,5	−14,5	−16	−16	−18	−18	−20	−20	−22	−22	−25	−25	−28	−28
+23	+23	+23	+26	+26	+28,5	+28,5	+31,5	+31,5	+35	+35	+40	+40	+45	+45
−23	−23	−23	−26	−26	−28,5	−28,5	−31,5	−31,5	−35	−35	−40	−40	−45	−45
+36	+36	+36	+40,5	+40,5	+44,5	+44,5	+48,5	+48,5	+55	+55	+62,5	+62,5	+70	+70
−36	−36	−36	−40,5	−40,5	−44,5	−44,5	−48,5	−48,5	−55	−55	−62,5	−62,5	−70	−70
+5	+5	+5	+5	+5	+7	+7	+8	+8	0	0	0	0	0	0
−24	−24	−24	−27	−27	−29	−29	−32	−32	−44	−44	−50	−50	−56	−56
+13	+13	+13	+16	+16	+17	+17	+18	+18	0	0	0	0	0	0
−33	−33	−33	−36	−36	−40	−40	−45	−45	−70	−70	−80	−80	−90	−90
+22	+22	+22	+25	+25	+28	+28	+29	+29	0	0	0	0	0	0
−50	−50	−50	−56	−56	−61	−61	−68	−68	−110	−110	−125	−125	−140	−140
−8	−8	−8	−9	−9	−10	−10	−10	−10	−26	−26	−30	−30	−34	−34
−37	−37	−37	−41	−41	−46	−46	−50	−50	−70	−70	−80	−80	−90	−90
0	0	0	0	0	0	0	0	0	−26	−26	−30	−30	−34	−34
−46	−46	−46	−52	−52	−57	−57	−63	−63	−96	−96	−110	−110	−124	−124
+9	+9	+9	+9	+9	+11	+11	+11	+11	−26	−26	−30	−30	−34	−34
−63	−63	−63	−72	−72	−78	−78	−86	−86	−136	−136	−155	−155	−174	−174
−22	−22	−22	−25	−25	−26	−26	−27	−27	−44	−44	−50	−50	−56	−56
−51	−51	−51	−57	−57	−62	−62	−67	−67	−88	−88	−100	−100	−112	−112
−14	−14	−14	−14	−14	−16	−16	−17	−17	−44	−44	−50	−50	−56	−56
−60	−60	−60	−66	−66	−73	−73	−80	−80	−114	−114	−130	−130	−146	−146
−5	−5	−5	−5	−5	−5	−5	−6	−6	−44	−44	−50	−50	−56	−56
−77	−77	−77	−86	−86	−94	−94	−103	−103	−154	−154	−175	−175	−196	−196
−41	−41	−41	−47	−47	−51	−51	−55	−55	−78	−78	−88	−88	−100	−100
−70	−70	−70	−79	−79	−87	−87	−95	−95	−122	−122	−138	−138	−156	−156
−33	−33	−33	−36	−36	−41	−41	−45	−45	−78	−78	−88	−88	−100	−100
−79	−79	−79	−88	−88	−98	−98	−108	−108	−148	−148	−168	−168	−190	−190
−50	−50	−50	−56	−56	−62	−62	−68	−68	−78	−78	−88	−88	−100	−100
−122	−122	−122	−137	−137	−151	−151	−165	−165	−188	−188	−213	−213	−240	−240
−68	−71	−75	−85	−89	−97	−103	−113	−119	−150	−155	−175	−185	−210	−220
−97	−100	−104	−117	−121	−133	−139	−153	−159	−194	−199	−225	−235	−266	−276
−60	−63	−67	−74	−78	−87	−93	−103	−109	−150	−155	−175	−185	−210	−220
−106	−109	−113	−126	−130	−144	−150	−166	−172	−220	−225	−255	−265	−300	−310

Systems of fits

Systems of fits are intended to assist in restricting the large number of possible tolerance classes for defining a particular fit, in order to save costs on production and measuring equipment.

Through the appropriate combination of external and internal tolerance interval positions, it is possible to achieve various fits:

- Clearance fits
- Transition fits
- Interference fits.

Shaft basis and hole basis

The choice of system of fits is based on the area of application:

- System of fits "shaft basis":
  Fits where the fundamental deviation of the shaft is zero (upper deviation is zero). Position of tolerance interval (fundamental deviation symbol) for the shaft always “h”, see Figure 13, Page 409
- System of fits "hole basis":
  Fits where the fundamental deviation of the hole is zero (lower deviation is zero). Position of tolerance interval (fundamental deviation symbol) for the hole always “H”, see Figure 14, Page 409

In the combination of an outer part and inner part of the same nominal size with the maximum and minimum sizes indicated by the relevant upper and lower limit deviations, the following fit types can be obtained:

- **Clearance fit:**
  Fit where, after combination of the parts, clearance always results
- **Transition fit:**
  Fit where, after combination of the parts, clearance or interference results (depending on the actual sizes)
- **Interference fit:**
  Fit where, after combination of the parts, interference always results (and pressure occurs between the fit surfaces).

The following diagrams show the ISO systems of fits "shaft basis" and "hole basis".

Figure 13
System of fits "shaft basis"

① Upper limit of size of the hole ULS_B with tolerance class A9
② Lower limit of size of the hole LLS_B with tolerance class A9
③ Nominal size
④ Housing
⑤ Shaft
⑥ Clearance fit
⑦ Transition fit
⑧ Interference fit
⑨ Position of tolerance interval not present in this tolerance grade

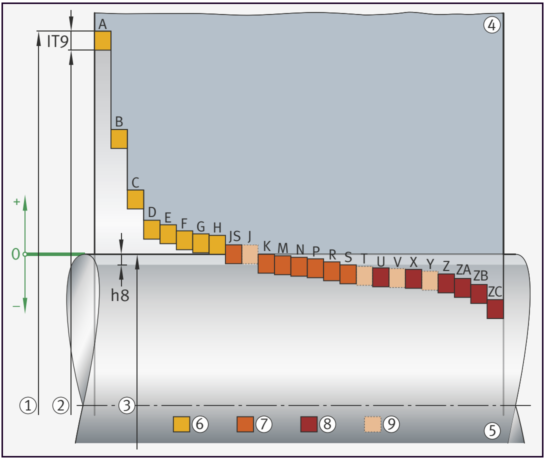
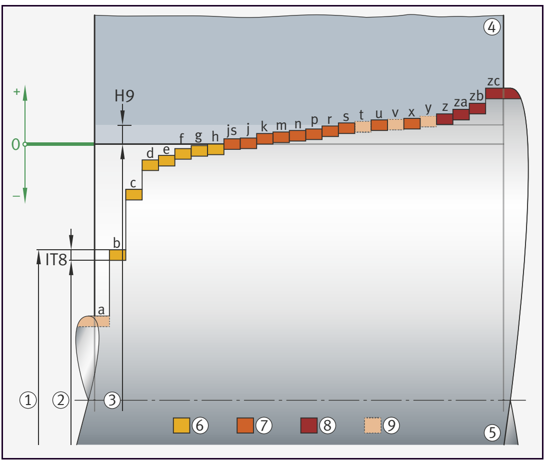


Figure 14
System of fits "hole basis"

① Upper limit of size of the shaft ULS_W with tolerance class b8
② Lower limit of size of the shaft LLS_W with tolerance class b8
③ Nominal size
④ Housing
⑤ Shaft
⑥ Clearance fit
⑦ Transition fit
⑧ Interference fit
⑨ Position of tolerance interval not present in this tolerance grade



Calculation of fits

Fits and fit tolerances are calculated as described below.

Maximum fit

The maximum fit P_o is calculated as follows:

Equation 1

$$P_o = ULS_B - LLS_W = ES - ei$$

The calculation is thus: maximum hole size minus the minimum shaft size. The results are interpreted as follows:

Equation 2

> 0	Maximum clearance
< 0	Minimum interference

Minimum fit

The minimum fit P_u is calculated as follows:

Equation 3

$$P_u = LLS_B - ULS_W = EI - es$$

The calculation is thus: minimum hole size minus the maximum shaft size. The results are interpreted as follows:

Equation 4

> 0	Minimum clearance
< 0	Maximum interference

In the case of clearance fits, the calculated values for the maximum and minimum fit are always positive while, in the case of interference fits, they are always negative.

Fit tolerance

The fit tolerance P_T is the sum of the dimensional tolerances of both form elements that give the fit. It is an absolute value without a mathematical sign.

The fit tolerance is calculated as follows:

Equation 5

$$\begin{aligned} P_T &= P_o - P_u = (ULS_B - LLS_W) - (LLS_B - ULS_W) \\ &= (ES - EI) + (es - ei) \end{aligned}$$

Application of ISO fits

Examples of the application of ISO fits are given below:

Hole basis	Application	Shaft basis
	Clearance fits	
H11/a11	Parts with **very large clearance** and **large tolerance**: locomotive and waggon construction, coupling pins, agricultural machinery	A11/h11
H11/c11	Parts with **large clearance** and **large tolerance**: agricultural and household machinery	C11/h11
H10/d9	Parts with **very abundant clearance**: transmission shafts, stuffing box parts, loose pulleys, countershafts	D10/h9
H8/e8	Parts with **abundant clearance**: machine tool shafts with several supports, plain bearings	E8/h8
H7/f7	Parts with **significant clearance**: main machine tool bearings, sliding sleeves on shafts, pistons in cylinders	F7/h7
H7/g6	**Without significant clearance**, capable of movement: sliding gear wheels, movable coupling parts, valve lever bearing arrangement	G7/h6
H7/h6	Lubricated by hand **capable of upward movement**: sleeve in the tailstock, centring flanges for couplings and pipelines	H7/h6
H6/h5	**Very small mean clearance**: for parts not moving against each other	H6/h5
	Transition fits	
H7/j6	**Joining by hand by means of light blows**: for belt pulleys, gear wheels and bearing bushes that can be easily dismounted	J7/h6
H7/k6	**Effective joining by means of hand-held hammer**: for belt pulleys, couplings and flywheels with a feather key joint	K7/h6
H7/m6	**Joining by means of hand-held hammer difficult but possible:** belt pulleys for single mounting only, couplings and gear wheels on electric motor shafts	M7/h6
H7/n6	**Joining possible by means of press**: for armatures on motor shafts and gear rings on gear wheels, bearing bushes in hubs	N7/h6
	Interference fits	
H7/r6 H7/s6	**Joining possible under high pressure or by heating:** circlip rings on flake graphite cast iron hubs, bearing bushes in housings (s6 for larger diameters, r6 for smaller diameters)	R7/h6 S7/h6
H8/u8 H8/x8	**Joining only achievable by means of press or temperature differential:** for transmission of high circumferential or longitudinal forces by frictional locking	U8/h8 X8/h8

Rolling bearing tolerances and ISO tolerances for shafts and housings

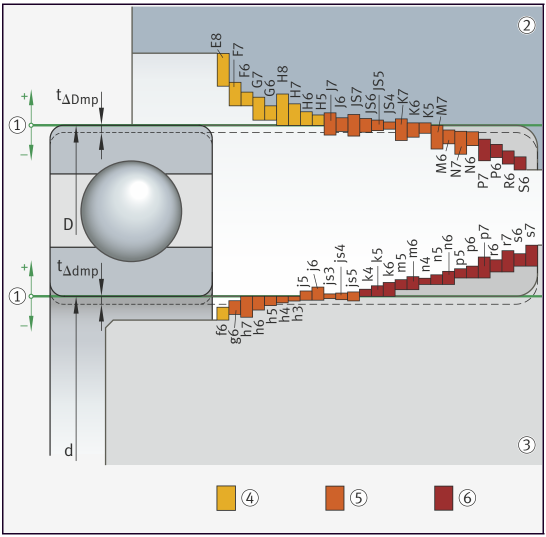
The tolerances for rolling bearings are defined for radial rolling bearings in accordance with ISO 492 (July 2014) and for axial rolling bearings in accordance with ISO 199 (July 2014). These give tolerances for the deviation of the bearing bore as $t_{\Delta dmp}$ and the deviation of the bearing outside diameter as $t_{\Delta Dmp}$. In both cases, the upper limit deviation of these tolerance intervals are additional to the nominal size. The lower limit deviations (in both cases towards minus) are defined by the accuracy grades detailed in accordance with ISO 492 and ISO 199.

In combination with the ISO tolerances for shafts and holes in accordance with DIN EN ISO 286, this gives approximately the fits shown in the following diagram.

Figure 15
Fits for rolling bearings

$t_{\Delta Dmp}$ = deviation of mean bearing outside diameter
$t_{\Delta dmp}$ = deviation of mean bearing bore diameter
D = nominal bearing outside diameter
d = nominal bearing bore diameter

① Zero line
② Housing
③ Shaft
④ Clearance fit
⑤ Transition fit
⑥ Interference fit



Mounting fits for rolling bearings

Mounting fits for rolling bearings are selected as a function of the conditions of rotation. The conditions of rotation indicate the movement of one bearing ring with respect to the load direction.

For further information on the conditions of rotation of rolling bearings, see the chapter Design elements, section Conditions of rotation, Page 591.

Geometrical and positional tolerances in drawings

Indications in drawings

The standard DIN EN ISO 1101 (September 2017) defines methods of indicating geometrical and positional tolerances in drawings.

Symbols for toleranced characteristics

For toleranced characteristics, the following symbols are defined:

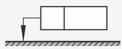
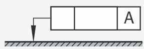
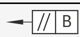
Types of elements and tolerances		Toleranced characteristic	Symbol
Elements without datum	Form tolerances	Straightness	⏤
		Flatness	⏥
		Roundness (circularity)	○
		Cylindricity (cylindrical form)	⌭
		Line profile	⌒
		Surface profile	⌓
Elements with datum	Orientation tolerances	Parallelism	//
		Perpendicularity	⊥
		Angularity	∠
	Orientation and location tolerances	Line profile	⌒
		Surface profile	⌓
Elements with/ without datum	Location tolerances	Position	⌖
Elements with datum		Concentricity and coaxiality	◎
		Symmetry	⌯
	Run-out tolerances	Circular run-out Radial run-out, axial run-out	↗
		Total run-out Total radial run-out, total axial run-out	⌰

Additional symbols The following additional symbols are defined in DIN EN ISO 1101:2017:

Description	Symbol
Modifiers for the combination of tolerance zones (including ISO 1660, ISO 2692 and ISO 5458)	
Combined zone	CZ
Separate zones	SZ
Modifiers for non-uniform tolerance zones (including ISO 1660, ISO 2692 and ISO 5458)	
Specified tolerance zone offset	UZ
Modifiers for constraints	
Unspecified linear offset tolerance zone (offset zone)	OZ
Unspecified inclination of tolerance zone (variable angle)	VA
Modifiers for associated toleranced features	
Minimax (Chebyshev) feature	Ⓒ
(Gaussian) least squares feature	Ⓖ
Minimum circumscribed feature	Ⓝ
Tangent feature	Ⓣ
Maximum inscribed feature	Ⓧ
Modifiers for derived toleranced features	
Derived feature	Ⓐ
Projected tolerance zone	Ⓟ
Modifiers for the association of reference features for geometrical evaluation	
Minimax (Chebyshev) feature without constraint	C
Minimax adjacent to the material-free side (Chebyshev) feature	CE
Minimax adjacent to the material side (Chebyshev) feature	CI
Least squares (Gaussian) feature without constraint	G
Least squares (Gaussian) feature adjacent to the material-free side	GE
Least squares (Gaussian) feature adjacent to the material side	GI
Minimum circumscribed feature	N
Maximum inscribed feature	X

Continuation of table, see Page 415.

Continuation of table Additional symbols from Page 414.

Description	Symbol
Modifiers for parameters	
Total range of deviations	T
Peak height	P
Valley depth	V
Standard deviation	Q
Modifiers for toleranced features	
Between	
United feature	UF
Lower diameter	LD
Major diameter	MD
Pitch diameter	PD
All around (profile)	
All over (profile)	
Tolerance indicators	
Geometrical specification indication without datum section	
Geometrical specification indication with datum section[1]	
Additional indications of features	
Any cross-section	ACS
Intersection plane indicator[1]	
Orientation plane indicator[1]	
Direction feature indicator[1]	
Collection plane indicator[1]	
Symbol for the theoretically exact dimension (TED)	
Theoretically exact dimension (TED)[1]	$\boxed{50}$

Continuation of table, see Page 416.

[1] The letters, values and characteristic symbols in these symbols are examples.

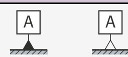
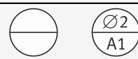
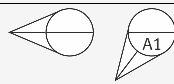
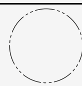
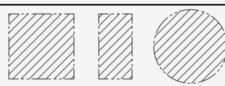
Continuation of table Additional symbols from Page 415.

Description	Symbol
Modifiers for the material requirement (in accordance with ISO 2692)	
Maximum material requirement	Ⓜ
Least material requirement	Ⓛ
Reciprocity requirement	Ⓡ
Modifier for the free state condition (in accordance with ISO 10579)	
Free state condition (non-rigid parts)	Ⓕ
Modifier for the size tolerance (in accordance with ISO 14405-1)	
Envelope requirement	Ⓔ

Indications and modifiers for datums, see also the table Datum features and datum point symbols.

Datums Datums are defined in the standard DIN ISO 5459:2013.

Datum features and datum point symbols
For datums in drawings, DIN ISO 5459:2013 defines the following datum features and datum point symbols (letters or values in these symbols are examples only):

Description	Symbol
Datum feature indicator	
Datum name	Capital letter (A, B, C, AA etc.)
Single datum target frame	
Moveable datum target frame	
Datum target point	
Closed datum target line	
Non-closed datum target line	
Datum target area	

Modifier symbols for datums

The following modifier symbols can, in accordance with DIN ISO 5459:2013, be associated with the datum letter:

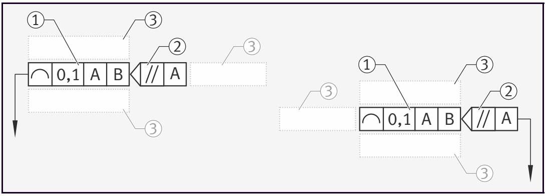
Description	Symbol
Pitch diameter	[PD]
Major diameter	[MD]
Lower diameter	[LD]
Any cross-section	[ACS]
Any longitudinal section	[ALS]
Contacting feature	[CF]
Distance variable (for a common datum)	[DV]
(Situation feature of type) Point	[PT]
(Situation feature of type) Straight line	[SL]
(Situation feature of type) Plane	[PL]
Only for constraint of direction	><
Projected (for secondary and tertiary datums)	Ⓟ
Least material requirement (in accordance with ISO 2692)	Ⓛ
Maximum material requirement (in accordance with ISO 2692)	Ⓜ

Tolerance indicator and additional indications

The indication of a geometrical product specification comprises a tolerance indicator, the optional plane and feature indications as well as optional adjacent indications, see Figure 16.

Figure 16
Components of a geometrical specification

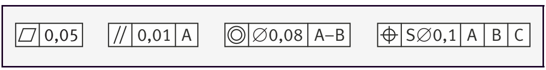
① Tolerance indicator
② Plane and feature indicator
③ Adjacent indications



The tolerance requirements are indicated in a rectangular frame which is divided into two or more compartments. This tolerance indicator contains, from left to right, the following compartments:

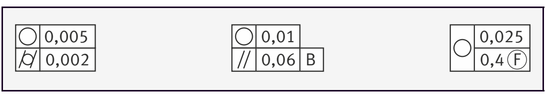
- Symbol compartment:
  with the symbol for the characteristic to be toleranced
- Compartment for zone, feature and characteristic:
  with the tolerance value in the unit used for dimensioning. This value is preceded by the symbol "∅" if the tolerance zone is circular or cylindrical or the notation "S∅" if the tolerance zone is spherical. Further indications with complementary modifiers are possible, see table Compartment for zone, feature and characteristic
- Datum compartment:
  if necessary, the letter or letters that identify the datum feature or features.

Figure 17
Tolerance indicators



If it is necessary to specify more than one tolerance characteristic for a feature, the tolerance indications should be indicated in tolerance frames one under the other:

Figure 18
Multiple tolerance indications per feature



Compartment for zone, feature and characteristic

Complementary modifiers for particular requirements are indicated within the tolerance indicator in accordance with the tolerance value in the "compartment for zone, feature and characteristic"; several such requirements may be present at the same time in the same tolerance frame.

The following table, see Page 419, shows the grouping of the modifiers and the sequence in which the modifiers are indicated.

With the exception of “Width and extent”, all modifiers are optional. Between indications with different numbering (no. 1 to 11), empty spaces must be present (except before letters in circles, columns no. 6, 7, 10, 11).

Tolerance zone					Toleranced feature				Characteristic		Material condition	Condition
					Filter[1]							
Shape	Width and extent	Comb.	Specified offset	Constraint	Type	Indices	Ass. tol. feature	Derived feature	Association	Parameter		
∅ S∅	0,02 0,02–0,01 0,1/75 0,1/75×75 0,2/∅4 0,2/75×30° 0,3/10°×30°	CZ SZ	UZ+0,2 UZ–0,3 UZ+0,1:+0,2 UZ+0,2:-0,3 UZ–0,2:–0,3	OZ VA ><	G S etc.	0,8 –250 0,8–250 500 –15 500–15 etc.	Ⓒ Ⓖ Ⓝ Ⓣ Ⓧ	Ⓐ Ⓟ Ⓟ25 Ⓟ32-7	C CE CI G GE GI X N	P V T Q	Ⓜ Ⓛ Ⓡ	Ⓕ
1		2[2]	3	4[2]	5		6	7[2]	8	9	10[2]	11

Source: DIN EN ISO 1101:2017.

[1] Filters: see DIN EN ISO 1101.

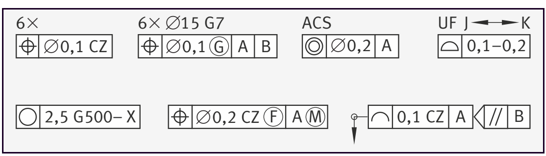
[2] More than one of the listed modifiers can be used at the same time.

Additional indications of features

Additional indications of features (intersection plane indicator, orientation plane indicator, direction feature indicator and/or collection plane indicator) are placed next to the tolerance indicator in compartment 2, see Figure 16, Page 418.

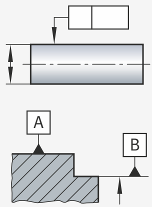
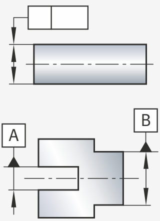
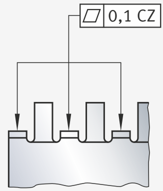
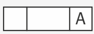
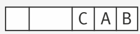
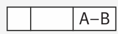
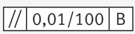
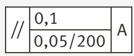
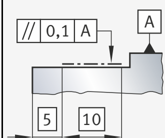
If a tolerance applies to more than one feature, the number of features is entered with the symbol "×" above the tolerance indicator and can be supplemented by the size of the feature. Further optional indications for describing the characteristics of the feature within the tolerance zone are also placed in compartment 3, see Figure 16, Page 418, (preferably above):
dimensional tolerance indications, variable width specification with the aid of the “between” symbol, additionally UF, ACS, LD, MD, PD.
The "all around" arrow is placed in the tolerance frame instead of the normal direction arrow if the profile characteristic applies to the entire contour line:

Figure 19
Additional tolerance indications



Features, datums and restrictions

Toleranced features, datum features and datum letters or restrictive instructions are indicated in drawings as follows:

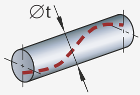
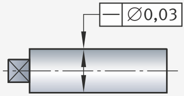
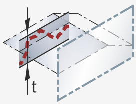
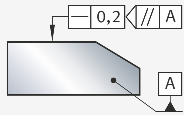
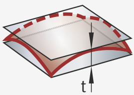
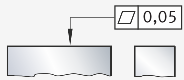
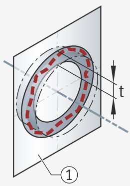
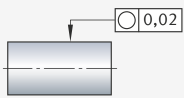
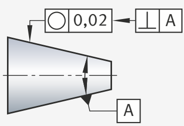
Toleranced features and datum features		
Line or area 	Axis or median plane 	Combined zone 
Single datum feature 	Multiple datum features with a fixed ranked-order (primary/secondary/ tertiary) 	Datums of equal significance forming a common datum feature 
Tolerance over a restricted length 	Smaller tolerance over a restricted length 	Tolerance over a restricted part of the feature 

Drawing entry and definitions

The standard DIN ISO 1101:2017 contains detailed definitions of geometrical and positional tolerances and their symbolic language.

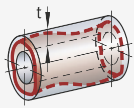
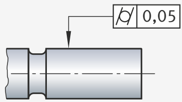
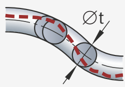
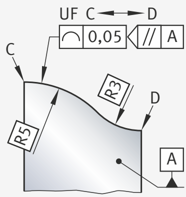
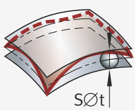
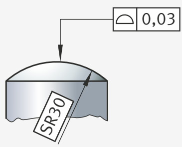
The following table gives a compilation of these definitions. An example is described for each toleranced characteristic. All further possible combinations can be derived from these examples.

The corresponding representation following the table shows an example of the indication of geometrical and positional tolerances in technical drawings.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	Definition
—	Straightness			The extracted median (actual) line of the cylinder to which the tolerance applies shall lie within a cylindrical tolerance zone of diameter t = 0,03 mm.
				The extracted median (actual) line to which the tolerance applies shall lie within a tolerance zone that is contained in any plane under consideration parallel to datum within two parallel straight lines at a distance of t = 0,2 mm apart.
▱	Flatness			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two parallel planes which are at a distance of t = 0,05 mm apart.
○	Roundness			The extracted (actual) circumferential line to which the tolerance applies shall lie within a tolerance zone that is contained in any cross-section within two concentric circles which are at a radial distance of t = 0,02 mm apart.
				Additional indication on taper (in this case, the directional feature indicator to perpendicularity): The circles are perpendicular to the datum axis A. ① Any cross-sectional plane (any cross-section)

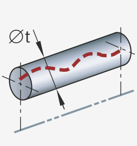
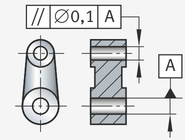
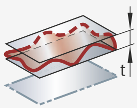
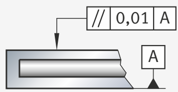
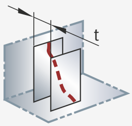
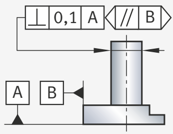
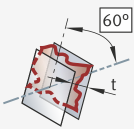
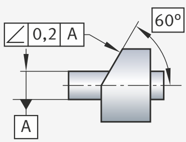
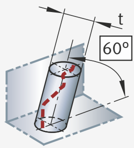
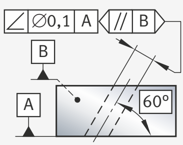
Continuation of table, see Page 422.

Continuation of table Drawing entry and definitions from Page 421.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	Definition
⌭	Cylindricity (cylindrical form)			The extracted cylindrical (actual) surface to which the tolerance applies shall lie within a cylindrical tolerance zone that is contained within two coaxial cylinders which have a radius differential of t = 0,05 mm.
⌒	Line profile			The extracted (actual) profile line to which the tolerance applies shall lie within a tolerance zone that is contained within any parallel plane to the datum plane A by two lines that enclose circles of diameter t = 0,05 mm. The centre points of these circles are situated on a geometrically true line.
⌓	Surface profile			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two surfaces that enclose spheres of diameter t = 0,03 mm. The centre points of these spheres are situated on a geometrically true surface.

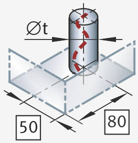
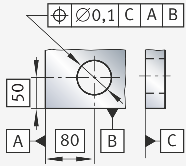
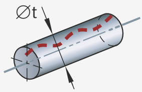
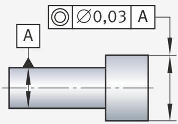
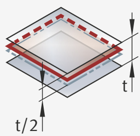
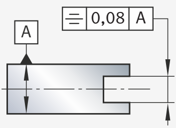
Continuation of table, see Page 423.

Continuation of table Drawing entry and definitions from Page 422.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	Definition
//	Parallelism			The extracted central (actual) line to which the tolerance applies shall lie within a cylinder-shaped tolerance zone parallel to the datum axis A which is of the diameter t = 0,1 mm.
				The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two parallel planes to the datum plane A which are at a distance of t = 0,01 mm apart.
⊥	Perpendicularity			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two parallel planes which are at a distance of t = 0,1 mm apart. The planes are perpendicular to the datum A and parallel to the secondary datum B.
∠	Angularity			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two inclined parallel planes at a specified angle to the datum which are at a distance of t = 0,2 mm apart.
				The extracted median (actual) line of the hole to which the tolerance applies shall lie within a cylindrical tolerance zone of diameter t = 0,1 mm. The tolerance zone lies parallel to the datum plane B and is inclined at the specified angle to the datum plane A.

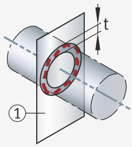
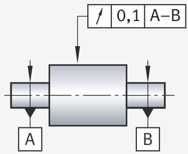
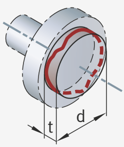
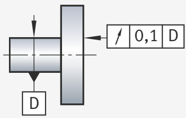
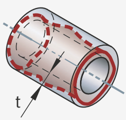
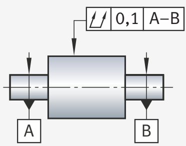
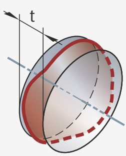
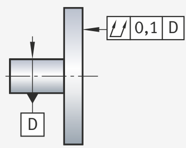
Continuation of table, see Page 424.

Continuation of table Drawing entry and definitions from Page 423.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	Definition
⌖	Position			The extracted central (actual) line of the bore to which the tolerance applies must be contained within a cylinder with the diameter t = 0,1 mm, whose axis coincides with the theoretically exact location of the axis of the bore in relation to the datum planes C, A and B.
◎	Coaxiality, concentricity			The extracted central (actual) line of the large cylinder shall lie within a cylindrical tolerance zone with the diameter t = 0,03 mm whose axis coincides with the datum axis A.
⌯	Symmetry			The extracted (actual) median plane of the slot to which the tolerance applies must be contained within two parallel planes which are at a distance of t = 0,08 mm apart and symmetrical to the datum median plane A.

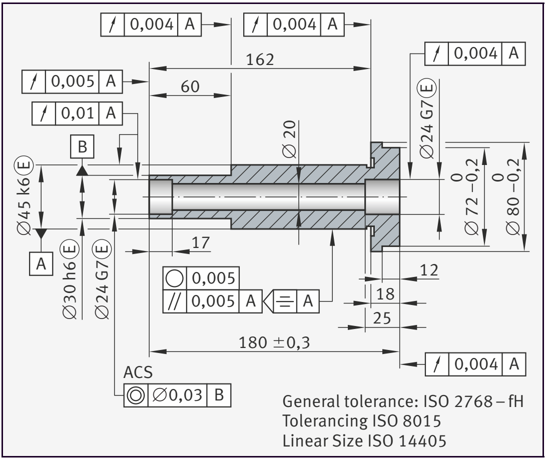
Continuation of table, see Page 425.

Continuation of table Drawing entry and definitions from Page 424.

Symbol and toleranced characteristic		Tolerance zone	Application examples	
			Drawing indication	**Definition**
↗	Radial run-out			The extracted central (actual) line of the bore to which the tolerance applies shall lie within a tolerance zone that is contained in any cross-section perpendicular to the datum axis A–B, within two concentric circles at a distance of t = 0,1 mm whose centre point lies on the datum axis. ① Cross-sectional plane with a variable position along the datum axis.
	Axial run-out			The extracted (actual) line to which the tolerance applies shall lie within a tolerance zone that is contained in any cylindrical cross-section of a variable diameter d whose axis coincides with the datum axis D, within two concentric circles at an axial distance of t = 0,1 mm.
⌰	Total radial run-out			The extracted (actual) line to which the tolerance applies shall lie within a tolerance zone that is contained within two coaxial cylinders which are at a radial distance of t = 0,1 mm, whose axis coincides with the common datum lines A–B.
	Total axial run-out			The extracted (actual) surface to which the tolerance applies shall lie within a tolerance zone that is contained within two parallel planes which are at a distance of t = 0,1 mm apart, that are perpendicular to datum axis D.

The following representation shows an example of the indication of geometrical and positional tolerances in technical drawings.

Figure 20
Drawing with geometrical and positional tolerances



Tolerancing principle

The tolerancing principle governs the relationship between dimensional, geometrical and positional tolerances. If the principle of independence applies, this means that dimensional, geometrical and positional tolerances are independent (DIN EN ISO 8015:2011-09). If the envelope principle in accordance with ISO 14405 (Linear Size ISO 14405 Ⓔ) applies, this means that all dimensional, geometrical and positional tolerances lie within the dimensional tolerance. If no other determinations are made, the principle of independence applies.

Principle of independence

Under the principle of independence, dimensional, geometrical and positional tolerances must be observed independently of each other if no mutual relationship is indicated. If no mutual relationship is indicated, each tolerance must be checked independently. If only Linear Size ISO 14405 is indicated in or on the title block and there are no special characteristics (modification symbols) on the dimensional feature, the two-point size always applies (DIN EN ISO 17450-1).

If the dimensional tolerance is defined as a two-point size, the local actual size must lie within the tolerance limits. This does not, however, ensure that no other geometrical deviations can be present.

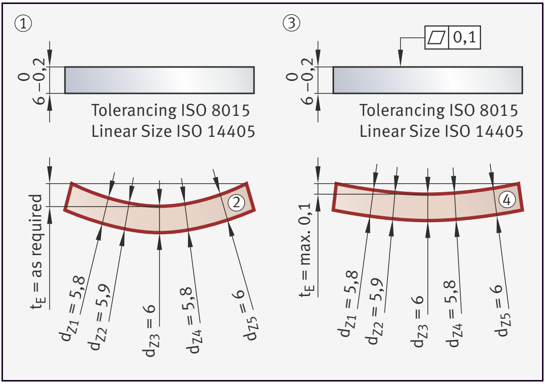
For example, if there is no special tolerance for the flatness, the requirements of the drawing in Figure 21 ①, Page 427 will still be fulfilled by a sheet metal piece with a considerable geometrical deviation whose local actual sizes conform to the tolerance limits.

If the geometrical deviation is restricted, a special tolerance must be applied to the flatness, see Figure 21 ③ and ④.

Figure 21
Drawing with principle of independence

d_Z = local actual size (two-point size)
t_E = flatness tolerance

① Drawing without flatness tolerance
② Permissible sheet metal piece with considerable geometrical deviation
③ Drawing with flatness tolerance
④ Sheet metal piece fulfils flatness tolerance



If the following statement is present in or adjacent to the title block of the drawing, the principle of independence applies:
Tolerancing ISO 8015, Linear Size ISO 14405
(or simply: Linear Size ISO 14405).

Unless a particular drawing indication is present, the principle of independence also applies worldwide in accordance with ISO 14405-1. If ASME drawings contain the following in or adjacent to the title block: Tolerancing ASME Y14.5, the envelope requirement applies here.

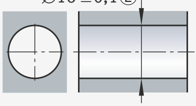
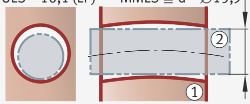
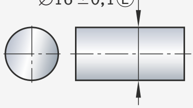
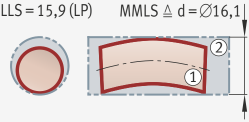
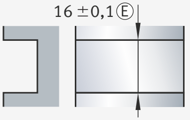
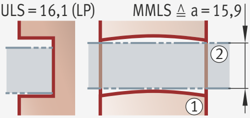
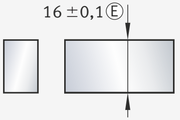
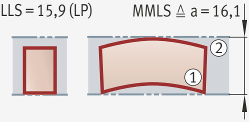
Envelope requirement

If the principle of independence is defined for a drawing, the envelope requirement should be defined for sizes (former description: fit surfaces) in order to ensure mating capability if no geometrical and positional tolerances are indicated. This stipulates that dimensional, geometrical and parallelism tolerances are in a particular relationship with each other. The application of the envelope requirement for the form element is defined by the symbol Ⓔ after the dimensional tolerance of the form element.

The envelope requirement is checked as a two-point size and by means of a gauge (envelope). This can also be carried out using a coordinate measuring machine.

The envelope requirement stipulates that the form element may not pierce the geometrically ideal envelope with a Maximum Material Limit (MMLS, maximum material limit of size) and that, at the same time, the local two-point size (LP) may not at any point lie outside the tolerance limits, see DIN EN ISO 14405-1:2017. Deviations of geometry and parallelism are thus also recorded automatically. According to the standard, the envelope requirement is the use of a combination of the two-point size and the minimum circumscribed lower limit of size LLS (in the case of a shaft or block) or the maximum inscribed upper limit of size ULS (in the case of a hole or slot).

The following table shows the geometrically ideal envelope for various form elements and the geometrical deviations subject to the envelope requirement:

Form element	Example drawing	Permissible workpiece, geometry of envelope	Recorded deviation
Circular cylinder (hole) ① Permissible workpiece ② Geometry of envelope	Hole ∅16 ±0,1Ⓔ 	Envelope: mandrel with d = LLS ULS = 16,1 (LP) MMLS ≙ d = ∅15,9 	Straightness, roundness, parallelism, cylindrical form
Circular cylinder (shaft) ① Permissible workpiece ② Geometry of envelope	Shaft ∅16 ±0,1Ⓔ 	Envelope: sleeve with d = ULS LLS = 15,9 (LP) MMLS ≙ d = ∅16,1 	Straightness, roundness, parallelism, cylindrical form
Parallel, flat inner surfaces ① Permissible workpiece ② Geometry of envelope	Slot 16 ±0,1Ⓔ 	Envelope: 2 parallel planes with a = LLS ULS = 16,1 (LP) MMLS ≙ a = 15,9 	Straightness, flatness, parallelism
Parallel, flat outer surfaces ① Permissible workpiece ② Geometry of envelope	Block 16 ±0,1Ⓔ 	Envelope: 2 parallel planes with a = ULS LLS = 15,9 (LP) MMLS ≙ a = 16,1 	Straightness, flatness, parallelism

In accordance with DIN EN ISO 1938-1:2015, the Maximum Material Limit of a Size (MMLS) is the limit size that corresponds to the Maximum Material State of the feature of size. The Least Material Limit of a Size (LMLS) is the limit size that corresponds to the Least Material State of the feature of size. MMLS and LMLS contain in each case the numerical value of the size and the defined allocation criteria.

For external sizes (shaft, block), the following applies:

- MMLS corresponds to the upper limit of size ULS; the material must lie within an envelope with d = ULS or a = ULS.
- LMLS (two-point size) corresponds to the minimum size LMS and must not be exceeded at any point on the shaft or block.

For internal sizes (hole, slot), the following applies:

- MMLS corresponds to the lower limit of size LLS; the material must lie within an envelope with d = LLS or a = LLS
- LMLS (two-point size) corresponds to the upper limit of size ULS and must not be exceeded at any point on in the hole or slot.

Envelope principle In accordance with the envelope principle, the envelope requirement is applied in the entire technical drawing if the following note appears in or on the title block: Linear Size ISO 14405 Ⓔ. A drawing entry for the individual sizes of the form elements using the symbol Ⓔ is not necessary then. The envelope requirement applies to all form elements such as circular cylinders, parallel surfaces or cones.

General rule In German drawings without reference to the standard (DIN) ISO 8015, the envelope principle applied automatically to all circular cylinders and parallel surfaces up to the year 2010 in accordance with the standard DIN 7167 (January 1987) without the presence of special indications in the drawing. After a revision of the standard ISO 8015 and application of the standard EN ISO 14405-1, the principle of independence has applied as a general rule with effect from 2010 in a drawing without reference to the standard ISO 8015.

General tolerances

Definition General tolerances in accordance with DIN ISO 2768 are used for the simplification of drawings. They are used for the central definition of tolerances for the entire drawing without the necessity of an individual tolerance entry. They are applied to all nominal sizes and form elements in a drawing in which no individual and deviating tolerances are indicated. With the selection of a tolerance class, the relevant accuracy normally expected of workshops and specific to the process is taken into consideration.

Indications in drawings The use of general tolerances is indicated in the drawing by the symbols for the relevant tolerance classes. The entry is made centrally in or adjacent to the title block of the drawing, for example ISO 2768 - mK. The defined tolerance classes, their symbols and the corresponding limit deviations are given in the following tables.

Angular and linear sizes The standard DIN ISO 2768-1 (June 1991) defines general tolerances for linear and angular sizes in production by means of methods involving cutting and forming. For other production methods, general tolerances are defined in further standards.

Angular sizes The tolerance classes for angular sizes are defined as follows:

Tolerance class		Limit deviations for linear sizes of the shorter angular leg mm				
Desig- nation	Description	incl. 10	over 10 incl. 50	over 50 incl. 120	over 120 incl. 400	over 400
f	fine	±1°	±30′	±20′	±10′	±5′
m	medium					
c	coarse	±1° 30′	±1°	±30′	±15′	±10′
v	very coarse	±3°	±2°	±1°	±30′	±20′

Linear sizes excluding those for broken edges The tolerance classes for linear sizes with the exception of those for broken edges (rounding radii and chamfer heights, see table Broken edges) are defined as follows:

Tolerance class		Limit deviations for nominal size ranges mm							
Symbol	Name	from 0,5[1] incl. 3	over 3 incl. 6	over 6 incl. 30	over 30 incl. 120	over 120 incl. 400	over 400 incl. 1 000	over 1 000 incl. 2 000	over 2 000 incl. 4 000
f	fine	±0,05	±0,05	±0,1	±0,15	±0,2	±0,3	±0,5	–
m	medium	±0,1	±0,1	±0,2	±0,3	±0,5	±0,8	±1,2	±2
c	coarse	±0,2	±0,3	±0,5	±0,8	±1,2	±2	±3	±4
v	very coarse	–	±0,5	±1	±1,5	±2,5	±4	±6	±8

[1] For nominal sizes below 0,5 mm, the limit deviations shall be indicated adjacent to the relevant nominal size.

Broken edges The tolerance classes for broken edges (rounding radii and chamfer heights) are defined as follows:

Tolerance class		Limit deviations for nominal size ranges mm		
Symbol	Name	from 0,5[1] incl. 3	over 3 incl. 6	over 6
f	fine	±0,2	±0,5	±1
m	medium			
c	coarse	±0,4	±1	±2
v	very coarse			

[1] For nominal sizes below 0,5 mm, the limit deviations shall be indicated adjacent to the relevant nominal size.

For workpiece edges with a geometrically indeterminate form, the standard DIN ISO 13715 (December 2000) is applied. It also shows symbols for the individual or collective indication of permissible chamfer dimensions in the drawing.

Geometry and position

The standard DIN ISO 2768-2 (April 1991) defines general tolerances for geometry and position.

Straightness and flatness

The tolerance classes for straightness and flatness are defined as follows:

Tolerance class	General tolerances for straightness and flatness for nominal size ranges mm					
	incl. 10	over 10 incl. 30	over 30 incl. 100	over 100 incl. 300	over 300 incl. 1000	over 1000 incl. 3000
H	0,02	0,05	0,1	0,2	0,3	0,4
K	0,05	0,1	0,2	0,4	0,6	0,8
L	0,1	0,2	0,4	0,8	1,2	1,6

Perpendicularity

The tolerance classes for perpendicularity are defined as follows:

Tolerance class	Perpendicularity tolerances for nominal size ranges of the shorter angular leg mm			
	incl. 100	over 100 incl. 300	over 300 incl. 1000	over 1000 incl. 3000
H	0,2	0,3	0,4	0,5
K	0,4	0,6	0,8	1
L	0,6	1	1,5	2

Symmetry

The tolerance classes for symmetry are defined as follows:

Tolerance class	Symmetry tolerances for nominal size ranges mm			
	incl. 100	over 100 incl. 300	over 300 incl. 1000	over 1000 incl. 3000
H	0,5	0,5	0,5	0,5
K	0,6	0,6	0,8	1
L	0,6	1	1,5	2

Run-out

The tolerance classes for run-out (radial run-out, axial run-out) are defined as follows:

Tolerance class	Run-out tolerances mm
H	0,1
K	0,2
L	0,5

Design elements

Definitions and principal functions

Design elements are machine elements of widely varying complexity that are always used in an identical or similar form in technical applications, where they fulfil an identical or similar function. They are among the most important elements used by design engineers to realise solutions.

Function-oriented approach

In accordance with this definition, it is possible to structure the very wide spectrum of machine elements in terms of their function, see Figure 1. This makes it easier for engineers to access this wide range in their design work on and with the aid of machine elements.

Classification according to principal function

This chapter is structured in accordance with the principal functions of design elements as they are found repeatedly in technical practice.

Design elements are described substantially in accordance with the following scheme:

- Characteristics
  (these describe the structure, geometry and quality of the product, in the way that these can be directly influenced by design engineers)
- Properties
  (these describe the behaviour of the product as it results from the interaction with other design elements and under the influence of operating conditions).

Overview through classification schemes

It would contradict the character of this Pocket Guide to give a detailed description of all common machine elements. For this reason, classification schemes are used to give an overview of the totality of design elements.

The detailed description focusses essentially on the elements that could traditionally be found in earlier editions of the Schaeffler Technical Pocket Guide.

The design elements with the function “Guiding elements (rotational and translational)” are covered in comprehensive detail.

Principal functions

The principal functions of the design elements are shown in Figure 1.

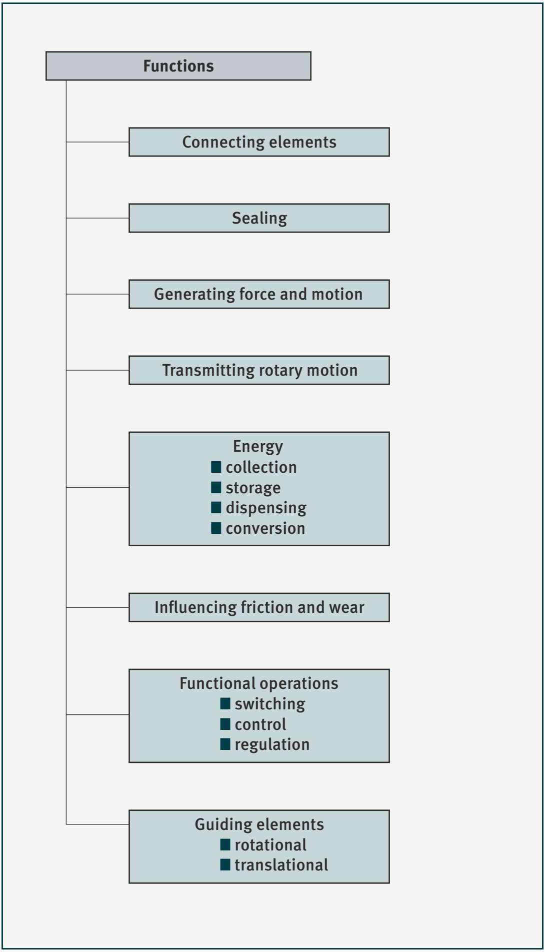


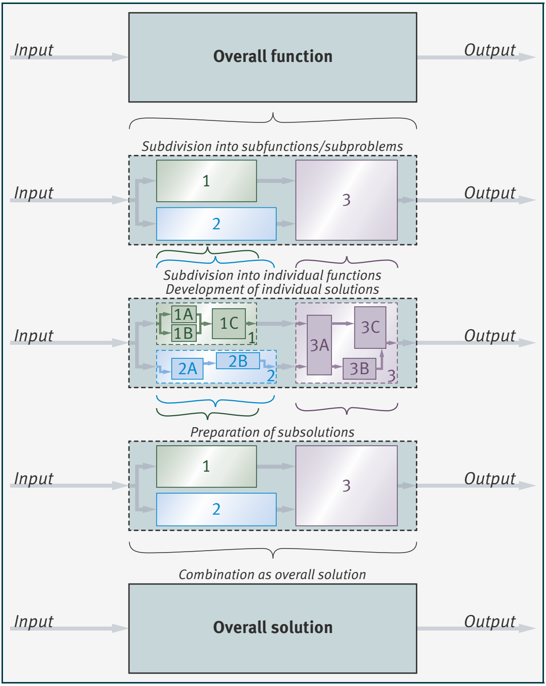
Figure 1
Classification according to principal functions – overview

Functional structure

Thinking and working on a function-oriented basis is an essential principle in modern development methodology.

From the starting point of the list of requirements, each technical system must fulfil an overall function that can be subdivided into subfunctions (functional structure). Through this "breaking down" into subordinate subfunctions, it can be ensured that the complex overall problem is classified into smaller subtasks that can be resolved more easily, giving subsolutions that can later be recombined to give an overall solution for a technical system.

Figure 2
Functional structure
Source: in accordance with VDI Guideline 2221

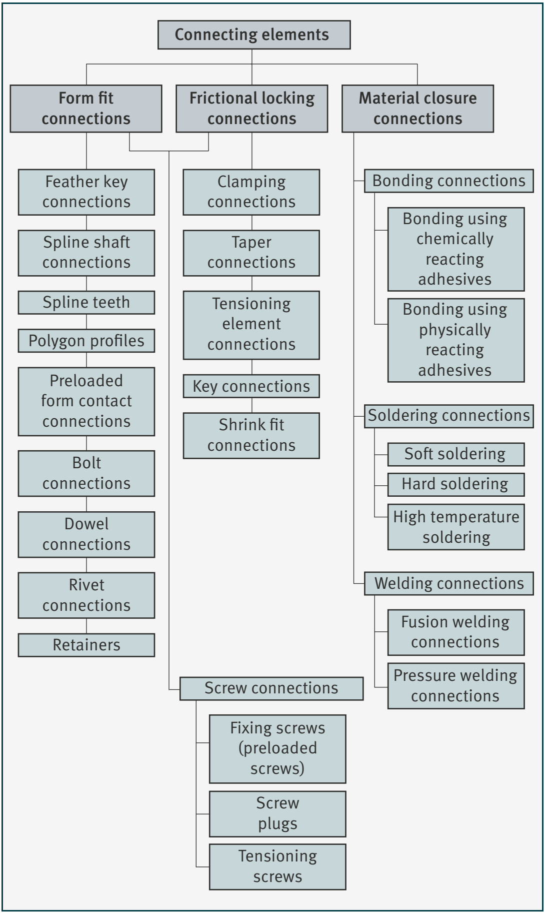


Working on a function-oriented basis also ensures that solutions from various domains (mechanics, electronics, software) can be found and followed up.

Connecting elements

Overview There is a large number of elements that can be used to connect design elements and combine design elements in more complex structures. The function of these connecting elements is to transmit forces and moments. Depending on the type of force transmission (operating principle), a distinction is made between connections based on material closure, form fit and frictional locking. Combinations of frictional locking and form fit are possible.

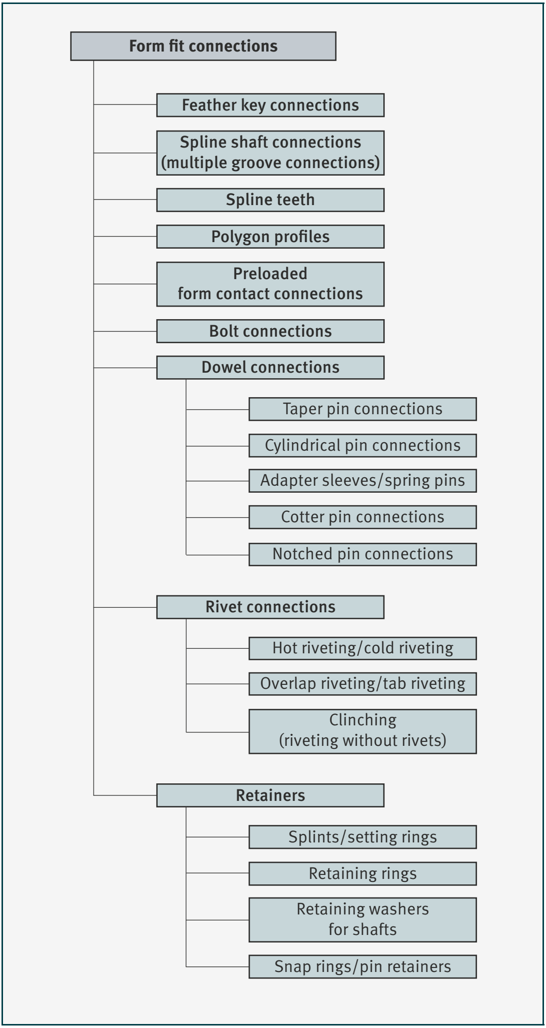
Figure 3
Overview
Connecting elements
(form fit, frictional locking, material closure)



Form fit connections

In the case of form fit connections, the connection is achieved by means of the form of the parts to be connected (= direct form fit) or by means of additional elements that act as “drivers” (= indirect form fit).

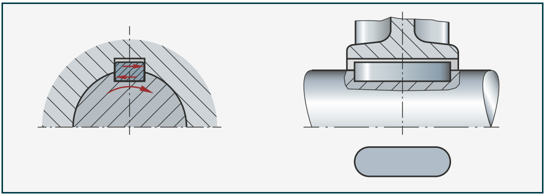
Figure 4
Form fit connections



Feather key connections

The feather key connection is the most important representative type of shaft/hub connection based on form fit. However, it is not suitable for the transmission of shock type torques and high, alternating torques.

Figure 5
Feather key connection



Characteristics:

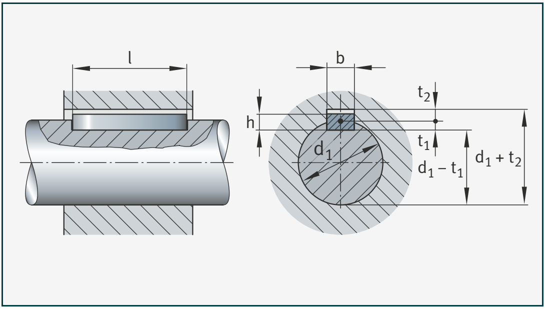
- The feather keys are simultaneously present in the shaft keyway and the hub keyway
- One feather key is normally used; up to a maximum of two can be arranged with an offset around the circumference of 120°
- The keyway is parallel to the axis
- A feather key used as a driver has a rectangular cross-section
- Designs of feather key with a round end or square end are possible
- The hub must be axially located.

Properties:

- For high torques
- Not suitable for changes of load direction and shocks
- Displacement of axis possible under certain preconditions (for indexing)
- Stress concentration on the shaft, due to notch effect
- Easy to mount and dismount, reusable.

Figure 6
Key values of a feather key connection

b = feather key width
h = feather key height
t_1 = shaft keyway width
t_2 = hub keyway width
l = effective feather key length (for round end design, deduct b)



The feather key connection transmits via its flanks the circumferential force generated by the torque to be transmitted and is thus subjected to contact pressure.

The circumferential force F_u can be calculated as follows:

Equation 1

$$F_u = \frac{2 \cdot M_t}{d}$$

where M_t is the transmissible torque and d is the shaft diameter, while contact pressure can be calculated as follows:

Equation 2

$$p = \frac{F_u}{(h - t_1) l \cdot i} \quad \text{or} \quad p = \frac{F_u}{0{,}45 \cdot h \cdot l \cdot i} \quad \text{to DIN 6892}$$

Legend

l mm
Effective feather key length

i
Number of feather keys

if i = 2
use only 75% of l.

Guide values for the permissible flank contact pressures can be found in the following table. For shaft calculation, take account of the reduction in strength as a result of the feather key connections (β_{kt} = 1,3 ... 2,0; β_{kb} = 2,1 ... 3,2).[1)]

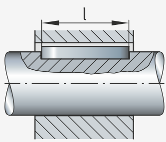
Material	Permissible contact pressure p_{per} under load type N/mm^2	
	Static	Pulsating
Steel, unhardened	100 ... 200	70 ... 150
Steel, hardened	150 ... 250	100 ... 170
Cast steel	100 ... 150	80 ... 100
Cast iron, malleable cast iron	80 ... 100	60 ... 80
Copper alloy (bronze, brass)	40 ... 50	30 ... 40
AlCuMg, aged	100 ... 160	70 ... 100
AlMg, AlMn, AlMgSi, aged	80 ... 150	60 ... 90
G AlSi, G AlSiMg	60 ... 70	40 ... 50

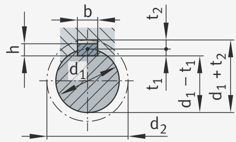
In general, the higher values apply in the case of higher values for yield stress, fracture strength and hardness of the materials, while the lower values apply accordingly in the case of lower strength values.

1) The values apply in the range R_m = 400 ... 1 200 N/mm^2 (in accordance with DIN 743).

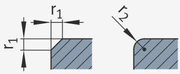
Feather keys, keyways, deep design

The following table shows feather keys, keyways (deep design) in accordance with DIN 6885 Part 1/Part 2:

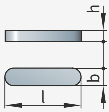




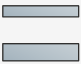
Edge break (all sides), chamfer or rounding (at manufacturer's discretion)



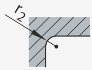
Type A, round end[1)]



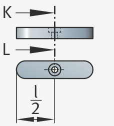
Type B, square end[1)]



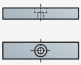
Rounding of keyway base for shaft and hub



Type C, round end Size 8 × 7 and above, with hole for 1 retaining screw above stepped line

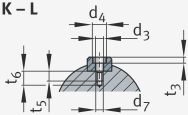


Type B, square end Size 8 × 7 and above, with hole for 1 retaining screw above stepped line



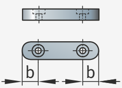
Hole for retaining screw

K – L

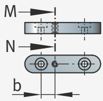


Type E, round end, with holes for 2 retaining screws below the stepped line

8 × 7 and 10 × 8

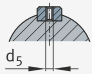


Size 12 × 8 and above, additionally with threaded hole for 1 or 2 extraction screws



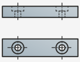
Hole for extraction screw

M – N

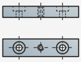


Type F, round end, with holes for 2 retaining screws below the stepped line

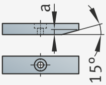
8 × 7 and 10 × 8



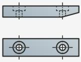
Size 12 × 8 and above, additionally with threaded hole for 1 or 2 extraction screws



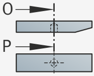
Type G, square end, with chamfer and hole for 1 retaining screw



Type H, square end, with chamfer and hole for 2 retaining screws

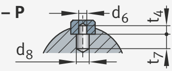


Type J, square end, with chamfer and hole for 1 adapter sleeve



Hole for adapter sleeve

O – P



Designation of feather key, type A, width b = 12 mm, height h = 8 mm, length l = 56 mm:
feather key A 12 × 8 × 56 DIN 6885.

Material: Part 1 for h ≤ 25 mm and Part 2 for all sizes E295, Part 1 for h ≥ 25 mm E335.

1) If feather keys of type A and B are to be supplied with holes for extraction screws (S), this must be specified at the time of ordering. In this case, the designation is: feather key AS 12 × 8 × 56 DIN 6885.

For feather keys (high design) in accordance with DIN 6885 Part 1/Part 2[1], the following values apply.

Feather key, cross-section (wedge steel to DIN 6880)		Width b	4	5	6
		Height h	4	5	6
Shaft diameter d_1		over	10	12	17
		incl.	12	17	22
Shaft	b, tight fit P9	loose fit N9	4	5	6
	t_1 with back clearance		2,5	3	3,5
Hub	b, tight fit P9	loose fit JS9	4	5	6
	t_2 in the case of back clearance		1,8	2,3	2,8
	for interference[2]		1,2	1,7	2,2
	a		–	–	–
	$d_2 = d_1 +$[3]		4	5	6
	Feather key r_1	min./max.	0,16/0,25	0,25/0,40	0,25/0,40
	Keyway r_2	max./min.	0,16/0,08	0,25/0,16	0,25/0,16
	Shaft t_1		3	3,8	4,4
	Hub t_2		1,1	1,3	1,7
	$d_2 = d_1 +$[1]		3	3,5	4
	l[4]		(10)	(12)	(16)
		over	8	10	14
		incl.	45	56	70
	Stepping of l	6 8 10 12 14 16 18 20 22 25 28 32			
	Feather key	d_3	–	–	–
		d_4	–	–	–
		d_5	–	–	–
		d_6 H12	–	–	–
		t_3	–	–	–
		t_4	–	–	–
	Shaft	d_1	–	–	–
		d_6	–	–	–
		t_5	–	–	–
		t_6	–	–	–
		t_7	–	–	–
Screw to DIN 84, DIN 7984 or DIN 6912, adapter sleeve to DIN 1481		–	–	–	–

[1] For Part 2 (type A, C and E only), the dimensions t_1, t_2 and d_2 in the part enclosed by broad lines apply; all other dimensions such as feather keys are in accordance with Part 1.

[2] t_2 in the case of interference is intended for exceptional cases in which the feather key is reworked (adapted).

[3] d_2 is the smallest diameter (inside dimension) of parts which can be slid concentrically over the feather key.

[4] The values stated within () are the smallest lengths according to Part 2 where these do not coincide with Part 1.

[5] Only up to 250 for feather keys in accordance with Part 2.

8 7	10 8	12 8	14 9	16 10	18 11	20 12	22 14	25 14
22 30	30 38	38 44	44 50	50 58	58 65	65 75	75 85	85 95
8	10	12	14	16	18	20	22	25
4	5	5	5,5	6	7	7,5	9	9
8	10	12	14	16	18	20	22	25
3,3	3,3	3,3	3,8	4,3	4,4	4,9	5,4	5,4
2,4	2,4	2,4	2,9	3,4	3,4	3,9	4,4	4,4
3 8	3 8	3 8	3,5 9	4 11	4,5 11	5 12	5,5 14	5,5 14
0,25/0,40 0,25/0,16	0,40/0,60 0,40/0,25	0,40/0,60 0,40/0,25	0,40/0,60 0,40/0,25	0,40/0,60 0,40/0,25	0,40/0,60 0,40/0,25	0,6/0,8 0,6/,04	0,6/0,8 0,6/,04	0,6/0,8 0,6/,04
5,4 1,7 4,5	6 2,1 5,5	6 2,1 6	6,5 2,6 7	7,5 2,6 8	8 3,1 8,5	8 4,1 11	10 4,1 12	10 4,1 12
(20) 18 90	(25) 22 110	(32) 28 140	(40) 36 160	– 45 180	– 50 200	– 56 220	– 63 250	– 70 280[5)]
36 40	45 50	56 63	70 80 90	100 110	125 140	160 180	200 220	250 280[5)]
3,4 6 M3 4	3,4 6 M3 4	4,5 8 M4 5	5,5 10 M5 6	5,5 10 M5 6	6,6 11 M6 8	6,6 11 M6 8	6,6 11 M6 8	9 15 M8 10
2,4 4	2,4 4	3,2 5	4,1 6	4,1 6	4,8 7	4,8 8	4,8 8	6 10
M3 4,5	M3 4,5	M4 5,5	M5 6,5	M5 6,5	M6 9	M6 9	M6 9	M8 11
4 7 M3 × 8	5 8 M3 × 10	6 10 M4 × 10	6 10 M5 × 10	6 10 M5 × 10	7 12 M6 × 12	6 11 M6 × 12	8 13 M6 × 16	9 15 M8 × 16
4 × 8	4 × 8	5 × 10	6 × 12	6 × 12	8 × 16	8 × 16	8 × 16	10 × 20

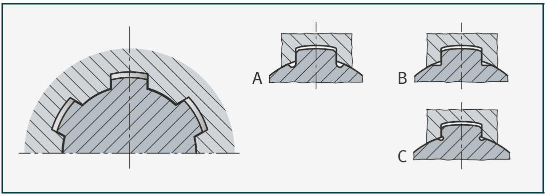
Spline shaft connections

Characteristics:
- The shaft and hub are provided with a large number of splines and grooves (6, 8, 10, ...)
- The splines are integral with the shaft, the grooves are integral with the hub
- The splines and grooves lie parallel to the axis.

Properties:
- For high, alternating and abruptly acting torques
- For alternating direction of rotation
- Displacement of axis possible under load (for indexing), otherwise axial location of the hub is necessary
- Stress concentration on the shaft, due to notch effect.

Figure 7
Spline shaft connection

A, B, C = various spline shaft types in accordance with DIN 5471



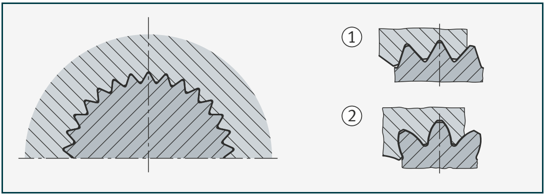
Spline teeth

Characteristics:
- A large number of teeth are integrated in the shaft and hub
- The teeth lie parallel to the axis
- The design normally has a spline angle of 60°, however other variants such as those with involute teeth are used.

Properties:
- For transmission of moderate loads
- For alternating direction of rotation
- Less stress concentration than in the case of a spline shaft
- Separable connection, axial retention necessary
- Centring only by tooth flanks and thus restricted running accuracy.

Figure 8
Spline teeth

① Notch tooth profile
② Involute tooth profile



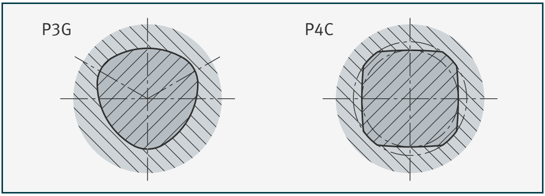
Polygon connections

Characteristics:
- Direct form contact, separable connection
- Non-circular profiles with defined profile geometry, standardised as P3G and P4C.

Properties:
- Highly suitable for transmission of high torques as well as alternating and shock type torques
- Profile P4C very suitable for axial displacement capability under load
- Very little stress concentration with tight fits
- Very smooth running due to self-centring
- High manufacturing costs
- Simple mounting and dismounting.

Figure 9
Polygon connection

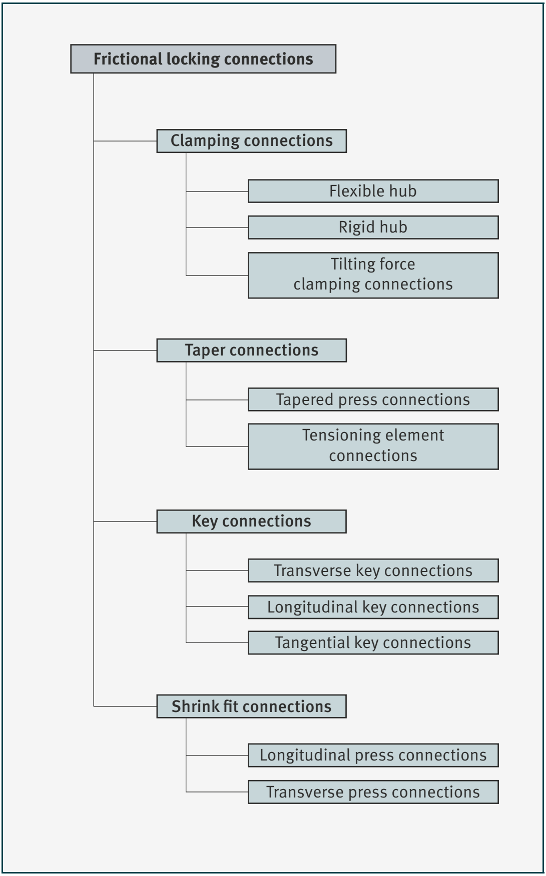
P3G = profile in accordance with DIN 32711
P4C = profile in accordance with DIN 32712



Frictional locking connections

In the case of these connections, forces are transmitted by frictional locking through pressing or clamping.

Figure 10
Frictional locking connections



Tapered press connections

Characteristics:

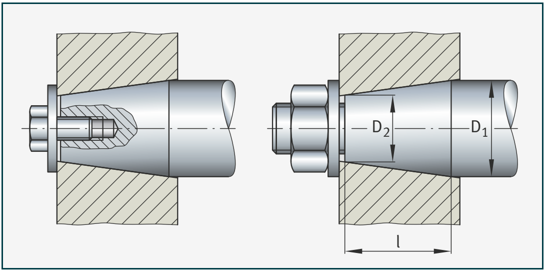
- The necessary joint pressure is achieved by sliding the hub in an axial direction onto the conical shaft seat
- Axial tensioning is achieved by means of a screw or nut
- Taper ratio $C = (D_1 - D_2)/l$
- Application of standardised taper ratios in accordance with DIN 254, for example:
  - tapered shaft ends 1:10
  - metric tool tapers 1:20
  - adapter sleeves 1:12 or 1:30.

Properties:

- Suitable for high torques, changes in load direction and shocks
- Very smooth running due to precise concentric seat
- Support of high axial forces possible
- No axial displacement of hub possible
- No precise axial positioning of hub possible
- Hub can be displaced in the direction of rotation
- High manufacturing costs, but simple mounting.

Figure 11
Taper ratio

D_1, D_2, l = taper dimensions



Axially preloaded tapered press joint

In the case of the axially preloaded tapered press joint, the joint pressure p_F is generated by sliding the hub in an axial direction onto the conical shaft seat with the force F_A.

Figure 12
Axially preloaded tapered press joint

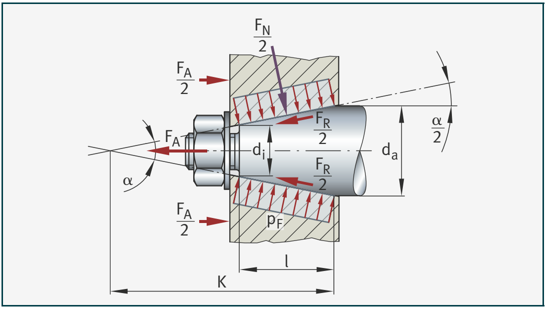
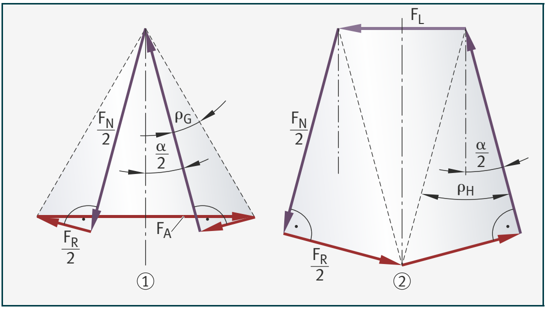


Figure 13
Forces in the axially preloaded tapered press joint

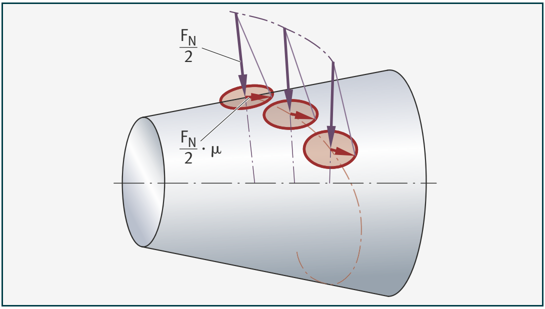
① Sliding-on process
② Dismounting



Taper to DIN 1448/49	$d_a/K = 1{:}10$, $\alpha = 5{,}7258° = 5°43'29''$
Sliding-on force (axial force)	$F_A = F_N\,[\sin(\alpha/2) + \mu_G \cos(\alpha/2)]$
Normal force	$F_N = p_F d_m\,\pi\, l/\cos(\alpha/2)$; $d_m = (d_a + d_i)/2$
Joint pressure	$p_F = F_N \cos(\alpha/2)/(d_m\,\pi l)$
Transmissible circumferential force	$F_U = \mu\, F_N/S$ S = safety against slipping, S = 1,2...1,5
Transmissible torque	$M_t = F_U d_m/2$
Dismounting force	$F_L = F_N\,[\sin(\alpha/2) + \mu_H \cos(\alpha/2)]$

In an axially preloaded tapered press joint, there is an equilibrium between the axial force F_A and the resultant of the normal force component $F_N \sin(\alpha/2)$ and the friction component $F_N \mu \cos(\alpha/2)$.

Figure 14
Forces acting on an axially preloaded tapered press joint during the sliding-on process under initial torque loading



During the first torque transmission, a circumferential force F_U is added to the forces and the equilibrium created by the joining process is altered. Since an additional circumferential force F_U is now acting additionally, the friction vector is rotated out of the axis direction into the direction of the resultant formed by F_U and $\mu_G \cdot F_N$, with the result that the axial sliding-on force is now counteracted by only one component of the friction force. As a result, the force F_A pushes the hub helically further onto the tapered stud until the new equilibrium state is reached. At the same time, the axial force F_A decreases while there is an increase in F_N and consequently p_F. This results in a connection with increased safety against slipping.

If an additional feather key (see DIN 1448/1449) or Woodruff key is used in the tapered press joint, this alone transmits the entire torque since it prevents the helical sliding-on process. In technical terms, a combination of a tapered press joint (with frictional locking) and a feather key (with form fit) is therefore not appropriate.

Cylindrical press connections

In the case of cylindrical press connections, the shaft and hub have an interference fit. The contact pressure necessary for frictional locking is achieved by the elastic deformation of both components after joining. Depending on the type of mounting, a distinction is made between longitudinal press connections, transverse press connections and oil press connections.

Characteristics:

- The shaft and hub have an interference fit before joining
- Frictional locking through elastic deformation of the shaft and hub
- No additional mechanical connecting elements
- No cross-sectional weakening
- For axial positioning, a means of restricting position (such as a shaft shoulder) is advantageous.

Properties:

- Large forces and moments can be transmitted
- Circumferential and longitudinal forces can be transmitted
- For alternating direction of rotation and abrupt operation
- Simple and economical manufacture
- Difficult dismounting
- Optimum force transmission under uniform flow of forces
- High geometrical strength and operational strength.

According to DIN EN ISO 286-1, an interference fit is defined as a fit where the maximum dimension of the hole (hub) is smaller than the minimum dimension of the shaft, in which case interference is then present. After joining, this interference between the hub and the shaft leads to a press fit with a normal force acting on the joined faces. The normal force generates an adhesion force by means of which longitudinal forces (parallel to the axis) and circumferential forces or torques (forces that act tangentially in the joint) can be transmitted from one part to another.

The joining process is carried out as follows:

- longitudinal press fit: longitudinal pressing of the inner part
- transverse press fit:
  - contraction of the outer part (with prior heating)
  - expansion of the inner part (with prior cooling)
  - expansion of the inner part and contraction of the outer part.

The joining temperature necessary for the transverse press fit can be determined on the basis of the interference U necessary for transmission of the torque or the axial force. In order to ensure reliable mounting, a mounting clearance S_M (advantageous value: $S_M = U/2$) must be taken into consideration.

If the outer part is heated, the following overtemperature for example is necessary for mounting:

Equation 3

$$U + S_M = \alpha \cdot d_{Ai} \cdot \Delta T$$

Equation 4

$$\Delta T = \frac{U + S_M}{\alpha \cdot d_{Ai}}$$

Legend

d_{Ai} Inside diameter of outer part	Steel $\alpha = (11 \ldots 12)\ 10^{-6}\ 1/K$[1]
α Coefficient of thermal expansion	Cast iron $\alpha = (9 \ldots 10)\ 10^{-6}\ 1/K$[1]
ΔT Overtemperature	Aluminium $\alpha = (23 \ldots 24)\ 10^{-6}\ 1/K$[1]

[1] α values are only valid for heating.

As a temperature source for heating of the outer part or cooling of the inner part, the following facilities can be used.

Heating facility

Heating facility	Area of application	Notes
Electric heating plates	Volume production parts (normally small)	Heating frequently incomplete, risk of local overheating!
Electric heating cores	Sleeves and hubs	Achievable joining temperature: up to ≈ 50 °C
Bath heating	Outer parts on whose joining surfaces oil may be present during joining	Natural organic heat transfer media up to 300 °C; paraffin-based or silicone-based oils up to 400 °C
Hot air ovens or hot air chambers	Outer parts whose joining surfaces must be dry and devoid of oxide layers	Normally for heating temperatures up to 400 °C; up to 650 °C possible in special ovens

The upper temperature is restricted by the increased loss of strength of the materials used.

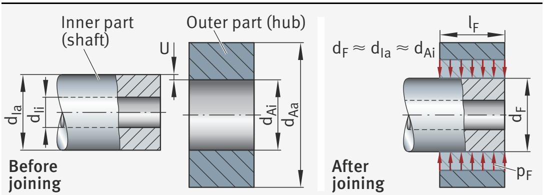
Means of supercooling

Means of supercooling	Chemical formula	Boiling point of gas	Notes
Carbon dioxide snow or dry ice	CO_2	−78,4 °C	Joined part cools relatively slowly; cooling speed is increased when spirit is used as a heat transfer medium. Addition of trichloroethylene prevents icing of the surfaces of the joined part
Liquid nitrogen	N_2	−195,8 °C	Ensure good ventilation during use in closed rooms! Otherwise, no particular risks

Due to the extreme risk of explosion, the use of liquid oxygen or liquid air is not advisable.

Calculation of cylindrical press fits

The following tables show the calculation of a cylindrical press fit for elastic load. In the case of an elasto-plastic load, see DIN 7190.



Diameter conditions		
Inner part I	(hollow shaft)	$Q_I = d_{Ii}/d_{Ia} \approx d_{Ii}/d_F < 1$
	(solid shaft)	$Q_I = 0$
Outer part A		$Q_A = d_{Ai}/d_{Aa} \approx d_F/d_{Aa} < 1$
Interference for the press fit determined from the fit data		$U = d_{Ia} - d_{Ai}$
Smoothing dimension (Rz = mean roughness depth)		$G = 0{,}8\,(Rz_{Ia} + Rz_{Ai})$
Effective adhesion dimension		$Z = U - G$
Constriction of the inner part as a result of the joint pressure		$\Delta d_{Ia} = -\dfrac{p_F\, d_F}{E_I}\left(\dfrac{1+Q_I^{\,2}}{1-Q_I^{\,2}} - \nu_I\right)$
Expansion of the outer part as a result of the joint pressure		$\Delta d_{Ai} = -\dfrac{p_F d_F}{E_A}\left(\dfrac{1+Q_A^{\,2}}{1-Q_A^{\,2}} + \nu_A\right)$

Continuation of table, see Page 451.

Continuation of table, Calculation of cylindrical press fits, from Page 450.

When the press fit is joined, the adhesion dimension is transformed into a constriction of the inner part and an expansion of the outer part, where the relationship is as follows: $Z = \lvert \Delta d_{Ia} \rvert + \lvert \Delta d_{Ai} \rvert$ $Z = p_F \left[\frac{d_F}{E_I} \left(\frac{1+Q_I{}^2}{1-Q_I{}^2} - \nu_I \right) + \frac{d_F}{E_A} \left(\frac{1+Q_A{}^2}{1-Q_A{}^2} + \nu_A \right) \right]$ From this relationship, the connection between the effective adhesion dimension and the joint pressure is derived. The smallest joint pressure results from the minimum interference of the fit data for the press fit.	
Axial force transmission Necessary joint pressure	$F_A = p_F\, d_F\, \pi\, l_F\, \mu / S$ $p_{F\,req} = F_A\, S/(d_F\, \pi\, l_F\, \mu)$
Torque transmission Necessary joint pressure	$M_t = p_F\, d_F\, \pi\, l_F\, \mu\, (d_F/2)/S$ $p_{F\,req} = 2\, M_t\, S/(d_F{}^2\, \pi\, l_F\, \mu)$

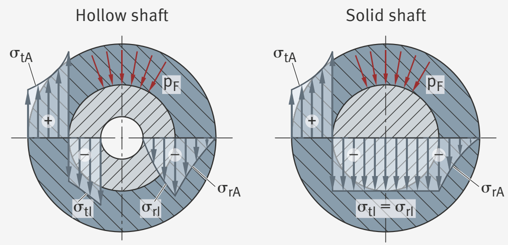
Type of press fits and stresses			Coefficients of adhesion μ (DIN 7190)	
			Dry	Lubricated
Coefficients of adhesion for transverse press fits in longitudinal and transverse directions				
Steel/steel combination	Hydraulic oil joint	Joined using mineral oil	–	0,12
		Degreased press surfaces, joined using glycerine	0,18	–
	Shrinkage joint	Heating up to 300 °C	–	0,14
		Degreased press surfaces, heating up to 300 °C	0,20	–
Steel/cast iron combination	Hydraulic oil joint	Joined using mineral oil	–	0,16
		Degreased press surfaces	0,16	–
Coefficients of adhesion for longitudinal press fits				
Shaft material		Chromium steel	–	–
Hub material		E335	0,11	0,08
		S235 JRG2	0,10	0,07
		EN-GJL-259	0,12 ... 0,14	0,06

Continuation of table, see Page 452.

Continuation of table, Calculation of cylindrical press fits, from Page 451.

Type of press fits and stresses	Coefficients of adhesion μ (DIN 7190)	
	Dry	Lubricated
Safety against slipping		
Transverse press joint	$S = 1{,}5 \dots 2{,}0$	
Longitudinal press joint	$S = 2{,}0 \dots 2{,}5$	

The stress profiles in the outer and inner parts of the cylindrical press joint (loading of thick-walled pipes under internal and external pressure) can be found in the compilation of the most important load types.

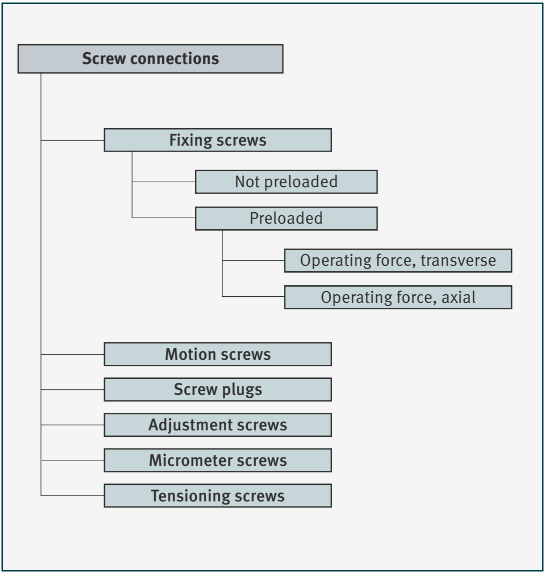


The loads that are critical for the press joint generally occur on the inner edge of the outer part (hub). In the case of a hollow shaft, it is also necessary to check the stress σ_{tIi} on the inner edge of the inner part.

Stresses on the inner edge of the outer part	
Tangential stress	$\sigma_{tAi} = p_F \dfrac{1+Q_A{}^2}{1-Q_A{}^2}$
Radial stress	$\sigma_{rAi} = -p_F$
Equivalent stress (DEH)	$\sigma_{vAi} = \sqrt{\sigma_{tAi}{}^2 + \sigma_{rAi}{}^2 - \sigma_{tAi} \cdot \sigma_{rAi}}$ $\sigma_{vAi} = \dfrac{2 \cdot p_F}{1-Q_A{}^2} < \sigma_{per}$

Screw connections Screw connections are a combination of form fit and frictional locking.

Figure 15
Screw connections



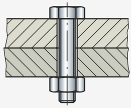
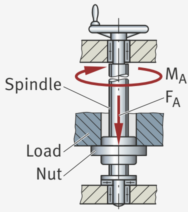
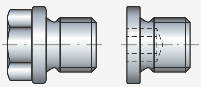
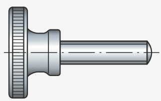
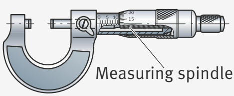
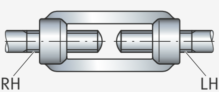
Form fit and frictional locking connections

Screw connections are based on a combination of a screw or grub screw with an external thread and a component with an internal thread (normally a nut) where form fit between the two is achieved in the thread.

In the thread, which can be seen in an unwound form as a skewed plane, rotation of the screw relative to the nut gives sliding of the thread flanks of the screw on the thread flanks of the nut and thereby longitudinal motion.

Design elements

Depending on the intended purpose, a distinction is made between various types of screw design.

Screw connection	Description
Fixing screw 	■ Separable connection of components ■ Design as through screw connection or fit screw connection possible
Motion screw 	■ Conversion of rotary motion into longitudinal motion and vice versa ■ Generation of high forces (design as ball screw possible for reduction of friction)
Screw plug 	■ Closing off of filling or outlet openings (sealing screw)
Adjustment screw 	■ Alignment of devices and instruments ■ Example: knurled screw
Micrometer screw 	■ Length measurement in micron range
Tensioning screw 	■ Generation of tensioning forces ■ Example: turnbuckle RH = right hand thread LH = left hand thread

Fixing screws

Screws are the most widely used machine elements for joining components. In comparison with welded, soldered, bonded and riveted joints, components can be separated non-destructively and joined again.
As a fixing element, the screw must perform the task of connecting components under the preload force applied during assembly and of maintaining this connection in the event of static and dynamic loads.

In contrast to the advantages of the screw connection, which lie in its simple assembly, non-destructive separability and ability to transmit large forces, there is the drawback that the **highly notched** screw may suffer fatigue fracture under dynamic loads or that an inadmissible breakdown of the preload force may occur in the joint as the result of settling phenomena at the contact points, or as a result of the nut working loose from the screw. A screw connection subjected to high loads will survive or fail depending on the ability of its screws to maintain or lose the preload force applied during assembly. Very often, the cause of a fatigue fracture in a screw can be attributed to a prior reduction in the preload force.
It is therefore imperative that a screw connection should be carefully designed and calculated.

Through screw connections

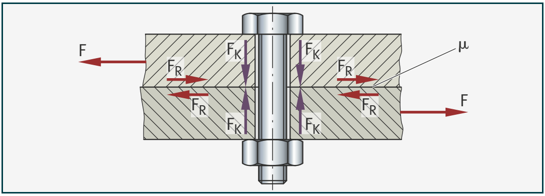
Characteristics:

- Frictional locking connection
- Clamping force generated by tightening of the nut
- Screw subjected to tensile load (torsion due to the tightening torque)
- Securing of screw necessary
- Additional centring necessary.

Properties:

- Separable connection
- Suitable for large forces
- Easy to fit
- Stress concentration due to hole in the flange (notch effect).

Figure 16
Through screw connection



Fit screw connections

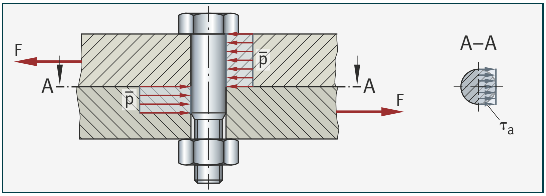
Characteristics:

- Form fit and frictional locking connection
- Screw subjected to shear load and contact pressure (torsion due to the tightening torque)
- Centring by means of fit between screw shank and flange
- Securing of screw necessary.

Properties:

- Separable connection
- Suitable for centring
- Easy to fit
- Stress concentration due to hole in the flange (notch effect)
- Significantly more expensive compared to through screw connection.

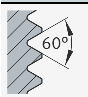
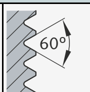
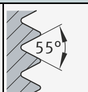
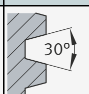
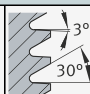
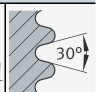
Figure 17
Fit screw connection



Base forms of the most common threads

Certain thread types defined in the DIN standards have proven themselves in accordance with the various conditions of use.

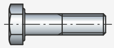
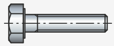
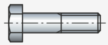
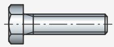
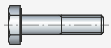
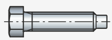
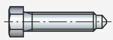
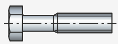
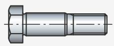
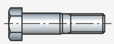
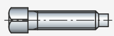
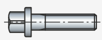
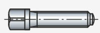
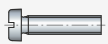
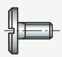
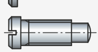
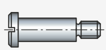
The base forms of the most common threads are shown in the following table.

Fixing thread			**Motion thread**		
					
Metric ISO thread		Whitworth pipe thread	Trapezoidal thread	Buttress thread	Round thread
Coarse pitch thread	Fine pitch thread				
DIN 13 DIN 14		DIN 2999 DIN 3858	DIN 103 DIN 263 DIN 380	DIN 513 DIN 2781	DIN 405 DIN 15403 DIN 20400

Unless otherwise indicated, these are right hand threads. A left hand thread is always marked with LH (= left hand).
The following sections specifically cover the use of screws as fixing elements.

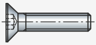
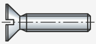
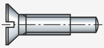
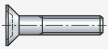
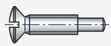
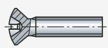
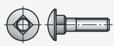
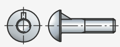
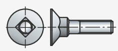
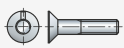
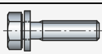
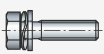
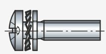
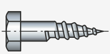
Overview of standardised screws

The following table gives an overview of standardised screws.

	DIN EN ISO 4014 DIN EN ISO 8765	Hexagon head screws Metric thread, metric fine pitch thread
	DIN EN ISO 4017 DIN EN ISO 8676	Hexagon head screws Full thread
	DIN EN ISO 4016	(Unfinished) hexagon head screws for steel structures
	DIN EN ISO 4018	(Unfinished) hexagon head screws Full thread
	DIN EN 14399-4	Hexagon head screws with large widths across flats
	DIN 561	Hexagon head set screws with full dog point
	DIN 564	Hexagon head set screws with half dog point and flat cone point
	DIN 24015 DIN 2510	Hexagon head screws with thin shank
	DIN 609	Hexagon fit screws
	DIN 7968	Hexagon fit screws for steel structures
	DIN 479	Square head screws with short dog point
	DIN 478	Square head screws with collar
	DIN 480	Square head screws with collar and short dog point with rounded end
	DIN EN ISO 4762 DIN 6912	Hexagon socket head cap screws
	DIN EN ISO 1207	Slotted cheese head screws
	DIN EN ISO 1580	Slotted pan head screws
	DIN 920	Slotted pan head screws with small head
	DIN 921	Slotted pan head screws with large head
	DIN 922	Slotted pan head screws with small head and full dog point
	DIN 923	Slotted pan head screws with shoulder
	DIN EN ISO 7045	Pan head screws with cross recess

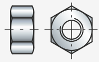
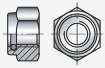
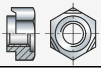
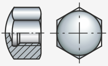
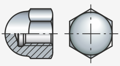
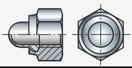
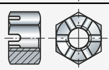
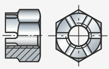
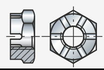
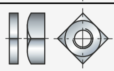
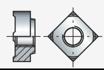
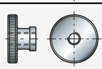
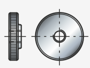
Continuation of table, see Page 458.

Continuation of table, Overview of standardised screws, from Page 457.

	DIN EN ISO 10642	Slotted countersunk head screws
	DIN EN ISO 2009	Slotted countersunk flat head screws
	DIN 925	Slotted countersunk head screws with full dog point
	DIN 7969	Slotted countersunk head screws (for steel structures)
	DIN EN ISO 7046	Countersunk flat head screws with cross recess
	DIN EN ISO 2010	Slotted raised countersunk head screws
	DIN 924	Slotted raised countersunk head screws with full dog point
	DIN EN ISO 7047	Slotted cross recess raised countersunk head screws
	DIN 603	Mushroom head square neck screws
	DIN 607	Cup head nib screws
	DIN 605 DIN 608	Flat countersunk square neck screws
	DIN 604	Flat countersunk nib screws
	DIN 404	Slotted capstan screws
	DIN EN ISO 10644 DIN EN ISO 10673	Screw and washer assemblies
	DIN 6900-2 DIN 6904	Screw and washer assemblies Curved spring washers
	DIN 6900-4 DIN 6907	Screw and washer assemblies Serrated lock washers
	DIN EN ISO 1479	Self-tapping screws
	DIN 7513 DIN 7516	Thread cutting screws
	DIN 571	Wood screws

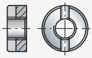
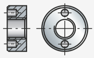
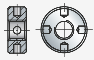
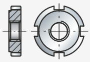
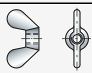
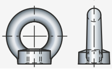
Overview of standardised nuts

The following table gives an overview of standardised nuts.

	DIN EN ISO 4032 DIN EN ISO 4034 DIN EN ISO 4035	(Unfinished) hexagon nuts Hexagon nuts Metric thread, metric fine pitch thread Flat hexagon nuts
	DIN EN ISO 7040 DIN EN ISO 7042 DIN EN ISO 10511	Hexagon nuts Self-locking
	DIN 929	Hexagon weld nuts
	DIN 917	Cap nuts Low type
	DIN 1587	Cap nuts High type
	DIN 986	Cap nuts Self-locking
	DIN 935 to M10	Hexagon castle nuts Metric thread, metric fine pitch thread
	DIN 935 to M10	Hexagon castle nuts Metric thread, metric fine pitch thread
	DIN 979	Hexagon thin castle nuts
	DIN 557 DIN 562	(Unfinished) square nuts Flat square nuts
	DIN 928	Square weld nuts
	DIN 466	Knurled nuts, high type
	DIN 467	Knurled nuts, flat type

Continuation of table, see Page 460.

Continuation of table, Overview of standardised nuts, from Page 459.

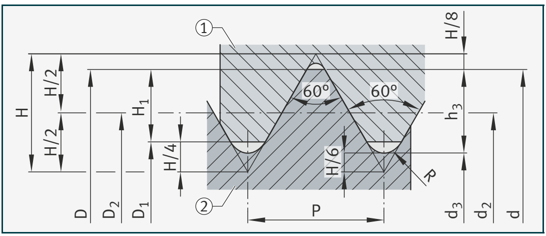
	DIN 546	Slotted round nuts
	DIN 547	Round nuts with drilled holes in one face
	DIN 548 DIN 1816	Round nuts with set pin holes in side
	DIN 981 DIN 1804	Locknuts
	DIN 315	Wing nuts
	DIN 582	Lifting eye nuts

Metric ISO threads

The following diagram and the subsequent table describe metric ISO threads in accordance with DIN 13-1 using the following example: coarse pitch threads with nominal thread diameter from 1 to 52 mm.

Figure 18
Nominal thread diameter $d = D = 12$ mm: M12

① Nut thread
② Bolt thread



Legend

$D_1 = d - 2\,H_1$	$H_1 = 0{,}541\,27\,P$
$d_2 = D_2 = d - 0{,}649\,52\,P$	$h_3 = 0{,}613\,43\,P$
$d_3 = d - 1{,}226\,87\,P$	$R = \dfrac{H}{6} = 0{,}144\,34\,P$
$H = 0{,}866\,03\,P$	

Table of coarse pitch threads with nominal thread diameter from 1 to 52 mm:

Nominal thread diameter d = D			Pitch	Flank diameter	Core diameter		Thread depth		Rounding
Series 1	Series 2	Series 3	P	$d_2 = D_2$	d_3	D_1	h_3	H_1	R
1	–	–	0,25	0,838	0,693	0,729	0,153	0,135	0,036
–	1,1	–	0,25	0,938	0,793	0,829	0,153	0,135	0,036
1,2	–	–	0,25	1,038	0,893	0,929	0,153	0,135	0,036
–	1,4	–	0,3	1,205	1,032	1,075	0,184	0,162	0,043
1,6	–	–	0,35	1,373	1,171	1,221	0,215	0,189	0,051
–	1,8	–	0,35	1,573	1,371	1,421	0,215	0,189	0,051
2	–	–	0,4	1,740	1,509	1,567	0,245	0,217	0,058
–	2,2	–	0,45	1,908	1,648	1,713	0,276	0,244	0,065
2,5	–	–	0,45	2,208	1,948	2,013	0,276	0,244	0,065
3	–	–	0,5	2,675	2,378	2,459	0,307	0,271	0,072
–	3,5	–	0,6	3,110	2,764	2,850	0,368	0,325	0,087
4	–	–	0,7	3,545	3,141	3,424	0,429	0,379	0,101
–	4,5	–	0,75	4,013	3,580	3,688	0,460	0,406	0,108
5	–	–	0,8	4,480	4,019	4,134	0,491	0,433	0,115
6	–	–	1	5,350	4,773	4,917	0,613	0,541	0,144
–	–	7	1	6,350	5,773	5,917	0,613	0,541	0,144
8	–	–	1,25	7,188	6,466	6,647	0,767	0,677	0,015
–	–	9	1,25	8,168	7,466	7,647	0,767	0,677	0,144
10	–	–	1,5	9,026	8,160	8,376	0,920	0,812	0,217
–	–	11	1,5	10,026	9,160	9,376	0,920	0,812	0,217
12	–	–	1,75	10,863	9,853	10,106	1,074	0,947	0,253
–	14	–	2	12,701	11,546	11,835	1,227	1,083	0,289
16	–	–	2	14,701	13,546	13,835	1,227	1,083	0,289
–	18	–	2,5	16,376	14,933	15,294	1,534	1,353	0,361
20	–	–	2,5	18,376	16,933	17,294	1,534	1,353	0,361
–	22	–	2,5	20,376	18,933	19,294	1,534	1,353	0,361
24	–	–	3	22,051	20,319	20,752	1,840	1,624	0,433
–	27	–	3	25,051	23,319	23,752	1,840	1,624	0,433
30	–	–	3,5	27,727	25,706	26,211	2,147	1,894	0,505
–	33	–	3,5	30,727	28,706	29,211	2,147	1,894	0,505
36	–	–	4	33,402	31,093	31,670	2,454	2,165	0,577
–	39	–	4	36,402	34,093	34,670	2,454	2,165	0,577
42	–	–	4,5	39,077	36,479	37,129	2,760	2,436	0,650
–	45	–	4,5	42,077	39,479	40,129	2,760	2,436	0,650
48	–	–	5	44,752	41,866	42,587	3,067	2,706	0,722
–	52	–	5	48,752	45,866	46,587	3,067	2,706	0,722

Selection of coarse pitch and fine pitch threads

A selection of coarse pitch and fine pitch threads in accordance with DIN 13-2 is given below.

Nominal thread diameter d = D			Pitch P for								
			Coarse pitch thread	Fine pitch thread							
Series 1	Series 2	Series 3		4	3	2	1,5	1,25	1	0,75	0,5
1	–	–	0,25	–	–	–	–	–	–	–	–
1,2	–	–	0,25	–	–	–	–	–	–	–	–
–	1,4	–	0,3	–	–	–	–	–	–	–	–
1,6	–	–	0,35	–	–	–	–	–	–	–	–
–	1,8	–	0,35	–	–	–	–	–	–	–	–
2	–	–	0,4	–	–	–	–	–	–	–	–
–	2,2	–	0,45	–	–	–	–	–	–	–	–
2,5	–	–	0,45	–	–	–	–	–	–	–	–
3	–	–	0,5	–	–	–	–	–	–	–	–
–	3,5	–	0,6	–	–	–	–	–	–	–	–
4	–	–	0,7	–	–	–	–	–	–	–	0,5
5	–	–	0,8	–	–	–	–	–	–	–	0,5
6	–	–	1	–	–	–	–	–	–	0,75	0,5[1)]
8	–	–	1,25	–	–	–	–	–	1	0,75	0,5[1)]
10	–	–	1,5	–	–	–	–	1,25	1	0,75	–
12	–	–	1,75	–	–	–	1,5	1,25	1	–	–
–	14	–	2	–	–	–	1,5	1,25	1	–	–
–	–	15	–	–	–	–	1,5	–	1	–	–
16	–	–	2	–	–	–	1,5	–	1	–	–
–	–	17	–	–	–	–	–	–	1	–	–
–	18	–	2,5	–	–	2	1,5	–	1	–	–
20	–	–	2,5	–	–	2	1,5	–	1	–	–
–	22	–	2,5	–	–	2	1,5	–	1	–	–
24	–	–	3	–	–	2	1,5	–	1	–	–
–	–	25	–	–	–	–	1,5	–	–	–	–
–	–	26	–	–	–	–	1,5	–	–	–	–
–	27	–	3	–	–	2	1,5	–	–	–	–

Continuation of table, see Page 463.

1) Not included in ISO 261:1998.

Continuation of table, Selection of coarse pitch and fine pitch threads, from Page 462.

Nominal thread diameter			Pitch P for								
d = D			Coarse pitch thread	Fine pitch thread							
Series 1	Series 2	Series 3		4	3	2	1,5	1,25	1	0,75	0,5
–	–	28	–	–	–	–	1,5	–	–	–	–
30	–	–	3,5	–	–	2	1,5	–	–	–	–
–	–	32	–	–	–	–	1,5	–	–	–	–
–	33	–	3,5	–	–	2	1,5	–	–	–	–
–	–	35	–	–	–	–	1,5	–	–	–	–
36	–	–	4	–	3	2	1,5	–	–	–	–
–	–	38	–	–	–	–	1,5	–	–	–	–
–	39	–	4	–	3	2	–	–	–	–	–
–	–	40	–	–	–	–	1,5	–	–	–	–
42	–	–	4,5	–	3	2	1,5	–	–	–	–
–	45	–	4,5	–	3	2	1,5	–	–	–	–
48	–	–	5	–	3	2	1,5	–	–	–	–
–	–	50	–	–	–	–	1,5	–	–	–	–
–	52	–	5	–	3	2	1,5	–	–	–	–
–	–	55	–	–	–	2	1,5	–	–	–	–
56	–	–	5,5	4	3	2	1,5	–	–	–	–
–	–	58	–	–	–	–	1,5	–	–	–	–
–	60	–	5,5	4	3	2	1,5	–	–	–	–
64	–	–	6	4	3	2	–	–	–	–	–
–	–	65	–	–	–	2	–	–	–	–	–
–	68	–	6	4	3	2	–	–	–	–	–

Grades for screws

Mechanical characteristics of screws are subdivided into grades. The grades are identified by two numbers that are separated by a full stop. The first number is 1/100 of the minimum tensile strength in N/mm^2. The second number is 10 times the ratio between the yield strength and the minimum tensile strength of the screw material. The following table shows an excerpt from DIN EN ISO 898-1.

		Grades									
							8.8				
		4.6	4.8	5.6	5.8	6.8	≤M16	>M16[1)]	9.8	10.9	12.9
Tensile strength R_m N/mm^2	nom.	400	400	500	500	600	800	800	900	1 000	1 200
	min.	400	420	500	520	600	800	830	900	1 040	1 220
Yield strength R_{eL} or 0,2% proof stress $R_{p0.2}$ or 0,0048 d proof stress for whole screws R_{pf} MPa = N/mm^2	nom.	240	320	300	400	480	640	640	720	900	1 080
	min.	240	340	300	420	480	640	660	720	940	1 100
Elongation at fracture A %	nom.	22	–	20	–	–	12	12	10	9	8
Vickers hardness HV F ≥ 98 N	min.	120	130	155	160	190	250	255	290	320	385
	max.	220	220	220	220	250	320	335	360	380	435
Brinell hardness HBW F = 30 D^2	min.	114	124	147	152	181	245	250	286	316	380
	max.	209	209	209	209	238	316	331	355	375	429
Notched bar impact work (ISO-U) Joule	min.	–	–	27	–	–	27	27	27	27	–

1) For bolting of steel structures, the limit is 12 mm.

Marking

At or above M5, the marking of the grade is applied directly on the screw head (end face or cylindrical face) or in the case of grub screws, on the shank. The markings must conform to the specifications in the following table. The full stop in the marking may be omitted in this case. Examples of marking are shown in Figure 19, Page 465.

	Grades								
	4.6	4.8	5.6	5.8	6.8	8.8	9.8	10.9	12.9
Marking of grade									
For screws with full load carrying capacity	4.6	4.8	5.6	5.8	6.8	8.8	9.8	10.9	12.9
For screws with reduced load carrying capacity	04.6	04.8	05.6	05.8	06.8	08.8	09.8	010.9	012.9

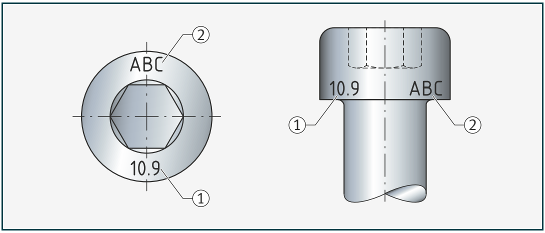


Figure 19
Examples of the marking of grades for screws of M5 and larger

① Marking of grade
② Manufacturer's symbol

In the case of smaller screws or a special head shape, marking may be carried out using symbols in accordance with the clockface system for marking of screws in DIN EN ISO 898-1.

Grades for nuts The following table lists nuts with coarse pitch thread in accordance with DIN EN 20898-2.

Nut grade	Matching screw				Nut	
					Type 1	Type 2
	Grade			Value	Value	
4	3.6	4.6	4.8	>M16	>M16	–
5	3.6	4.6	4.8	≤M16	≤M39	–
	5.6	5.8	–	≤M39		
6	6.8			≤M39	≤M39	–
8	8.8			≤M39	≤M39	>M16 ≤M39
9	9.8			≤M16	–	≤M16
10	10.9			≤M39	≤M39	–
12	12.9			≤M39	≤M16	≤M39

Nuts with a nominal height $\geq 0{,}8 \cdot D$ (effective thread height $\geq 0{,}6 \cdot D$) are identified by a number corresponding to the highest screw grade with which the nut may be matched.

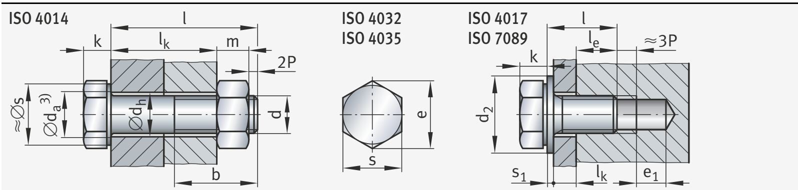
Type 1: Nut height in accordance with DIN EN 20898-2, nominal height $\geq 0{,}8 \cdot D$

Type 2: Nut height in accordance with DIN EN 20898-2 (approx. 10% greater than Type 1)

A screw with a thread M5 to M39, matched with a nut of the corresponding grade, will give a connection that can be subjected to a load up to the test force defined for the screw without stripping of the thread.

Connections using hexagon head screws

The following table shows design dimensions for connections using hexagon head screws in a selection taken from various DIN standards:



DIN EN ISO DIN EN DIN	4014, 4032 etc. 475, ISO 272		4014	4014	4017	4014	4014	4032	4035	 935	1234
Thread	Width across flats A/F	Width across corners	Height of head	Nominal length range	Nominal length range	Length of thread for $l \leqq 125$ mm	Length of thread for $l > 125 \dots 200$ mm	Height of nut Type 1	Height of nut Low type	Hexagon castle nut	Splint
d	s	e	k	l[1]	l[1]	b	b	m[2]	m	h	$d_1 \times l_1$
M3	5,5	6,01	2	20 ... 30	6 ... 30	12	18	2,4	1,8	–	–
M4	7	7,66	2,8	25 ... 40	8 ... 40	14	20	3,2	2,2	5	1 × 10
M5	8	8,79	3,5	25 ... 50	10 ... 50	16	22	4,7	2,7	6	1,2 × 12
M6	10	11,05	4	30 ... 60	12 ... 60	18	24	5,2	3,2	7,5	1,6 × 14
M8	13	14,38	5,3	40 ... 80	16 ... 80	22	28	6,8	4	9,5	2 × 16
M10	16	17,77	6,4	45 ... 100	20 ... 100	26	32	8,4	5	12	2,5 × 20
M12	18	20,03	7,5	50 ... 120	25 ... 120	30	36	10,8	6	15	3,2 × 22
M14	21	23,38	8,8	60 ... 140	30 ... 140	34	40	12,8	7	16	3,2 × 25
M16	24	26,75	10	65 ... 160	30 ... 200	38	44	14,8	8	19	4 × 28
M20	30	33,53	12,5	80 ... 200	40 ... 200	46	52	18	10	22	4 × 36
M24	36	39,98	15	90 ... 240	50 ... 200	54	60	21,5	12	27	5 × 40
M30	46	51,28	18,7	110 ... 300	60 ... 200	66	72	25,6	15	33	6,3 × 50
M36	55	61,31	22,5	140 ... 360	70 ... 200	–	84	31	18	38	6,3 × 63

All dimensions in mm; exception: A_p: mm^2.

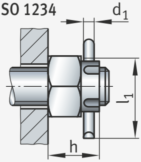
Source: Roloff/Matek, ME-Tabellenbuch, Vieweg+Teubner, 19. Auflage 2009.

1) Stepping of length l:
... 6 8 10 12 16 20 25 30 35 40 45 50 55 60 65 70 80 90 100 110 120 130 140 150 160 180 200 220 240 260 280 300 320 340 ... 500.

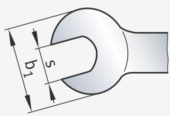
2) Higher resistance to stripping due to larger nut heights in accordance with DIN EN ISO 4033 with $m/d \approx 1$.

3) The transition diameter d_a restricts the maximum transition from the radius to the flat head seat. In accordance with DIN 267-2, the following applies in general for the product classes A(m) and B(mg) up to M18: d_a = through hole "medium" + 0,2 mm and for M20 to M39: d_a = through hole "medium" + 0,4 mm. For the product class C(g), the same equations apply with the through hole "coarse".

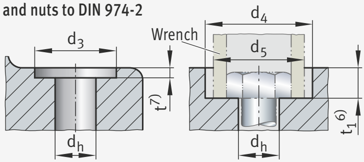
DIN 935
ISO 1234



DIN 3110



Counterbores for normal hexagon head screws and nuts to DIN 974-2



7089, 7090		20273				76	3129	3110	974-2				
Washers		Through hole[4] Series: Fine	Medium	Coarse	Head or nut seating surface	Blind hole transition (standard)	Socket wrench insert Outside diameter	Open end wrench Width	Series 1: For socket wrenches, socket wrench inserts to DIN 3124	Series 2: For offset ring wrenches, socket wrench inserts to DIN 3129	Series 3: For countersinks under restricted space conditions	For screws to ISO 4014 and ISO 4017 without support parts	For nuts to ISO 4032 and ISO 4035 without support parts and thread overhang
d_2	s_1	d_h	d_h	d_h	A_p[5]	e_1	d_5	b_1	d_3	d_3	d_3	t_1	t_1
7	0,8	3,2	3,4	3,6	7,5	2,8	9,7	19	11	11	9	2,6	2,8
9	0,8	4,3	4,5	4,8	11,4	3,8	12,8	20	13	15	10	3,4	3,6
10	1	5,3	5,5	5,8	13,6	4,2	15,3	22	15	18	11	4,1	5,1
12	1,6	6,4	6,6	7	28	5,1	17,8	27	18	20	13	4,6	5,6
16	1,6	8,4	9	10	42	6,2	21,5	34	24	26	18	6,1	7,4
20	2	10,5	11	12	72,3	7,3	27,5	38	28	33	22	7,3	9
24	2,5	13	13,5	14,5	73,2	8,3	32,4	44	33	36	26	8,4	11,4
28	2,5	15	15,5	16,5	113	9,3	36,1	49	36	43	30	9,7	13,4
30	3	17	17,5	18,5	157	9,3	42,9	56	40	46	33	10,9	15,4
37	3	21	22	24	244	11,2	50,4	66	46	54	40	13,4	18,4
44	4	25	26	28	356	13,1	64,2	80	58	73	48	16,1	22,3
56	4	31	33	35	576	15,2	76,7	96	73	82	61	20,1	26,6
66	5	37	39	42	856	16,8	87,9	–	82	83	73	23,9	32

[4] For screws of the mainly used product class A(m), to be designed using the series “medium” so that $d_h \approx d_a$.

[5] Ring-shaped seating surface determined with minimum diameter d_w of the seating surface and through hole of series “medium”. Possible reduction by the chamfering of the through hole.

[6] The countersink depth for flush closing is determined from the sum of the maximum values of the screw head height and the height of the support parts with the addition of: 0,4 mm for M3 to M6; 0,6 mm for M8 to M20; 0,8 mm for M24 to M27 and 0,1 mm at or greater than M30.
The countersink depth on the nut side must be defined including the overhang of the screw end in a suitable manner.

[7] t needs be no larger than necessary for the production by machining of a circular surface perpendicular to the axis of the through hole.

Thread runout and thread undercut

Thread runouts and thread undercuts are described in DIN 76-1. This standard is valid for screws and design parts with a metric ISO thread (coarse pitch/fine pitch thread) in accordance with DIN 13-1 and DIN ISO 261:1999.

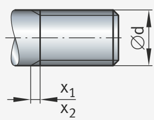
For the thread undercut on external and internal threads, type letters are included. As a result, there is no need to enter the dimensions. If no type letters are stated, standard case A or standard case C applies. Example of the designation of a thread undercut of type B: thread undercut DIN 76 – B.

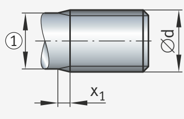
Metric external threads

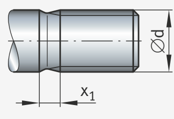
The following two tables show thread runouts and thread undercuts for metric external threads in accordance with DIN 76-1.

Thread runout

x_1 Standard case
x_2 Short
(1) Shank diameter = flank diameter

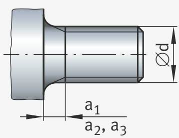


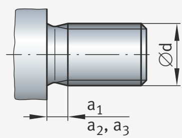




Spacing between the last full thread turn and the seating surface
(for parts with full thread)

a_1 Standard case
x_2 Short
a_3 Long

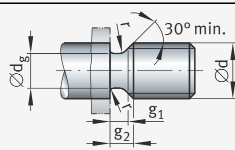




Thread undercut

Type A:
g_1 and g_2
Standard case

Type B:
g_1 and g_2
Short

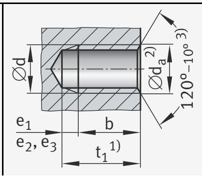
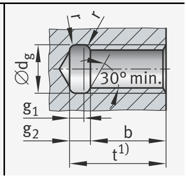


Thread pitch P	Nominal thread diameter d (coarse pitch thread)	Thread runout		Spacing			Thread undercut					
		x_1 1) max.	x_2 2) max.	a_1 3) max.	a_2 4) max.	a_3 5) max.	d_g	g_1 min.		g_2 max.		r approx.
		Standard	Short	Standard	Short	Long	h13 6)	A 7) Standard	B 8) Short	A 7) Standard	B 8) Short	
0,5	3	1,25	0,7	1,5	1	–	d – 0,8	1,1	0,5	1,75	1,25	0,2
0,6	3,5	1,5	0,75	1,8	1,2	–	d – 1	1,2	0,6	2,1	1,5	0,4
0,7	4	1,75	0,9	2,1	1,4	–	d – 1,1	1,5	0,8	2,45	1,75	0,4
0,75	4,5	1,9	1	2,25	1,5	–	d – 1,2	1,6	0,9	2,6	1,9	0,4
0,8	5	2	1	2,4	1,6	3,2	d – 1,3	1,7	0,9	2,8	2	0,4
1	6; 7	2,5	1,25	3	2	4	d – 1,6	2,1	1,1	3,5	2,5	0,6
1,25	8	3,2	1,6	3,75	2,5	5	d – 2	2,7	1,5	4,4	3,2	0,6
1,5	10	3,8	1,9	4,5	3	6	d – 2,3	3,2	1,8	5,2	3,8	0,8
1,75	12	4,3	2,2	5,25	3,5	7	d – 2,6	3,9	2,1	6,1	4,3	1

1) The thread runout x_1 is always valid if no other indications are given in the individual standards and drawings.
2) The thread runout x_2 is only valid for the cases in which a short thread runout is necessary for technical reasons.
3) The spacing a_1 is always valid if no other indications are given in the individual standards and drawings.
4) The spacing a_2 is valid for slotted screws and crosshead screws and those cases in which a short spacing is necessary for technical reasons.
5) The spacing a_3 is only valid for screws in product class C.
6) Tolerance class h12 for threads up to nominal diameter 3 mm.
7) The thread undercut of type A is always valid if no other indications are given in the individual standards and drawings. In a variation from ISO 4755, g_2 = 3,5P applies instead of 3P.
8) The thread runout of type B (short) is only valid for special cases in which a short thread runout is necessary for technical reasons. This thread undercut requires special tools for thread production. It is not included in ISO 4755.

Metric internal threads (threaded blind holes)

The following two tables show thread runouts and thread undercuts for metric internal threads (threaded blind holes) in accordance with DIN 76-1.

Threaded blind hole with thread runout		Threaded blind hole with thread undercut	
e_1 Standard case e_2 Short e_3 Long		Type C Standard case Type D Short	

[1] Permissible deviation for the calculated dimension t, t_1: +0,5 · P.

[2] $d_{a\,min} = 1 \cdot d$; $d_{a\,max} = 1{,}05 \cdot d$; countersunk diameter for nuts: see dimensional standards.

[3] The special cases 90°, 60°, ... must be indicated in the drawing. Recommendation of 60° for set screws with thread runout and for centring holes, slight countersink for set screws made from light metal.

Thread pitch P	**Nominal thread diameter d** (coarse pitch thread)	**Thread runout** including blind hole overhang			**Thread undercut**					
		e_1[1]	e_2[2]	e_3[3]	d_g	g_1 min.		g_2 max.		r approx.
		Guide values				C[4]	D[5]	C[4]	D[5]	
		Standard	**Short**	**Long**	**H13**	**Standard**	**Short**	**Standard**	**Short**	
0,5	3	2,8	1,8	4,5	d + 0,3	2	1,25	2,7	2	0,2
0,6	3,5	3,4	2,1	5,4	d + 0,3	2,4	1,5	3,3	2,4	0,4
0,7	4	3,8	2,4	6,1	d + 0,3	2,8	1,75	3,8	2,75	0,4
0,75	4,5	4	2,5	6,4	d + 0,3	3	1,9	4	2,9	0,4
0,8	5	4,2	2,7	6,8	d + 0,3	3,2	2	4,2	3	0,4
1	6; 7	5,1	3,2	8,2	d + 0,5	4	2,5	5,2	3,7	0,6
1,25	8	6,2	3,9	10	d + 0,5	5	3,2	6,7	4,9	0,6
1,5	10	7,3	4,6	11,6	d + 0,5	6	3,8	7,8	5,6	0,8
1,75	12	8,3	5,2	13,3	d + 0,5	7	4,3	9,1	6,4	1

[1] The thread runout e_1 is always valid if no other indications are given in the individual standards and drawings.

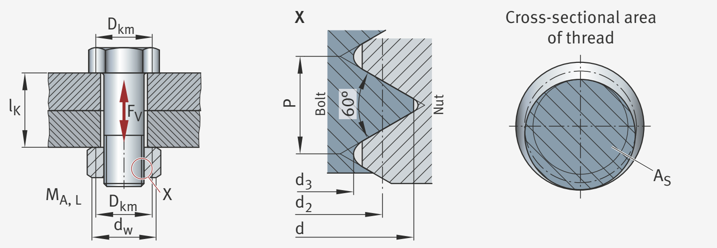
[2] The thread runout e_2 is only valid for the cases in which a short overhang is necessary for technical reasons.

[3] The thread runout e_3 is only valid for the cases in which a long overhang is necessary for technical reasons.

[4] The thread undercut of type C is always valid if no other indications are given in the individual standards and drawings.

[5] The thread runout of type D (short) is only valid for special cases in which a short thread undercut is necessary for technical reasons.

General calculation of fixing screws Screw connections can be calculated with the aid of the following relationships:



Nominal thread diameter Flank diameter Core diameter Pitch	d d_2 d_3 P } See DIN 13 page 1 ... 12 or table, Page 461
Pitch angle	$\tan \varphi = P/(d_2 \cdot \pi)$; $\varphi = \arctan P/(d_2 \cdot \pi)$
Cross-sectional area of thread	$A_S = \frac{\pi}{4}\left(\frac{d_2 + d_3}{2}\right)^2$
Mean head (nut) seat ∅	$D_{km} = (d_w + d_h)/2$, see Page 490
Coefficients of friction in thread (vee thread)	$\mu_G{}' = \tan \rho_G{}' = \frac{\mu_G}{\cos(\beta/2)}$ $\rho_G{}'$ = friction angle
in head (nut) seating surface	μ_K
Tightening torque	$M_A = M_G + M_K$ $M_A = F_V \frac{d_2}{2}\left[\tan(\varphi + \rho_G{}') + \mu_K \frac{D_{km}}{d_2}\right]$
Loosening torque	$M_L = F_V \frac{d_2}{2}\left[\tan(-\varphi + \rho_G{}') + \mu_K \frac{D_{km}}{d_2}\right]$
Thread torque during tightening	$M_G = F_V \frac{d_2}{2} \tan(\varphi + \rho_G{}')$

Continuation of table, see Page 472.

Continuation of table, General calculation of fixing screws, from Page 471.

Normal stress in thread	$\sigma_N = F_V/A_S$
Torsional stress in thread	$\tau = \dfrac{M_G}{W_P}$ where $W_P = \dfrac{\pi\left[(d_2 + d_3)/2\right]^3}{16}$
Equivalent stress in thread during tightening	$\sigma_V = \sqrt{\sigma^2{}_N + 3\tau^2} \leq \nu \cdot R_e$
Utilisation of yield stress	$\nu = 0{,}6 \ldots 0{,}9$
Coefficients of friction depending on surface treatment and lubrication[1)] (μ_G in thread, μ_K in head seating surface)	$\mu_G = 0{,}08 \ldots 0{,}20$ $\mu_K = 0{,}08 \ldots 0{,}20$

[1)] For values, see VDI Guideline 2230: Systematic calculation of high duty bolted joints.

Coefficients of friction μ_G and μ_K

In the case of various surface and lubrication conditions, the values for the coefficients of friction μ_G are as follows:

μ_G					External thread (screw)								
Thread	Material	Surface	Thread production method		Steel								
				Surface	Black annealed or phosphated				Electroplated zinc (Zn6)		Electroplated cadmium (Cd6)		Ad-hesive
				Thread production method	Rolled			Cut	Cut or rolled				
					Dry	Oiled	MoS_2	Oiled	Dry	Oiled	Dry	Oiled	Dry
Internal thread (nut)	Steel	Bright	Cut	Dry	0,12 to 0,18	0,10 to 0,16	0,08 to 0,12	0,10 to 0,16	–	0,10 to 0,18	–	0,08 to 0,14	0,16 to 0,25
	Steel	Electroplated zinc	Cut	Dry	0,10 to 0,16	–	–	–	0,12 to 0,20	0,10 to 0,18	–	–	0,14 to 0,25
	Steel	Electroplated cadmium	Cut	Dry	0,08 to 0,14	–	–	–	–	–	0,12 to 0,16	0,12 to 0,14	–
	GG/GTS	Bright	Cut	Dry	–	0,10 to 0,18	–	0,10 to 0,18	–	0,10 to 0,18	–	0,08 to 0,16	–

In the case of various surface and lubrication conditions, the values for the coefficients of friction μ_G are as follows:

<table>
<tr><th>μ_K</th><th colspan="3">Seating surface</th><th colspan="10">Screw head</th></tr>
<tr><th rowspan="5">Seating surface</th><th colspan="3">Material</th><th colspan="10">Steel</th></tr>
<tr><th rowspan="4">Material</th><th colspan="2">Surface</th><th colspan="6">Black annealed or phosphated</th><th colspan="2">Electroplated zinc (Zn6)</th><th colspan="2">Electroplated cadmium (Cd6)</th></tr>
<tr><th rowspan="3">Surface</th><th>Pro-duction</th><th colspan="3">Pressed</th><th colspan="2">Turned</th><th>Ground</th><th colspan="4">Pressed</th></tr>
<tr><th rowspan="2">Production</th><th>Lubri-cation</th><th>Dry</th><th>Oiled</th><th>MoS_2</th><th>Oiled</th><th>MoS_2</th><th>Oiled</th><th>Dry</th><th>Oiled</th><th>Dry</th><th>Oiled</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="6">Mating position</td><td rowspan="4">Steel</td><td rowspan="2">Bright</td><td>Cut</td><td rowspan="6">Dry</td><td>–</td><td>0,16 to 0,22</td><td>–</td><td>0,10 to 0,18</td><td>–</td><td>0,16 to 0,22</td><td>0,10 to 0,18</td><td>–</td><td>0,08 to 0,16</td><td>–</td></tr>
<tr><td rowspan="3">Machined by cutting</td><td>0,12 to 0,18</td><td>0,10 to 0,18</td><td>0,08 to 0,12</td><td>0,10 to 0,18</td><td>0,08 to 0,12</td><td>– –</td><td colspan="2">0,10 to 0,18</td><td>0,08 to 0,16</td><td>0,08 to 0,14</td></tr>
<tr><td>Electroplated zinc</td><td colspan="2">0,10 to 0,16</td><td>–</td><td>0,10 to 0,16</td><td>–</td><td>0,10 to 0,18</td><td>0,16 to 0,20</td><td>0,16 to 0,18</td><td>–</td><td>–</td></tr>
<tr><td>Electroplated cadmium</td><td colspan="6">0,08 to 0,16</td><td>–</td><td>–</td><td>0,12 to 0,20</td><td>0,12 to 0,14</td></tr>
<tr><td rowspan="2">GG/GTS</td><td rowspan="2">Bright</td><td>Ground</td><td>–</td><td>0,10 to 0,18</td><td>–</td><td>–</td><td>–</td><td colspan="3">0,10 to 0,18</td><td>0,08 to 0,16</td><td>–</td></tr>
<tr><td>Machined by cutting</td><td>–</td><td>0,14 to 0,20</td><td>–</td><td>0,10 to 0,18</td><td>–</td><td>0,14 to 0,22</td><td>0,10 to 0,18</td><td>0,10 to 0,16</td><td>0,08 to 0,16</td><td>–</td></tr>
</table>

Systematic calculation of heavily loaded screw connections

The following section presents the calculation of heavily preloaded screw connections (HV connections) in accordance with VDI 2230.

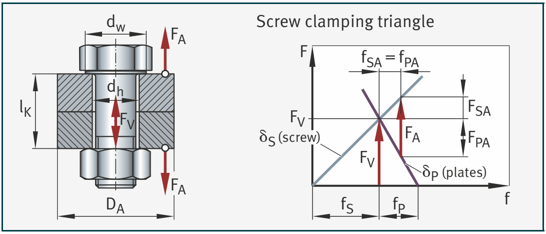


Figure 20
Screw connection and tensioning diagram

Typical applications include:

- flange connections in pipework construction
- location of crown wheels, clutch discs etc. (force locking transmission of traction forces)
- location of the cylinder head and connecting rod in the engine (flange connections with simultaneous occurrence of static and dynamic forces).

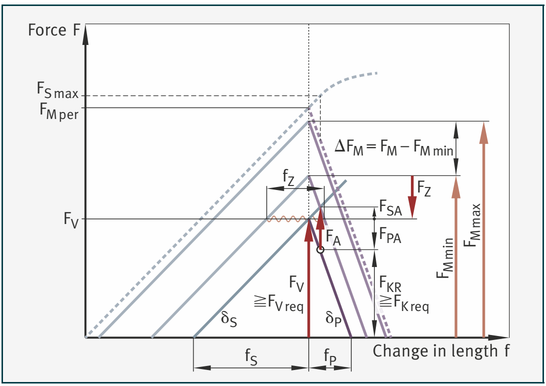
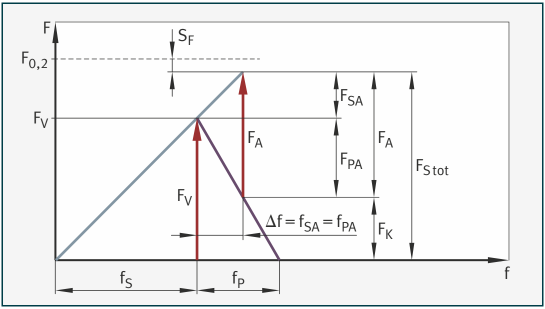


Figure 21
Dimensioning: Principal and other important values

The tensioning forces must be continuously high enough that, under the influence of operating forces, there is no unilateral lifting at the parting line and no displacement movement. The preload forces required in this case can be a multiple of the operating forces occurring.

Figure 22
Screw clamping triangle in operation

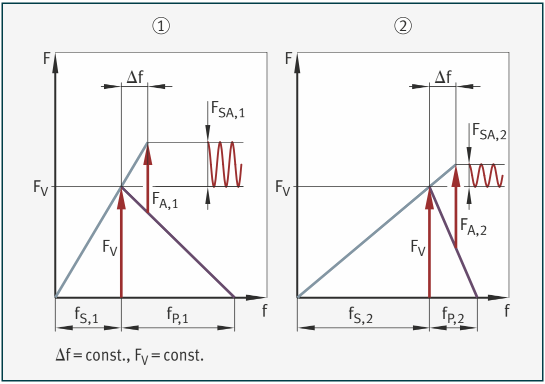
F_{PA} = proportion of the operating force that relieves parts of load
F_{PA} = proportion of the operating force that places additional load on screws
F_K = clamping force



The durability of a screw connection is all the greater, the more compliant the screw and the more rigid the flange.

Figure 23
Influence of the rigidity of screw and flange on the screw clamping triangle

① Rigid screw, compliant flange
② Compliant screw, rigid flange: advantageous for durability



The basis for screw calculation is the dimensioning formula:

Equation 5

$$
\begin{aligned}
F_{M\,max} &= \alpha_A \cdot F_{M\,min} \\
&= \alpha_A \cdot \left[F_{K\,req} + (1-\Phi) \cdot F_A + F_Z + \Delta F_{Vth}\right]
\end{aligned}
$$

Values and designations

The following table shows a selection of suitable values and designations for the calculation of heavily preloaded screw connections.

Value	Designation
A_D	Sealing surface (largest parting line area minus the through hole for the screw)
A_P	Area of screw head or nut seating surface
A_S	Cross-sectional area of screw thread in accordance with DIN 13-28
A_0	Smallest applicable cross-sectional area of screw
D_A	Substitute diameter of main body in the parting line; if the parting line area deviates from a circular form, a mean diameter must be used
D_{Km}	Effective diameter for the frictional torque in the screw head or nut seating surface
d	Screw diameter = major thread diameter (nominal diameter)
d_h	Bore diameter of clamped parts
d_w	Outside diameter of the flat head seating surface of the screw (at the entry of the radius transition from the head); in general, outside seating diameter
d_0	Diameter at the smallest applicable cross-section of the screw
d_2	Pitch diameter of screw thread
F_A	Axial force; a component of the freely aligned operating force F_B aligned to the screw axis and proportionally related to a screw
F_B	Freely aligned operating force at a connection
F_K	Clamping force
$F_{K\,req}$	Clamping force required for sealing functions, frictional locking and connection of the unilateral lifting at the parting line
F_{KP}	Minimum clamping force for ensuring sealing function
F_{KQ}	Minimum clamping force for transmission of a transverse force and/or a torque by frictional locking
F_{KR}	Residual clamping force at the parting line at application or alleviation of load by F_{PA} and after settling in operation
F_M	Mounting preload force
ΔF_M	Differential between mounting preload force F_M and minimum preload force $F_{M\,min}$

Continuation of table, see Page 477.
Source: VDI 2230.

Continuation of table on Values and designations from Page 476.

Value	Designation
F_{Mm}	Mean mounting preload force
$F_{M\,max}$	Maximum mounting preload force for which a screw must be designed in order that, despite inaccuracy of the tightening method and the anticipated settling rates in operation, the necessary clamping force at the connection is achieved and maintained
$F_{M\,min}$	Requisite minimum mounting preload force; smallest mounting preload force that can occur at $F_{M\,max}$ as a result of inaccuracy of the tightening method and maximum friction
$F_{M\,Tab}$	Tabular value for mounting preload force (see table for R7 – Determining the mounting loading $F_{M\,per}$ and checking the screw size on Page 484)
$F_{M\,per}$	Permissible mounting preload force
F_{mGM}	Stripping force of nut or internal thread
F_{mGS}	Stripping force of pin thread
F_{mS}	Breaking force of the free loaded screw thread
F_{PA}	Proportion of the axial force that changes the load on the clamped parts, additional plate force
F_Q	Transverse force; an operating force aligned perpendicular to the screw axis or its component under a freely aligned operating force F_B
F_S	Screw force
F_{SA}	Axial additional screw force
F_{SAo}	Upper (maximum) axial additional screw force
F_{SAu}	Lower (minimum) axial additional screw force
F_V	Preload force, general
ΔF_{Vth}	Change in the preload force due to a temperature other than room temperature; additional thermal force
$\Delta F'_{Vth}$	Change in the preload force due to a temperature other than room temperature (simplified); approximated additional thermal force
F_Z	Preload force loss due to settling in operation
f	Elastic length change under a force F
f_Z	Plastic deformation through settling, settling rate
k_τ	Reduction coefficient
l_K	Clamping length

Continuation of table, see Page 478.
Source: VDI 2230.

Continuation of table on Values and designations from Page 477.

Value	Designation
M_A	Tightening torque in mounting for preloading a screw to F_M
M_G	Active part of the tightening torque in the thread (thread torque)
M_Y	Torque about the screw axis
P	Pitch of thread
p	Contact pressure
p_G	Limit contact pressure, maximum permissible pressure under screw head, nut or washer
p_i	Internal pressure to be sealed off
p_M	Contact pressure in the mounted state
q_F	Number of force-transmitting (F_Q) internal parting lines involved in any sliding/shearing of the screw
q_M	Number of torque-transmitting (M_Y) internal parting lines involved in any sliding
$R_{p0,2}$	0,2% proof stress of screw in accordance with DIN EN ISO 898-1
Rz	Averaged roughness depth from at least two individual measurement distances
r_a	Frictional radius on the clamped parts under the influence of M_Y
W_P	Polar section modulus of screw cross-section
α_A	Tightening factor
δ_P	Elastic compliance of clamped parts under concentric clamping and concentric loading
δ_S	Elastic compliance of screw
μ_G	Friction coefficient in thread
μ_K	Friction coefficient in head seat
μ_T	Friction coefficient in parting line
σ_a	Fatigue loading of screw
$\sigma_{red,B}$	Equivalent stress in operating condition
σ_z	Tensile stress in screw in operating condition
τ	Torsional stress in thread due to M_G
Φ	Force ratio, relative compliance ratio
Φ_{en}	Force ratio under concentric clamping and eccentric force application via the clamped parts

Source: VDI 2230.

Calculation in accordance with VDI 2230 is carried out in individual steps (R0 to R13).

R0 – Determining the nominal diameter
Selection of screw type and grade. With the aid of the following table, this gives a screw diameter for the first draft (estimate of the diameter range of the screws).

Force N	Nominal diameter d mm Grade 8.8	 10.9	 12.9
250	–	–	–
400	–	–	–
630	–	–	–
1000	3	3	3
1600	3	3	3
2500	4	3	3
4000	5	4	4
6300	6	5	4
10000	8	6	5
16000	10	8	6
25000	12	10	8
40000	14	12	10
63000	16	14	12
100000	20	18	16
160000	24	22	20
250000	30	27	24
400000	36	33	30
630000	–	39	36

Source: VDI 2230.

R1 – Determining the tightening factor α_A
The tightening factor α_A takes account of the scatter in the achievable mounting preload force between $F_{M\,min}$ and $F_{M\,max}$:

Equation 6

$$\alpha_A = \frac{F_{M\,max}}{F_{M\,min}}$$

It is determined with the aid of guide values from the following table. The scatter is calculated as follows:

Equation 7

$$\frac{\Delta F_M}{2 \cdot F_{Mm}} = \frac{\alpha_A - 1}{\alpha_A + 1}$$

Table for determining the tightening factor α_A:

Tight-ening factor α_A	Scat-ter %	Tightening method	Setting method	Comments
1,1 to 1,2	± 5 to ± 9	Tightening with control or inspection of elongation by means of ultrasound	Sound run time	■ Calibration values necessary ■ If $l_K/d < 2$, pay attention to progressive increase in errors ■ Smaller errors with direct mechanical coupling, larger errors with indirect coupling
1,1 to 1,3	± 5 to ± 13	Mechanical elongation by pressure screws arranged in the nut or screw head	Specification for elongation of the screw, setting by means of the extraction torque of the pressure screws	■ Hardened seating washer for supporting the pressure screws ■ from approx. M24
1,2 to 1,5	± 9 to ± 20	Mechanical elongation by multi-piece nut with threaded bush	Torque of tightening tool	■ Substantially torsion-free screw mounting ■ from approx. M30

Continuation of table, see Page 481.

Source: VDI 2230.

Note: Smaller tightening factors are possible in specific cases. They require increased work in relation to the setting method, the quality of the tool or the quality of connectors and components.

Continuation of table on R1 – Determining the tightening factor α_A from Page 480.

Tightening factor α_A	Scatter %	Tightening method	Setting method	Comments
1,1 to 1,5	±5 to ±20	Tightening with mechanical elongation measurement or inspection	Direct method: setting by means of length measurement Indirect method: axial clearance to control pin used up	■ Necessary: Precise determination of the proportional axial elastic compliance of the screw ■ Scatter essentially dependent on the accuracy of the measurement method ■ Calibration necessary for low values ■ If $l_K/d < 2$, pay attention to progressive increase in errors
1,1 to 1,4	±5 to ±17	Hydraulic friction-free and torsion-free tightening	Setting by means of compression or elongation measurement or prevailing angle of nut	■ If $l_K/d \geqq 5$, smaller values can be achieved, in the case of machined screw and plates $\alpha_A = 1{,}05$ is possible ■ In the case of standard screws and nuts $\alpha_A \geqq 1{,}2$ ■ Smaller clamping length ratios lead to larger α_A values ■ Spring-back losses occur that are not considered in the tightening factor ■ Application from M20
1,2 to 2,0	±9 to ±33	Impulse wrench with hydraulic impulse cell, torque-controlled and/or angle-controlled	Setting by means of rotation angle or prevailing torque	■ Small values only with presetting to the screw joint by means of rotation angle, compressed air servo valve and impulse count ■ In special cases, over-elastic mounting is also possible
1,2 to 1,4	±9 to ±17	Yield strength-controlled tightening, motorised or manual	Specification of relative torque/ angle coefficient	The scatter in preload force is essentially determined by the scatter in yield strength of the screw batch fitted. The screws are dimensioned in this case for $F_{M\,min}$; design of the screws for $F_{M\,max}$ with the tightening factor α_A is therefore omitted for these tightening methods.
1,2 to 1,4	±9 to ±17	Angle-controlled tightening, motorised or manual	Test-based determination of initial tightening torque and rotation angle (steps)	
1,4 to 1,6	±17 to ±23	Torque-controlled tightening using hydraulic tool	Setting by means of pressure measurement	At or above approx. M30

Continuation of table, see Page 482.

Source: VDI 2230.

Note: Smaller tightening factors are possible in specific cases. They require increased work in relation to the setting method, the quality of the tool or the quality of connectors and components.

Continuation of table on R1 – Determining the tightening factor α_A from Page 481.

Tightening factor α_A	Scatter %	Tightening method	Setting method	Comments	
1,4 to 1,6	±17 to ±23	Torque-controlled tightening with a torque wrench, signal-emitting wrench or motorised screwdriver with dynamic torque measurement	Test-based determined of the nominal tightening torques on the original screw mounting part (for example by elongation measurement of the screw)	Low values: Large number of setting or control tests (for example 20) required; small scatter of outputted torque (for example ±5%) necessary	Low values: ■ for small rotation angles, (relatively rigid connections) ■ for relatively low hardness of the mating position[1] ■ for mating positions without a tendency to "fretting" (for example with phosphate coating or adequate lubrication) High values: ■ for large rotation angles, (relatively compliant connections) as well as fine pitch threads ■ for high hardness of the mating position combined with a rough surface
1,6 to 2,0[2] 1,7 to 2,5[3]	±23 to ±33 ±26 to ±43	Torque-controlled tightening with a torque wrench, signal-emitting wrench or motorised screwdriver with dynamic torque measurement	Determination of the nominal tightening torque by estimation of the friction coefficient (high influence of surface and lubrication conditions)	■ Low values: for measuring torque wrench with uniform tightening and for precision screwdriver ■ High values: for signal-emitting or buckling torque wrenches	
2,5 to 4	±43 to ±60	Tightening with impact wrench, "choke" wrench or impulse wrench; tightening by hand	Setting of the screwdriver by the retightening torque that results from the nominal tightening torque (for the estimated friction coefficient) and an allowance; hand tightening on a subjective basis	Low values: ■ for large number of setting tests (retightening torque) ■ on horizontal axis of screw characteristic diagram ■ for clearance-free impulse transmission Method only suitable for preliminary tightening; with tightening by hand, risk of stretching with M10 and smaller	

Source: VDI 2230.

[1] Mating position: clamped part whose surface is in contact with the tightening element of the connection (screw head or nut).

[2] Friction coefficient class B

[3] Friction coefficient class A

Note: Smaller tightening factors are possible in specific cases. They require increased work in relation to the setting method, the quality of the tool or the quality of connectors and components.

R2 – Determining the requisite minimum clamping force
This gives frictional locking for transmission of the transverse force F_Q and/or a torque M_Y about the screw axis:

Equation 8

$$F_{KQ} = \frac{F_{Q\ max}}{q_F \cdot \mu_{T\ min}} + \frac{M_{Y\ max}}{q_M \cdot r_a \cdot \mu_{T\ min}}$$

or for sealing against a medium:

Equation 9

$$F_{KP} = A_D \cdot p_{i\ max}$$

R3 – Subdivision of operating force
Force ratio:

Equation 10

$$\Phi = \frac{F_{SA}}{F_A}$$

R4 – Changing the preload force
Due to settling, there is a preload force loss:

Equation 11

$$F_Z = \frac{f_Z}{\delta_S + \delta_P}$$

Guide values for the settling rates can be found in the following table.

Mean roughness depth in accordance with ISO 4287[1)] Rz µm	Load	Guide values for settling rates		
		in thread µm	per head or nut seating surface µm	per internal parting line µm
<10	Tension/ compression	3	2,5	1,5
	Thrust	3	3	2
10 to <40	Tension/ compression	3	3	2
	Thrust	3	4,5	2,5
40 to <160	Tension/ compression	3	4	3
	Thrust	3	6,5	3,5

Source: VDI 2230.

1) Mean value of the maximum roughness depths Rt from at least two individual measurement distances. If five individual measurement distances are used, Rz corresponds with good approximation to the old R_z in accordance with DIN 4768.

R5 – Determining the minimum mounting preload force

The minimum mounting preload force $F_{M\ min}$ is determined as follows:

Equation 12

$$F_{M\ min} = F_{K\ req} + \left(1 - \Phi^{*}_{en}\right) \cdot F_{A\ max} + F_Z + \Delta F'_{Vth}$$

R6 – Determining the maximum mounting preload force

The maximum mounting preload force $F_{M\ max}$ is determined as follows:

Equation 13

$$F_{M\ max} = \alpha_A \cdot F_{M\ min}$$

R7 – Determining the mounting loading $F_{M\ per}$ and checking the screw size

With 90% utilisation of the minimum yield strength $R_{p0,2\ min}$, the mounting preload force based on $F_{M\ per} = F_{M\ Tab}$ can be taken from the following table. The table is valid for coarse pitch threads; for further tables, see VDI 2230.

Coarse pitch thread															
Size	Grade	Mounting preload force $F_{M\ Tab}$ kN							Tightening torque M_A Nm						
		μ_G =							$\mu_K = \mu_G$ =						
		0,08	0,10	0,12	0,14	0,16	0,20	0,24	0,08	0,10	0,12	0,14	0,16	0,20	0,24
M4	8.8	4,6	4,5	4,4	4,3	4,2	3,9	3,7	2,3	2,6	3,0	3,3	3,6	4,1	4,5
	10.9	6,8	6,7	6,5	6,3	6,1	5,7	5,4	3,3	3,9	4,6	4,8	5,3	6,0	6,6
	12.9	8,0	7,8	7,6	7,4	7,1	6,7	6,3	3,9	4,5	5,1	5,6	6,2	7,0	7,8
M5	8.8	7,6	7,4	7,2	7,0	6,8	6,4	6,0	4,4	5,2	5,9	6,5	7,1	8,1	9,0
	10.9	11,1	10,8	10,6	10,3	10,0	9,4	8,8	6,5	7,6	8,6	9,5	10,4	11,9	13,2
	12.9	13,0	12,7	12,4	12,0	11,7	11,0	10,3	7,6	8,9	10,0	11,2	12,2	14,0	15,5
M6	8.8	10,7	10,4	10,2	9,9	9,6	9,0	8,4	7,7	9,0	10,1	11,3	12,3	14,1	15,6
	10.9	15,7	15,3	14,9	14,5	14,1	13,2	12,4	11,3	13,2	14,9	16,5	18,0	20,7	22,9
	12.9	18,4	17,9	17,5	17,0	16,5	15,5	14,5	13,2	15,4	17,4	19,3	21,1	24,2	26,8
M7	8.8	15,5	15,1	14,8	14,4	14,0	13,1	12,3	12,6	14,8	16,8	18,7	20,5	23,6	26,2
	10.9	22,7	22,5	21,7	21,1	20,5	19,3	18,1	18,5	21,7	24,7	27,5	30,1	34,7	38,5
	12.9	26,6	26,0	25,4	24,7	24,0	22,6	21,2	21,6	25,4	28,9	32,2	35,2	40,6	45,1
M8	8.8	19,5	19,1	18,6	18,1	17,6	16,5	15,5	18,5	21,6	24,6	27,3	29,8	34,3	38,0
	10.9	28,7	28,0	27,3	26,6	25,8	24,3	22,7	27,2	31,8	36,1	40,1	43,8	50,3	55,8
	12.9	33,6	32,8	32,0	31,1	30,2	28,4	26,6	31,8	37,2	42,2	46,9	51,2	58,9	65,3
M10	8.8	31,0	30,3	29,6	28,8	27,9	26,3	24,7	36	43	48	54	59	68	75
	10.9	45,6	44,5	43,4	42,2	41,0	38,6	36,2	53	63	71	79	87	100	110
	12.9	53,3	52,1	50,8	49,4	48,0	45,2	42,4	73	73	83	93	101	116	129
M12	8.8	45,2	44,1	43,0	41,9	40,7	38,3	35,9	63	73	84	93	102	117	130
	10.9	66,3	64,8	63,2	61,5	59,8	56,3	52,8	92	108	123	137	149	172	191
	12.9	77,6	75,9	74,0	72,0	70,0	65,8	61,8	108	126	144	160	175	201	223

Continuation of table, see Page 485.

Source: VDI 2230.

Continuation of table on R7 – Determining the mounting loading $F_{M\ per}$ and checking the screw size from Page 484.

Coarse pitch thread															
Dimension	Grade	Mounting preload force $F_{M\ Tab}$ kN							Tightening torque M_A Nm						
		μ_G =							$\mu_K = \mu_G$ =						
		0,08	0,10	0,12	0,14	0,16	0,20	0,24	0,08	0,10	0,12	0,14	0,16	0,20	0,24
M14	8.8	62,0	60,6	59,1	57,5	55,9	52,6	49,3	100	117	133	148	162	187	207
	10.9	91,0	88,9	86,7	84,4	82,1	77,2	72,5	146	172	195	218	238	274	304
	12.9	106,5	104,1	101,5	98,8	96,0	90,4	84,4	171	201	229	255	279	321	356
M16	8.8	84,7	82,9	80,9	78,8	76,6	72,2	67,8	153	180	206	230	252	291	325
	10.9	124,4	121,7	118,8	115,7	112,6	106,1	99,6	224	264	302	338	370	428	477
	12.9	145,5	142,4	139	135,4	131,7	124,1	116,6	262	309	354	395	433	501	558
M18	8.8	107	104	102	99	96	91	85	220	259	295	329	360	415	462
	10.9	152	149	145	141	137	129	121	314	369	421	469	513	592	657
	12.9	178	174	170	165	160	151	142	367	432	492	549	601	692	769
M20	8.8	136	134	130	127	123	116	109	308	363	415	464	509	588	655
	10.9	194	190	186	181	176	166	156	438	517	592	661	725	838	933
	12.9	227	223	217	212	206	194	182	513	605	692	773	848	980	1 092
M22	8.8	170	166	162	158	154	145	137	417	495	567	634	697	808	901
	10.9	242	237	231	225	219	207	194	595	704	807	904	993	1 151	1 284
	12.9	283	277	271	264	257	242	228	696	824	945	1 057	1 162	1 347	1 502
M24	8.8	196	192	188	183	178	168	157	529	625	714	798	875	1 011	1 126
	10.9	280	274	267	260	253	239	224	754	890	1 017	1 136	1 246	1 440	1 604
	12.9	327	320	313	305	296	279	262	882	1 041	1 190	1 329	1 458	1 685	1 877
M27	8.8	257	252	246	240	234	220	207	772	915	1 050	1 176	1 292	1 498	1 672
	10.9	367	359	351	342	333	314	295	1 100	1 304	1 496	1 674	1 840	2 134	2 381
	12.9	429	420	410	400	389	367	345	1 287	1 526	1 750	1 959	2 153	2 497	2 787
M30	8.8	313	307	300	292	284	268	252	1 053	1 246	1 428	1 597	1 754	2 031	2 265
	10.9	446	437	427	416	405	382	359	1 500	1 775	2 033	2 274	2 498	2 893	3 226
	12.9	522	511	499	487	474	447	420	1 755	2 077	2 380	2 662	2 923	3 386	3 775
M33	8.8	389	381	373	363	354	334	314	1 415	1 679	1 928	2 161	2 377	2 759	3 081
	10.9	554	543	531	517	504	475	447	2 015	2 392	2 747	3 078	3 385	3 930	4 388
	12.9	649	635	621	605	589	556	523	2 358	2 799	3 214	3 601	3 961	4 598	5 135
M36	8.8	458	448	438	427	415	392	368	1 825	2 164	2 482	2 778	3 054	3 541	3 951
	10.9	652	638	623	608	591	558	524	2 600	3 082	3 535	3 957	4 349	5 043	5 627
	12.9	763	747	729	711	692	653	614	3 042	3 607	4 136	4 631	5 089	5 902	6 585
M39	8.8	548	537	525	512	498	470	443	2 348	2 791	3 208	3 597	3 958	4 598	5 137
	10.9	781	765	748	729	710	670	630	3 345	3 975	4 569	5 123	5 637	6 549	7 317
	12.9	914	895	875	853	831	784	738	3 914	4 652	5 346	5 994	6 596	7 664	8 562

Source: VDI 2230.

In order that the screw size determined approximately in step R0 can be reused, the following must apply:

Equation 14

$$F_{M\ Tab} \geq F_{M\ max}$$

Otherwise, a larger screw diameter must be selected and the calculations carried out again starting from step R2.

R8 – Determining the operating loading $\sigma_{red,B}$
For connections in which the yield strength of the screw is not to be exceeded under load, the maximum screw force should be as follows:

Equation 15

$$F_{S\ max} = F_{M\ per} + \Phi^{*}_{en} \cdot F_{A\ max} - \Delta F_{Vth}$$

For the maximum tensile stress, the following applies:

Equation 16

$$\sigma_{z\ max} = \frac{F_{S\ max}}{A_0}$$

and for the maximum torsional stress:

Equation 17

$$\tau_{max} = \frac{M_G}{W_P}$$

where $$M_G = F_{M\ per} \frac{d_2}{2} \left(\frac{P}{\pi \cdot d_2} + 1{,}155\, \mu_{G\ min} \right)$$

$$W_P = \frac{\pi}{16} d_0^3$$

For reduced stress or equivalent stress in operating condition (operating loading), the following applies:

Equation 18

$$\sigma_{red,B} = \sqrt{\sigma^2_{z\ max} + 3(k_\tau \cdot \tau_{max})^2}$$

The following must apply:

Equation 19

$$\sigma_{red,B} < R_{p0,2\ min}$$

R9 – Determining the fatigue loading σ_a

The fatigue behaviour is checked by means of:

Equation 20

$$\sigma_a = \frac{F_{SAo} - F_{SAu}}{2A_S}$$

R10 – Determining the maximum contact pressure p_{max}

The mounting preload force or maximum force in operation should not cause contact pressures that lead to creep processes and thus a loss in preload force.

For the mounted state, the following should apply:

Equation 21

$$p_{M\ max} = \frac{F_{M\ per}}{A_{P\ min}} \leq p_G$$

For yield strength-controlled or angle-controlled tightening methods, the following applies with the values for $F_{M\ Tab}$ from the table on R7 – Determining the mounting loading $F_{M\ per}$ and checking the screw size from Page 484:

Equation 22

$$p_{max} = \frac{F_{M\ Tab}}{A_{P\ min}} \cdot 1{,}4$$

R11 – Determining the minimum screw depth

In order to prevent failure due to stripping of the threads mating with each other, adequate overlap is necessary between the screw and nut threads.

The following applies:

Equation 23

$$F_{mS} \leq \min\left(F_{mGM}, F_{mGS}\right)$$

R12 – Determining the security against sliding S_G and the shear loading $\tau_{Q\ max}$

Transverse forces occurring in a screw connection must be transmitted by frictional locking.

For the minimum residual clamping force, the following applies:

Equation 24

$$F_{KR\ min} = \frac{F_{M\ per}}{\alpha_A} - \left(1 - \Phi^*_{en}\right) \cdot F_{A\ max} - F_Z - \Delta F_{Vth}$$

For the clamping force necessary to transmit the transverse forces, the following applies:

Equation 25

$$F_{KQ\ req} = \frac{F_{Q\ max}}{q_F \cdot \mu_{T\ min}} + \frac{M_{Y\ max}}{q_M \cdot r_a \cdot \mu_{T\ min}}$$

R13 – Determining the tightening torque M_A

The tightening torque is calculated as follows:

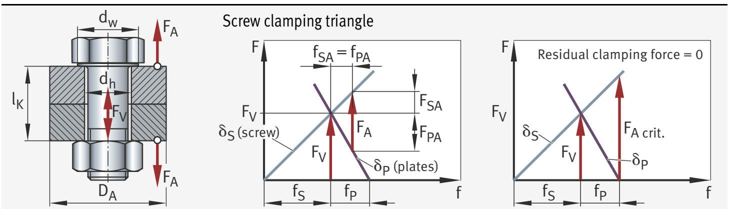
Equation 26

$$M_A = F_{M\ per}\left(0{,}16 \cdot P + 0{,}58 \cdot d_2 \cdot \mu_{G\ min} + \frac{D_{Km}}{2} \cdot \mu_{K\ min}\right)$$

For torque-controlled tightening, the torque required can be taken from the relevant tables.

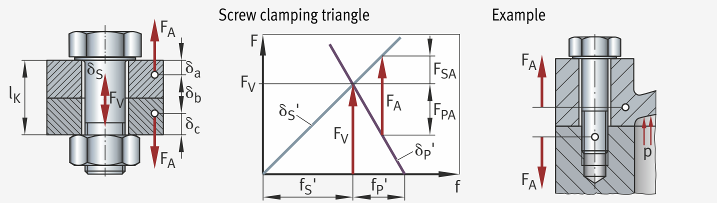
The following tables show some calculation examples.

Example 1:



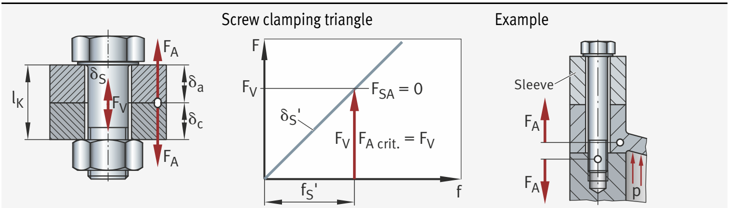
Application of a static operating force F_A under the screw head and the nut		
Additional loading of the screw due to F_A	$F_{SA} = F_A \cdot \delta_P/(\delta_S + \delta_P)$	
Load alleviation of clamped parts due to F_A	$F_{PA} = F_A \cdot \delta_S/(\delta_S + \delta_P)$	
Additional elongation of the screw due to F_A	$f_{SA} = F_{SA} \cdot \delta_S$	$f_{SA} = f_{PA}$
Spring-back of clamped parts due to F_A	$f_{PA} = F_{PA} \cdot \delta_P$	$f_{PA} = f_{SA}$
Critical operating force at which the residual clamping force of the parts becomes zero	$F_{A\ crit.} = F_V\,(1 + \delta_S/\delta_P)$	

Example 2:



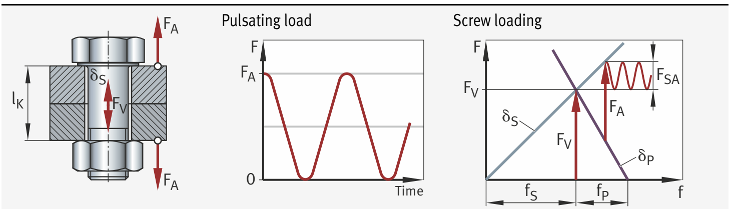
Application of the operating force F_A within the clamped parts	
Apparent elastic compliance of screw	$1/\delta_S' = 1/\delta_S + 1/\delta_a + 1/\delta_c$
Apparent elastic compliance of clamped parts	$\delta_P' = \delta_b$
With δ_S' and δ_P', the relationships on the previous page apply	

Example 3:



Application of the operating force F_A in the parting line of the clamped parts	
Apparent elastic compliance of screw	$1/\delta_S' = 1/\delta_S + 1/\delta_a + 1/\delta_c$
Apparent elastic compliance of clamped parts	$\delta_P' \rightarrow \infty$
Critical operating force at which the residual clamping force of the parts becomes zero	$F_{A\,crit.} = F_V$

Example 4:



Pulsating load on the screw connection **Application of the operating force under the screw head and the nut**	
Oscillating loading	$F_{dyn} = (F_V + F_{SA}/2) \pm F_{SA}/2$

Contact pressure in the head and nut seating surfaces

The contact pressure at the head and nut seating surfaces is calculated for standardised hexagon head and hexagon socket screws on the basis of utilisation of the yield stress of the screws.

Example 1: Seating surfaces for hexagon head screws to DIN EN ISO 4014 and nuts to DIN EN ISO 4032

Dimensions d mm	Width across flats s_{max} mm	Diameter of seating plate $d_{w\,min}$ mm	Through hole[1)] d_h mm	Seating surface A_P mm^2	Cross-sectional area of thread A_S mm^2	Contact pressure under head $p = \frac{A_S}{A_P} \cdot 0{,}7\,R_{p0,2}$ $\frac{N}{mm^2}$ 8.8	 10.9	 12.9
M3	5,5	4,6	3,4	7,54	5,03	299	439	514
M4	7	5,9	4,5	11,4	8,78	344	505	591
M5	8	6,9	5,5	13,6	14,2	467	686	802
M6	10	8,9	6,6	28	20,1	322	473	553
M8	13	11,6	9	42	36,6	390	573	670
M10	16	14,6	11	72,3	58	359	527	617
M10	17	15,6	11	96,1	58	270	397	465
M12	18	16,6	13,5	73,2	84,3	516	757	886
M12	19	17,4	13,5	94,6	84,3	399	586	686
M14	21	19,6	15,5	113	115	456	670	784
M14	22	20,5	15,5	141	115	365	535	627
M16	24	22,5	17,5	157	157	448	658	770
M18	27	25,3	20	188	192	471	670	784
M20	30	28,8	22	244	245	463	660	772
M22	32	30	24	254	303	550	784	917
M22	34	31,7	24	337	303	416	592	693
M24	36	33,6	26	356	353	459	653	764
M27	41	38	30	459	459	497	707	828
M30	46	42,7	33	561	561	450	640	749

1) In accordance with DIN EN 20273.

Example 2: Seating surfaces for hexagon socket head screws to DIN EN ISO 4762, DIN 6912

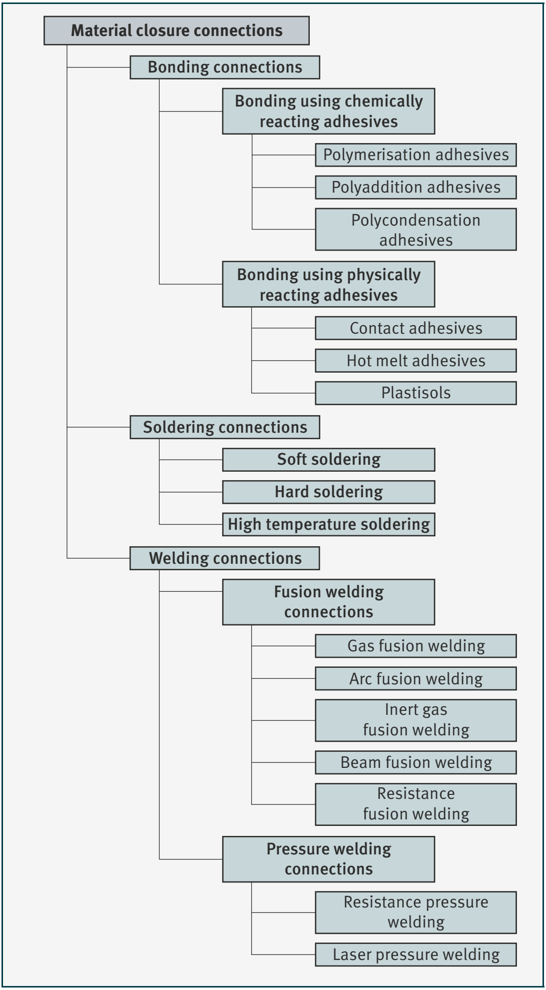
Dimensions	Width across flats	Diameter of seating plate	Through hole[1]	Seating surface	Cross-sectional area of thread	Contact pressure under head		
d	s_{max}	$d_{w\,min}$	d_h	A_P	A_S	$p = \frac{A_S}{A_P} \cdot 0{,}7\,R_{p0,2}$ $\frac{N}{mm^2}$		
mm	mm	mm	mm	mm^2	mm^2	**8.8**	**10.9**	**12.9**
M3	5,5	5,07	3,4	11,1	5,03	203	298	349
M4	7	6,53	4,5	17,6	8,78	224	329	385
M5	8,5	8,03	5,5	26,9	14,2	237	348	407
M6	10	9,38	6,6	34,9	20,1	258	379	444
M8	13	12,33	9	55,8	36,6	294	432	505
M10	16	15,33	11	89,5	58	290	426	499
M12	18	17,23	13,5	90	84,3	420	616	721
M14	21	20,17	15,5	131	115	394	579	677
M16	24	23,17	17,5	181	157	389	571	668
M18	27	25,87	20	211	192	420	298	699
M20	30	28,87	22	274	245	413	588	688
M22	33	31,81	24	342	303	409	583	682
M24	36	34,81	26	421	353	388	552	646
M27	40	38,61	30	464	459	547	651	762
M30	45	43,61	33	638	561	406	578	677

[1] In accordance with DIN EN 20273.

Material closure connections

In the case of material closure connections, the parts are connected by joining using either an additional material of a characteristic type (welding) or an additional material of a non-characteristic type (soldering, adhesive bonding). These belong to the class of non-separable connections (in other words, they cannot be separated without the occurrence of damage).

Figure 24
Material closure connections

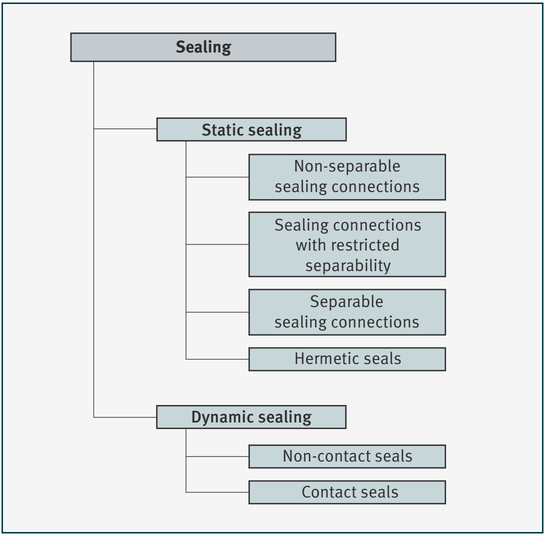


Sealing

Overview

The principal function of seals is the separation of two functionally different areas under equal or differing pressure such that there is no exchange of solid, liquid or gaseous media between these or at least that such exchange is within permissible limits (permissible leakage quantity).

Figure 25
Sealing



Examples:

- Prevention of the loss of operating materials (for example the egress of oil from bearings or of air from pneumatic lines)
- Prevention of the ingress of foreign bodies (for example, into bearings)
- Separation of different operating materials (for example, of bearing grease and lye in washing machines).

Some specific examples of sealing elements can be found in the section on rolling bearings.

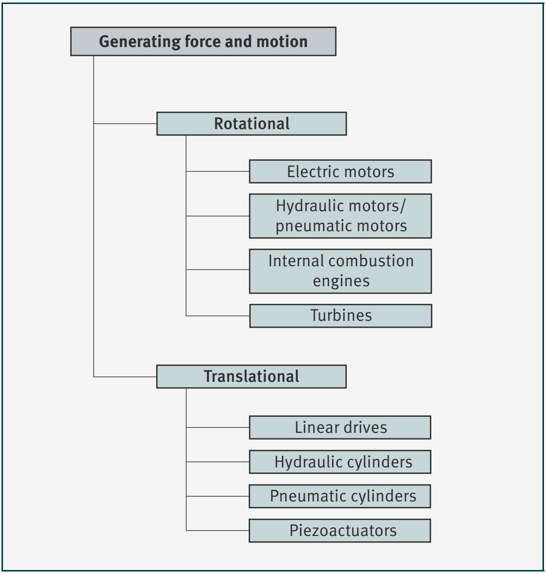
Generating force and motion

Overview

In technical systems such as machinery, plant and vehicles, it is always necessary to consider suitable drive systems as well as the actuation and adjustment functions involved. For this purpose, there is a wide spectrum available of different subsystems with a very wide range of functionality, complexity and performance capability.

Elements for generating force and motion convert particular initial forms of energy (such as flow energy of fluids, chemically bound energy in internal combustion engines, electrical energy in electric motors) into mechanical, kinetic energy (of rotational or translational character). This is always associated with conversion losses.

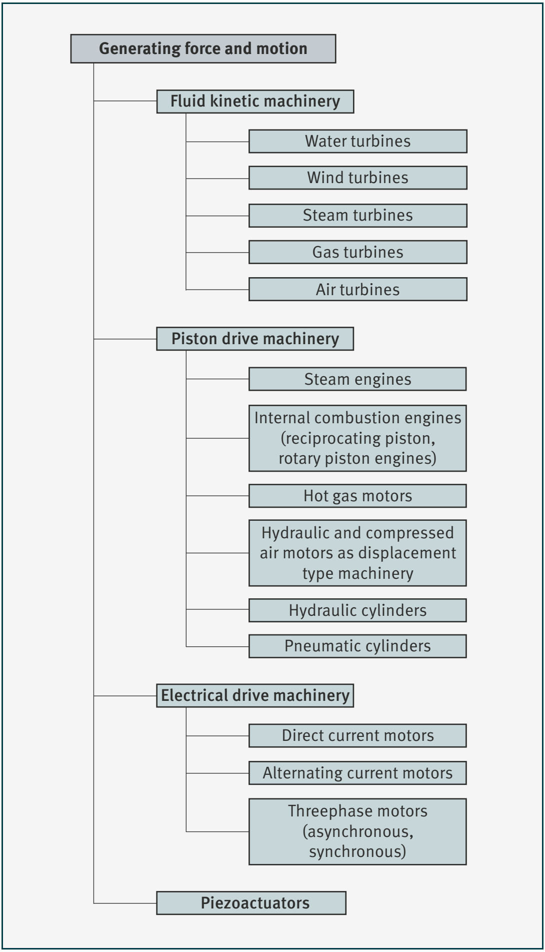
Figure 26
Generating force and motion – subdivision by type of motion



Subdivision by operating principle

In technical systems, prime movers provide the necessary drive for a wide range of driven machines. For the wide range of possible requirements, a wide spectrum of drive machines is available. These can be subdivided according to their operating principle.

Figure 27
Generating force and motion – subdivision by operating principle

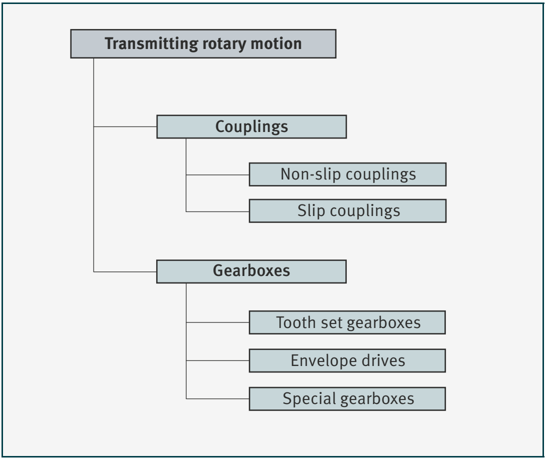


Transmitting rotary motion

Overview In technical systems (machines, plant, vehicles) it is always necessary to transmit mainly rotary motion between the drive machine and the driven machine. Couplings and gearboxes are essential components of drive systems (comprising the three subsystems: drive machine or prime mover, drive train, driven machine).

In the drive train of such a drive system, couplings and gearboxes with their various characteristics and features play a decisive role alongside the shafts, bearings and seals. In the transmission of power in the drive system, the couplings mainly take up the function of routing power, while the gearboxes are able to convert torque and speed.

Figure 28
Transmitting rotary motion



Couplings Couplings are design elements based on form fit or force locking for the connection of rotating shafts abutting each other (with or without alignment) or rotating bodies as well as the transmission of power (torque and speed). They thus have a routing function.

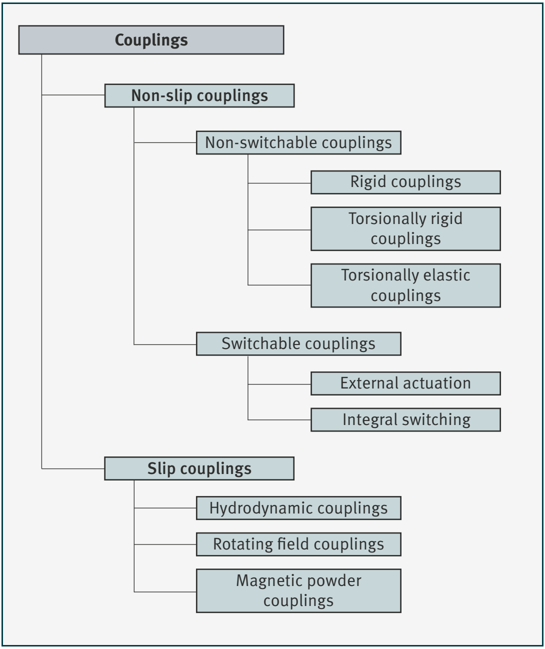
In addition to this principal function, they can take up additional functions, such as: compensation of offset (axial, radial, angular or torsional), disengagement (closure or interruption as necessary of the power flow) as well as influencing the dynamic characteristics of a drive system (for example, reduction or damping of torque shocks and the displacement of natural frequencies).

Brakes can be defined as special examples of switchable couplings in which one half of the coupling is stationary.

Characteristics:
The wide range of functions leads to a large number of different types and designs that are available in the market in various sizes depending on the torque to be transmitted. A distinction must be made between couplings based on form fit, force locking and material closure.

Properties:
Couplings are characterised by the fact that, in stationary operation, the input torque of the coupling is always equal to the output torque. In contrast, the output speed can lie between zero and the drive speed depending on the design.

Figure 29
Couplings



Torsionally rigid couplings transmit not only the necessary torque but also – without change – fluctuations in torque, shocks and vibrations that may arise from the drive machine or driven machine. In contrast, torsionally elastic couplings transmit torque by means of elastic metal or rubber spring elements, in which case they act as torsion springs and can thus reduce or damp shocks.

Switchable couplings allow the interruption and restoration of the connection between drive components as required in operational behaviour. Depending on the type of actuation, a distinction is made between switchable couplings with external actuation (by mechanical, electromagnetic, hydraulic or pneumatic means) or couplings with integral switching (actuated by speed, torque or direction).

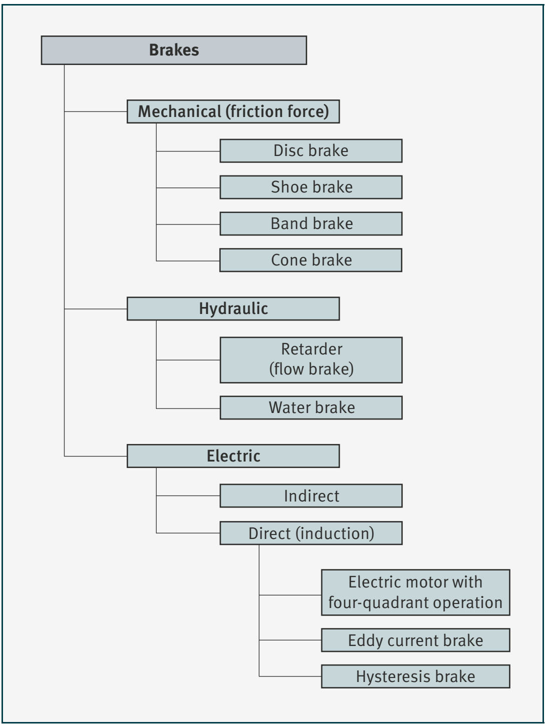
In the case of slip couplings, there is always a difference between the input speed and output speed.

Brakes

Brakes are couplings with a stationary output component. Their functions can include the following:

- locking (in one direction)
- holding (in both directions)
- stopping (halting motion)
- controlling (velocity)
- loading of prime movers (power brake).

Figure 30
Brakes



Gearboxes Gearboxes fulfil the function of speed and torque conversion.

Systematic subdivision and overview of the wide range of gearboxes is possible and advisable on the basis of the following criteria[1)]:

- Kinematics
  - uniform
  - non-uniform
- Physical principle
  - mechanical
  - hydraulic/pneumatic
  - electrical
- Operating principle
  - form fit
  - force locking
- Type of transmission
  - constant
  - stepped
  - stepless.

In practice, gearboxes fulfil different adjustment tasks, such as:

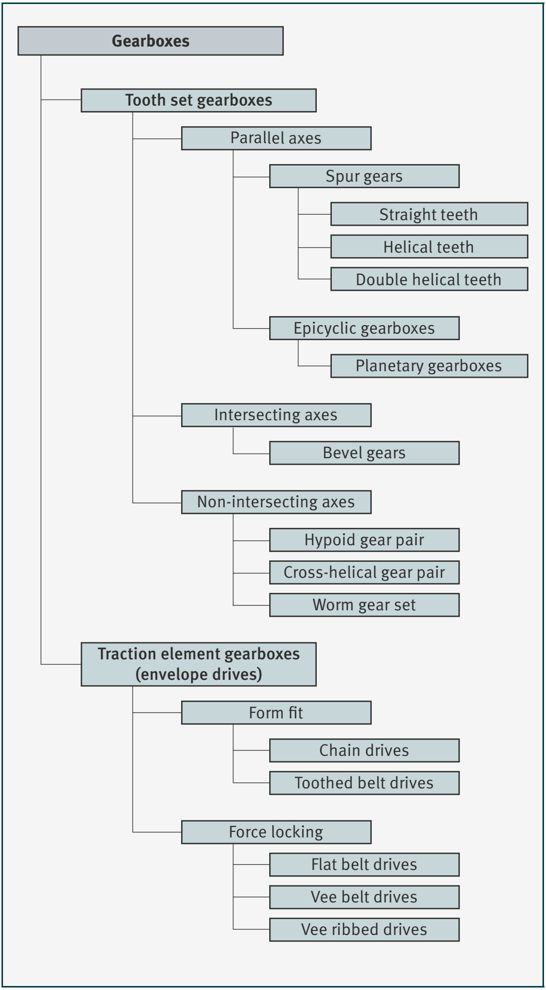
- Kinematic adjustment
  - speed
  - velocity
  - type of motion
- Geometrical adjustment
  - centre distance
  - angular position
- Adjustment of characteristics
  - operating point
  - characteristic pattern
- Adjustment of power flow
  - addition
  - division
  - conversion.

Depending on the intended purpose, a distinction can also be made between gearboxes in mobile power transmission (vehicle engineering) and in stationary power transmission (machine tools, industrial plant, wind turbines).

[1)] W. Steinhilper; B. Sauer: Konstruktionselemente des Maschinenbaus Band 2, Springer Verlag 2005.

In the case of gearboxes with a uniform transmission ratio, a distinction must be made between tooth set gearboxes and traction element gearboxes. Tooth set gearboxes can be subdivided in accordance with the arrangement of axes and the type of tooth set. Traction element gearboxes can be realised by means of form fit or force locking.

Figure 31
Gearboxes



Tooth set gearboxes
Characteristics:
Toothed gears are drive elements that transmit power in the form of rotary motion with form fit from one shaft to the other. Toothed gears are defined by the form of the base elements, the characteristics of the flank lines and the profile form[1].

Depending on the type of tooth set and the gear form or arrangement of the axes relative to each other, a distinction is made between the following:

- spur gearbox
- bevel gearbox
- helical gearbox
- hypoid gearbox
- worm gearbox.

The material predominantly used for the toothed gears is steel, which can be subjected to heat treatment in order to increase the load carrying capacity of the flanks. In pairs of toothed gears with a high sliding motion component, the mating gear is normally made from bronze in order to reduce friction. Where small loads are present, plastics can be used.

Properties:
Tooth set gearboxes are suitable for the transmission of power at a consistently high level of efficiency, while it is only in such gearboxes with a high sliding motion component that efficiency remains low. Through the combination of toothed gears with different numbers of teeth, it is possible to achieve different transmission ratios (i = ratio between input speed and output speed). Depending on the ratio between the input speed and output speed, it is possible to achieve transmission ratios to a slower or faster speed. As a result, tooth set gearboxes are highly suitable for the conversion of speeds and torques.

Envelope drives
Characteristics:
Envelope drives (also known as traction element gearboxes) comprise two or more discs or wheels that are not in contact with each other but are wrapped by means of a traction element (belt or chain)[1].

[1] W. Steinhilper; B. Sauer: Konstruktionselemente des Maschinenbaus Band 2, Springer Verlag 2005.

A distinction must be made between traction element gearboxes based on force (frictional) locking and based on form fit:

- Force locking
  - flat belt
  - vee belt
  - vee-ribbed belt
  - round belt
- Form fit
  - toothed belt
  - roller chain, pin chain, bush chain
  - toothed chain.

Properties:
Envelope drives are suitable for the conversion of torques and speeds as well as changes in direction of rotation and also for the spanning of large centre distances.

Special gearboxes
Special gearboxes are characterised by the implementation of very high transmission ratios in a small design envelope.

Examples of special gearboxes include:

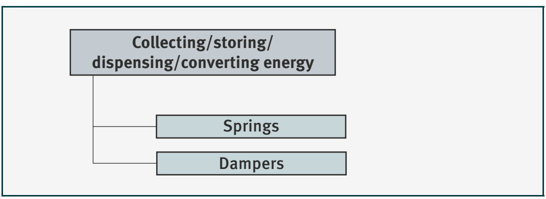
- cycloid gearboxes, such as Cyclo
- wave gearboxes, such as Harmonic Drive.

Collecting/storing/dispensing/converting energy

Overview

This category is defined as comprising elements that, due to their elastic material behaviour, appropriate form or by using the compressibility of fluids, are able to collect and store energy, dispense energy or convert energy.

Figure 32
Collecting/storing/dispensing/converting energy



Springs Springs are elastic elements that are characterised by their ability to store potential energy through contraction, expansion or torsion of the spring body and provide the energy at a later point in time in the form of work (minus the associated friction losses).

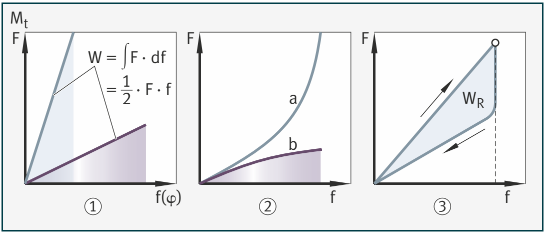
Accordingly, the use of springs covers the following functions:

- Work storage:
  the storage of potential energy (e.g. the tension spring in an air gun, the springs in mechanical timepieces)
- Force-travel converter:
  the conversion of force into travel (e.g. spring scales, expansion screws, springs in switchable and slip couplings)
- Energy converter:
  the damping of impacts and vibrations, the conversion of impact energy into thermal energy (e.g. shock absorber, buffer, rubber-bonded-to-metal elements).

The following section considers mechanical springs only. The behaviour of a spring is described by the spring diagram or the spring characteristic. This is defined as the dependence of the spring force F (or the spring torque M_t) on the deformation (change in length f or torsion angle φ).

Figure 33
Spring characteristics

① Linear characteristic line
② Curved characteristic line
a = progressive
b = degressive
③ Characteristic pattern of damping springs



For linear spring characteristics, as displayed by most metal springs, the dependence relationship is as follows:

Equation 27

$$F = c \cdot f \qquad \text{or} \qquad M_t = c_t \cdot \varphi$$

Spring rate:

Equation 28

$$c = \frac{F}{f} \qquad \text{or} \qquad c_t = \frac{M_t}{\varphi}$$

In the case of non-linear springs, a spring rate c (spring stiffness) can be specified for the operating point by means of the tangent gradient:

Equation 29

$$c = \frac{dF}{df} \qquad c_t = \frac{dM_t}{d\varphi}$$

The elastic spring work W is the energy that is stored in a spring as potential energy when under an external load. It is given by the content of the area underneath the spring characteristic line:

Equation 30

$$W = \int_0^f F\,df \quad \text{or} \quad W = \int_0^\varphi M_t\,d\varphi$$

In the case of springs with a linear characteristic line, the following elastic spring work is stored between the unloaded and the loaded state:

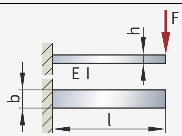
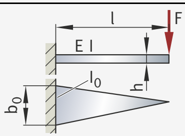
Equation 31

$$W = \frac{1}{2}\cdot c\cdot f^2 = \frac{1}{2}F\cdot f \quad \text{or} \quad W = \frac{1}{2}\cdot c\cdot\varphi^2 = \frac{1}{2}M_t\cdot\varphi$$

If a spring is repeatedly subjected to load and then relieved, and if its damping capacity is sufficient (material damping or external friction), the characteristic line for loading will differ from that for relief. The area enclosed by these two characteristic lines is a measure of the damping work W_R.

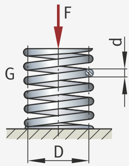
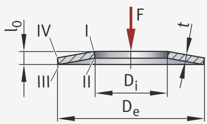
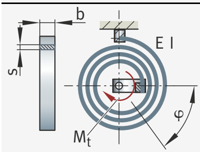
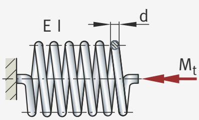
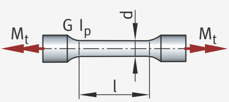
Spring rates for metallic springs

The following table shows spring rates, deformations and loads for metallic springs.

Spring type	Spring rate c, c_t Deformation f, φ	Principal load	Degree of utilisation
Rectangular spring			
 	$c = \frac{3EI}{l^3} = \frac{bh^3E}{4l^3}$ $f = \frac{Fl^3}{3EI} = \frac{4Fl^3}{bh^3E}$	$\sigma_b = \frac{M_b}{W} = \frac{6Fl}{bh^2}$	$\eta_A = \frac{1}{9}$
Triangular spring			
 	$c = \frac{2EI_0}{l^3} = \frac{b_0 h^3E}{6l^3}$ $f = \frac{Fl^3}{2EI_0} = \frac{6Fl^3}{b_0 h^3E}$	$\sigma_b = \frac{M_b}{W_0} = \frac{6Fl}{b_0h^2}$ $b(x) = \frac{b_0}{l}\cdot x$	Identical load $\eta_A = \frac{1}{3}$

Continuation of table, see Page 505.

Continuation of table, Spring rates for metallic springs, from Page 504.

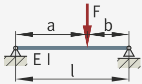
Spring type	Spring rate c, c_t Deformation f, φ	Principal load	Degree of utilisation
Cylindrical coil spring			
	$c = \dfrac{Gd^4}{8iD^3}$ $f = \dfrac{8FiD^3}{Gd^4}$ i = number of turns	$\tau = \dfrac{M_t}{W_p} = \dfrac{8FD}{\pi d^3}$	$\eta_A = \dfrac{1}{2}$
Disc spring			
	$c \approx \dfrac{4E}{1-\nu^2} \dfrac{t^3}{K_1 D_e{}^2}$ [1)] for $(l_0 - t)/t \leqq 0{,}4$ $D_e/D_i = 2$; $K_1 = 0{,}69$	$\sigma_{I,II} \approx \pm F \dfrac{K_3}{t^2}$ $\sigma_{III,IV} \approx \pm F \dfrac{K_3}{t^2} \dfrac{D_i}{D_e}$ $K_3 = 1{,}38$	$\eta_A < \dfrac{1}{3}$
Spiral spring			
	$c_t = \dfrac{EI}{l} = \dfrac{Ebs^3}{12 \cdot l}$ $\varphi = \dfrac{M_t l}{EI} = \dfrac{12 M_t l}{bs^3 E}$ l = length of spring	$\sigma_b = \dfrac{M_t}{W} = \dfrac{6M_t}{bs^2}$ $M_b = M_t = \text{const.}$	Rectangular cross-section b, s $\eta_A = \dfrac{1}{3}$
Cylindrical helical torsion spring (leg spring)			
	$c_t = \dfrac{EI}{l} = \dfrac{E\pi d^4}{64 \cdot l}$ $\varphi = \dfrac{M_t l}{EI} = \dfrac{64 M_t l}{\pi d^4 E}$ l = straightened length of turns	$\sigma_b = \dfrac{M_t}{W} = \dfrac{32 M_t}{\pi d^3}$ $M_b = M_t = \text{const.}$	Circular cross-section d $\eta_A = \dfrac{1}{4}$
Torsion bar spring			
	$c_t = \dfrac{GI_p}{l} = \dfrac{G\pi d^4}{32 \cdot l}$ $\varphi = \dfrac{M_t l}{GI_p} = \dfrac{32 M_t l}{\pi d^4 G}$	$\tau = \dfrac{M_t}{W_p} = \dfrac{16 M_t}{\pi d^3}$	$\eta_A = \dfrac{1}{2}$

[1)] For more precise calculation, see DIN 2092.

Spring rates for some elastic systems

The spring rates for some elastic systems are shown below.

Bending bar supported on both sides

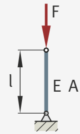


$$c = \frac{3EIl}{a^2b^2}$$

Special case: a = b = l/2

$$c = \frac{48EI}{l^3}$$

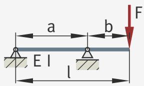
Articulated column



$$c = \frac{EA}{l}$$

E·A = longitudinal stiffness

Torsion bar with protruding end

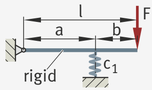


$$c = \frac{3EI}{(a+b)\,b^2}$$

Special case: a = b = l/2

$$c = \frac{12EI}{l^3}$$

Rotatable bar, spring-supported

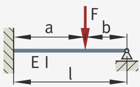


$$c = c_1\left(\frac{a}{l}\right)^2$$

Special case = a = b = l/2

$$c = \frac{c_1}{4}$$

Torsion bar clamped and supported

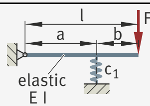


$$c = \frac{12EIl^3}{a^3b^2\left(3l+b\right)}$$

Special case: a = b = l/2

$$c = \frac{768EI}{7\cdot l^3}$$

Rotatable bar, spring-supported

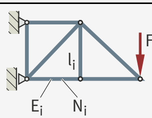


$$\frac{1}{c_{tot}} = \frac{1}{c_1}\left(\frac{l}{a}\right)^2 + \frac{(a+b)\,b^2}{3EI}$$

Torsion bar clamped and guided

$$c = \frac{12EI}{l^3}$$

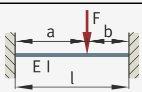
Latticework



$$c = \quad 1 \quad F\sum_{i=1}^{n}\frac{E_i\,A_i}{N_i\,\overline{N}_i\,l_i}$$

$\overline{N}_i$ = normal forces as a result of “1”

Torsion bar clamped on both sides

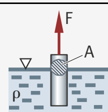


$$c = \frac{3EIl^3}{a^3b^3}$$

Special case: a = b = l/2

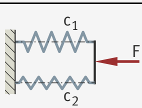
$$c = \frac{192EI}{l^3}$$

Lift



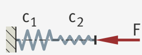
$c = A \cdot \rho \cdot g$

Springs in parallel



$c_{tot} = c_1 + c_2$

Springs in series

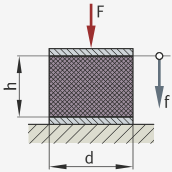
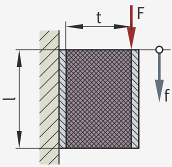
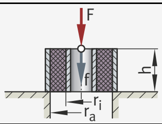
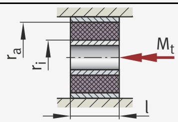
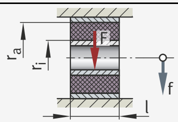


$$\frac{1}{c_{tot}} = \frac{1}{c_1} + \frac{1}{c_2}$$

$$c_{tot} = \frac{c_1c_2}{c_1 + c_2}$$

Spring rate, deformation and loading of rubber springs

The following table gives spring rates, deformations and loads for rubber springs.

Spring type	Deformation f/ spring rate c	Load
Woodruff key (compression)		
	$f = \frac{Fh}{EA} = \frac{4Fh}{E\pi d^2}$ $c = \frac{F}{f} = \frac{EA}{h} = \frac{E\pi d^2}{4h}$ Shape factor $k = \frac{d}{4h}$	$\sigma_d = E\varepsilon = \frac{F}{A}$ $F_{per} = \frac{\pi d^2}{4}\sigma_{per}$ $\varepsilon = \frac{f}{h}$
Rectangular spring (thrust)		
	$f = \frac{Ft}{GA} = \frac{Ft}{Gbl}$ $c = \frac{F}{f} = \frac{Gbl}{t}$ Width b	$\tau = G\gamma = \frac{F}{A}$ $F_{per} = G\,b\,l\,\gamma_{per}$ $\gamma = \frac{f}{t}$
Sleeve type spring (thrust)		
	$f = \frac{F}{2\pi hG}\ln\frac{r_a}{r_i}$ $c = \frac{F}{f}$	$\tau_i = \frac{F}{A_i} = \frac{F}{2\pi r_i h}$ $F_{per} = 2\pi\, r_i\, h\, G\, \gamma_{per}$
Sleeve type spring (rotational thrust)		
	$\varphi = \frac{M_t}{4\pi lG}\left(\frac{1}{r_i^2} - \frac{1}{r_a^2}\right)$ $c_t = \frac{M_t}{\varphi}$	$\tau_i = \frac{F}{A_i} = \frac{M_t}{2\pi r_i^2 l}$ $M_{t\,per} = 2\pi\, G\, r_i^2\, l\, \gamma_{per}$
Sleeve type spring (compression, thrust)		
	$f = \frac{F}{\pi l(E+G)}\ln\frac{r_a}{r_i}$ $c = \frac{F}{f}$	$\sigma_{dmax} = \frac{2}{\pi}\frac{F_{max}}{lr_i}$ $F_{per} = \frac{\pi l r_i}{2}\sigma_{dper}$

Calculation of rubber springs

The following table shows reference values for the approximate calculation of rubber springs.

Shore hardness Sh (A)	Modulus of elasticity E_{st} N/mm² Compression k = 1/4	Modulus of elasticity E_{st} N/mm² Compression k = 1,0	Shear modulus G_{st} N/mm²	Permissible static deformation under permanent load % Compression	Permissible static deformation under permanent load % Thrust Tension	Permissible static stress under permanent load N/mm² Compression k = 1/4	Permissible static stress under permanent load N/mm² Compression k = 1,0	Permissible static stress under permanent load N/mm² Thrust Tension
30	1,1	4,5	0,3	10 ... 15	50 ... 75	0,18	0,7	0,20
40	1,6	6,5	0,4		45 ... 70	0,25	1,0	0,28
50	2,2	9,0	0,55		40 ... 60	0,36	1,4	0,33
60	3,3	13,0	0,8		30 ... 45	0,50	2,0	0,36
70	5,2	20,0	1,3		20 ... 30	0,80	3,2	0,38

Permissible alternating stresses 1/3 to 1/2 of the permissible static stresses.

Properties of elastomers for rubber springs

The following table gives some properties of elastomers for rubber springs.

Elastomers	Styrene butadiene rubber	Natural rubber (polyisoprene)	Butyl rubber	Ethylene propylene diene rubber	Chloro butadiene rubber	Acrylonitrile butadiene rubber	Polyurethane rubber	Silicone rubber	Polyacrylate rubber (PA)	Fluorocarbon rubber
Designation	SBR	NR	JIR	EPDM	CR	NBR	AU, EU	VMQ	ACM	FKM
Example trade name	Buna, Hüls	Rubber	Butyl	Buna AP	Neoprene	Perbunan	Vulkollan	Silopren	Cyanacryl	Viton
Density g/cm³	0,92	0,95	0,93	–	1,23	0,98	1,26	1,19	–	–
Tensile strength (DIN 53504) N/mm²	≤ 24	≤ 28	≤ 15	18	20 ... 27	22 ... 27	30 ... 32	≤ 10	15	15
Elongation at fracture Maximum value (DIN 53504)	700	1000	900	800	800	800	600	500	–	–
Shore hardness Sh (DIN 53505) (A)	40 ... 95	30 ... 98	40 ... 90	40 ... 90	40 ... 95	40 ... 95	65 ... 95	40 ... 90	55 ... 85	60 ... 90
Operating temperature range °C	−30 ... +90	−40 ... +70	−25 ... +110	−35 ... +130	−25 ... +100	−25 ... +100	−15 ... +80	−60 ... +200	−15 ... +150	−20 ... +220

Continuation of table, see Page 509.

Continuation of table, Properties of elastomers for rubber springs, from Page 508.

Elastomers	Styrene butadiene rubber	Natural rubber (polyisoprene)	Butyl rubber	Ethylene propylene diene rubber	Chloro butadiene rubber	Acrylonitrile butadiene rubber	Polyurethane rubber	Silicone rubber	Polyacrylate rubber (PA)	Fluorocarbon rubber
Designation	SBR	NR	JIR	EPDM	CR	NBR	AU, EU	VMQ	ACM	FKM
Example trade name	Buna, Hüls	Rubber	Butyl	Buna AP	Neoprene	Perbunan	Vulkollan	Silopren	Cyanacryl	Viton
Oil resistance	Slight	Slight	Slight	Moderate	Moderate	Good	Very good	Good	Very good	Very good
Petrol resistance	–	–	–	–	–	Good	Good	Moderate	Very good	Very good
Ozone resistance	Slight	Slight	Very good	Excellent	Good	Slight	Very good	Very good	Very good	Very good
Creep resistance	Very good	Excellent	Medium	Good	Good	Very good	Good	Good	Good	Good
Rebound elasticity	Good	Very good	Slight	Good	Good	Good	Very good	Good	Slight	Slight
Damping	Good	Moderate	Excellent	Good	Good	Very good	Good	Good	Very good	Excellent
Abrasion resistance	Very good	Very good	–	–	Good	–	Very good	–	–	–
Adherence to metal	Good	Excellent	Moderate	Moderate	Good	Very good	Very good	Medium	Medium	Good
Special properties	–	3)	1) 2)	–	–	–	5)	4)	5)	2)
Processibility	–	–	–	–	–	–	–	–	Bright production possible	Difficult
Electrical insulation capability	Good	Very good	Very good	Very good	Slight	–	Slight	Good	Poor	Good
Price	Low	Low	Low	Low	Fairly low	Fairly low	Medium	High	High	Very high

Source: Dubbel, Taschenbuch für den Maschinenbau, 21. Auflage.

1) Gas permeability very low.
2) Acid resistance good.
3) Flammable.
4) Flame resistant.
5) Sensitive to water at 40 °C.

Dampers Dampers (vibration dampers) are elements that convert motion energy into heat energy. They are generally used in order to cause decay of the vibration of a sprung mass. In special cases, shock dampers can also be used to give gentle braking of a mass performing linear or rotary motion.

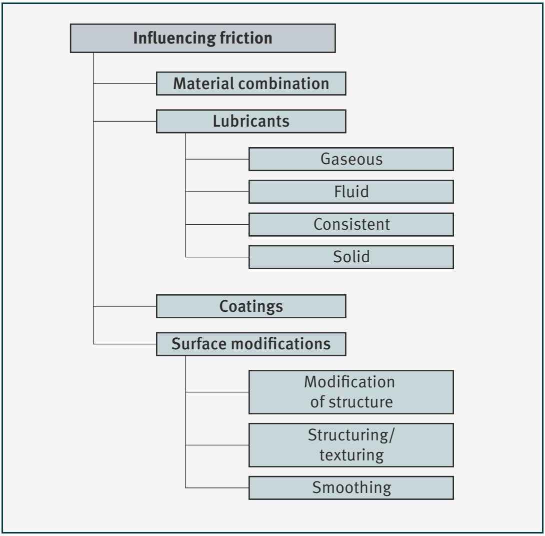
Various types of damping can be used here:

- material damping
- friction damping
- viscosity damping
- eddy current damping
- radiation damping
- turbulence damping.

Influencing friction and wear

Overview In most cases, friction and the resulting wear are undesirable. The aim is therefore to reduce friction through suitable material combinations for components and the use of lubricants, coatings or surface modifications.

Figure 34
Influencing friction and wear



Lubricants Depending on the application, lubricants in fluid, consistent, gaseous or solid form are used. These are intended principally to decrease friction and reduce wear.

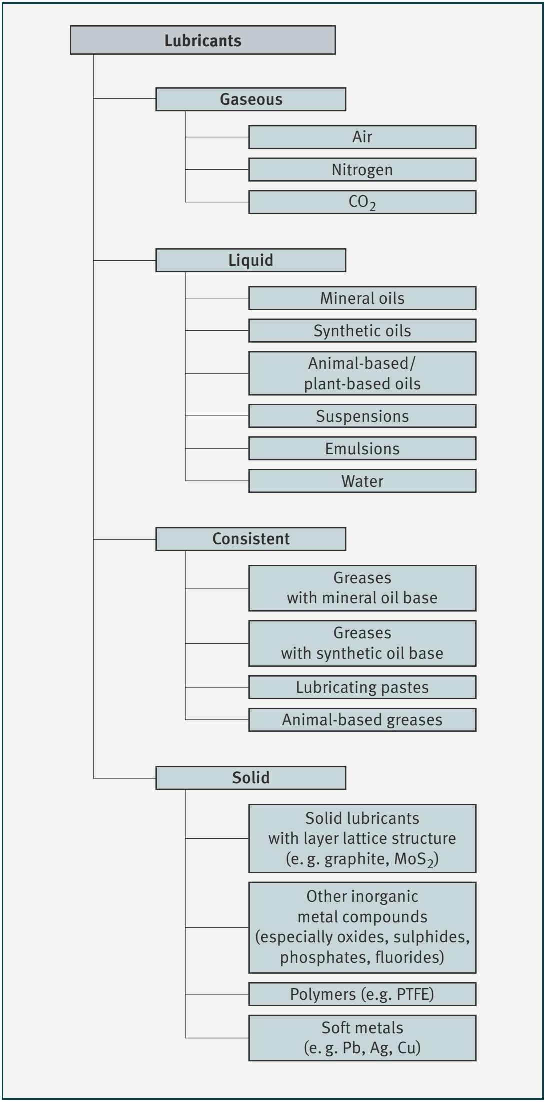
Lubricants provide complete separation (full lubrication) or partial separation (partial lubrication) of surfaces moving relative to each other and subjected to load.

Fluid lubricants are mainly mineral oils and synthetic oils as well as animal-based and plant-based oils.

Consistent lubricants are greases comprising mineral and synthetic oils (thickened using soaps).

Examples of solid lubricants include graphite, molybdenum disulphide (MoS_2) and polytetrafluorethylene (PTFE).

Figure 35
Lubricants



Coatings

Introduction

An increasing number of components in technical systems are subjected to surface loading that leads to the operating limits of most metallic materials being breached.

The use of technologies from the field of industrial surface technology – in this case, mainly coating – makes it possible to refine the surfaces in such a way that the operating limits can be considerably expanded in comparison with their untreated original condition. Furthermore, it is possible to achieve functional integration or additional characteristics.

In the selection and use of surface technology, the decisive factor for success is a holistic approach that takes the entire technical system into consideration.

The component surface – an active area

Technical components are used to fulfil particular functions within a higher level machine or installation. The component comprises a particular material and has a corresponding geometry as well as a production history. The geometry can be further subdivided into the component volume and the component surface. These perform various subfunctions: while the **component surface** in the form of an **active area** supports the external loads and transmits these to the interior regions, the actual load-bearing function is performed preferably by the component volume.

The life of technical components is frequently determined not only by the strength but also by fatigue or wear of the surface. Since these phenomena take place on the surface, it is necessary for logical reasons to mainly address this area in order to solve the rating life issue.

Surface loading

The component surface generally represents the area subjected to the heaviest loading. This is where normal and frictional forces or heat flows are introduced. Electric potentials build up here or electric currents are transmitted. In many cases, wear or corrosion on the component surface determine the life of the entire component. In industrial and mobile applications, this surface loading originates essentially from the following categories:

- Tribological surface loading:
  Friction and wear (abrasive or adhesive) lead to damage in machine elements. Friction can be influenced and wear reduced by a specifically modified surface topography and coatings.

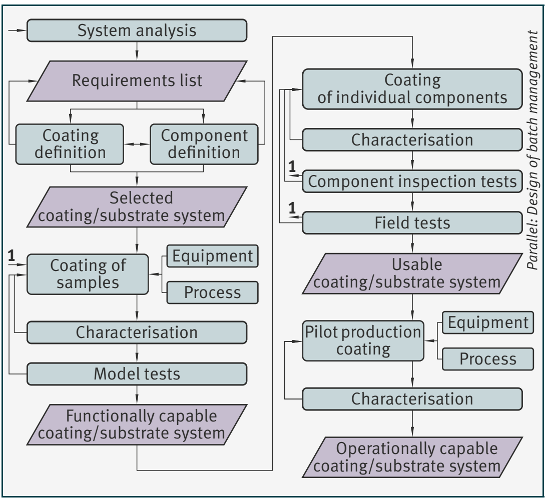
- Corrosive surface loading:
  Corrosion as a chemical or electrochemical reaction occurs in metallic materials in the presence of humidity, contact with water or aggressive media (alkalis or acids). A protective function can be provided here by suitable coatings.
- Electrical surface loading:
  As a result of current transfer or the buildup and discharge of potentials, electrical surface loading can occur. This can have effects on intervening media, which may bring about the premature ageing of lubricants. Surface engineering measures facilitate the application of insulating or current-conducting coatings.

Coatings as design elements – "Tailored Coatings"

The surfaces of components are therefore – with different functions – of enormous significance. Coatings for the refinement of surfaces must therefore be considered in terms of their different composition and application as design elements.

If this approach is followed logically, the selection and design of coatings must be carried out in the same meticulous manner as in the case of normal machine elements.

Figure 36
Systematic coatings development process



For a coatings development process, a systematic procedure is recommended in accordance with the **flow diagram** shown here.

When all the boundary conditions are taken into consideration, this gives the most suitable "tailored coating" for the application.

Available coatings

For the various types of surface loading, a wide range of different coatings is available.

Surface technology offers a large quantity of coatings as well as process engineering approaches to solutions for the production of the different coatings. The selection of the coating methods to be used is influenced by the material characteristics of the substrates and coatings, the geometry of the components and economic aspects.

In essence, the coating methods that are used for the production of coatings against tribological, corrosive or electrical surface loading can be subdivided into two groups:

1. Methods for the modification and transformation of the surface zone of the substrate
2. Methods for the creation of overlay coatings.

From an industrial perspective, the following coating methods are relevant especially for the production of large quantities in volume processes:

- thermochemical diffusion methods
- conversion methods
- chemical/electrochemical methods
- PVD method (Physical Vapor Deposition) or PACVD method (Plasma Assisted Chemical Vapor Deposition), also known as: PECVD method (Plasma Enhanced Chemical Vapor Deposition)
- thermal spraying
- painting.

The coating materials that can be produced for the aforenamed areas of application by means of the above methods are explained, in greater detail and in conjunction with the applications, from Page 519.

Tribological coatings

Tribological coatings can reduce fatigue close to the surface and wear. They can be used to specifically influence friction and thus make a contribution to energy efficiency and CO_2 reductions.

In order to prevent destruction of the surface, good surface quality (small roughness peaks, proportionally large load-bearing area) is advantageous. High friction can be reduced by means of friction-reducing coatings such as DLC (Diamond Like Carbon) or PTFE (polytetrafluoroethylene). Protection against **abrasive wear** requires a high surface hardness. The contact partners can be protected here by particularly hard coatings. The PVD and PACVD methods can be used to deposit coatings with hardnesses $>$2 000 HV. Furthermore, electroplated coatings such as chromium or NiP can prevent abrasive wear, since their hardness is greater than that of the base material.

Adhesive wear occurs principally in contact partners with similar bonding characteristics, such as metal/metal. In order to prevent this wear, it is sufficient to change the type of bonding close to the surface by the coating of one contact partner. A typical example of adhesive wear is slippage damage. This wear can be reduced by, for example, the targeted oxidation of the material surface by means of black oxide coating. In this case, a metallic surface is converted into a surface (metal oxide) with heteropolar bonding. Through coating with an amorphous carbon layer, a covalent bonding character can be achieved on the surface.

In order to prevent wear by means of tribochemistry, solutions can be used that are similar to those for the prevention of adhesive wear. The chemical reactions can be suppressed by means of a suitable coating. An example of this is the phosphating of a surface.

Due to the increasing requirements in relation to performance capability and resource efficiency as well as the ever smaller availability of space, increasing importance is being attached to thin layers produced using highly eco-friendly vacuum plasma techniques. A general classification of these coatings is shown in Figure 37.

Figure 37
Classification of tribological thin film coating systems

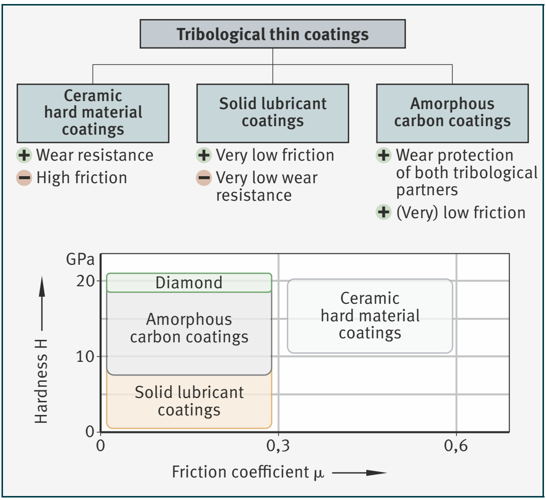
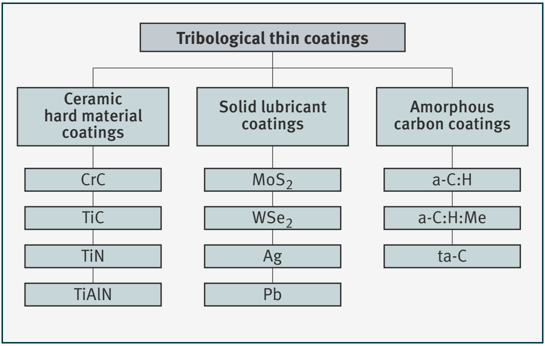


Figure 38
Examples of tribological thin film coating systems



Anti-corrosion coatings

In contact with water or humidity, steels tend to undergo corrosion. In many cases, corrosion-resistant high grade steels cannot be quenched and tempered to the requisite hardnesses or lose their corrosion-inhibiting properties during the hardening process. In this case, assistance is possible using anti-corrosion coatings, mainly based on zinc and zinc alloys.

Figure 39
Bearing rings after 24 h on a test rig in the salt spray test

① Coated with zinc alloy coating
② Uncoated

Insulation coatings

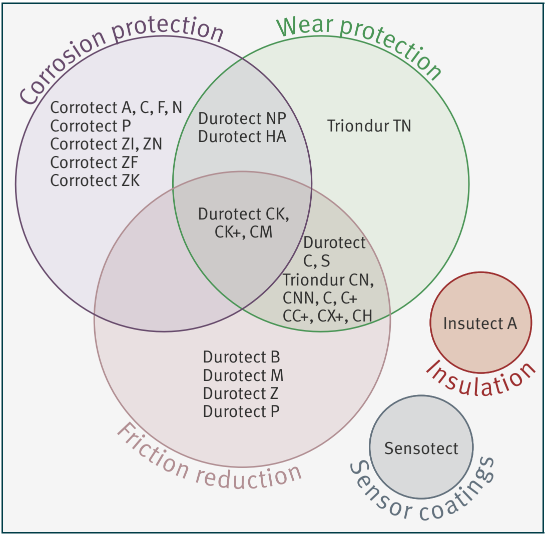
Rolling bearings can be damaged by current transmission. The damage resulting from current passage can lead to failure of the bearing. In order to give protection against electric current, there are various established solutions that are used according to the size and type of bearing. Solutions in the form of coatings or even glass fibre reinforced plastic housings are used.

Modular coating concept

For the selection of coatings for different problem areas, Schaeffler has developed a modular coating concept.

The modular coating concept shown is intended to give easier selection of suitable coatings.

Figure 40
Modular coating concept



Application examples

Schaeffler has developed suitable coatings for various applications. The resulting recommendations are presented below.

Recommendations for tribological coatings

Friction reduction and wear protection

The quality of a rolling bearing is determined to a significant extent by its smooth running and wear resistance. A low friction coefficient reduces not only energy consumption but also the requirement for lubricant. This is associated with lower mechanical wear, while the operating life of the bearing increases. The different wear types (abrasive, adhesive, tribochemical) require different measures.

Protection against **abrasive** wear:

- High surface quality (high hardness, small roughness peaks) necessary.
- Protection of contact partners by particularly hard coatings (hardness in excess of 2 000 HV) that are applied by means of PVD (Physical Vapor Deposition) or PACVD (Plasma Assisted Chemical Vapor Deposition).
- High friction can be reduced by means of friction-reducing coatings such as DLC (Diamond Like Carbon) or PTFE (polytetrafluoroethylene).
- This is also possible using electroplated coatings such as chromium or NiP, since their hardness is greater than that of the base material.

Protection against **adhesive** wear:

- Occurs principally in contact partners with similar bonding characteristics, such as metal/metal.
- In order to prevent this wear (typically: slippage damage), it is sufficient to change the type of bonding close to the surface by the coating of one contact partner.
- Remedy by targeted oxidation of the metal surface by means of black oxide coating.
- Through coating with an amorphous carbon layer, a covalent bonding character can be achieved on the surface.

Protection against wear by means of **tribochemistry**:

- Solutions similar to those for the prevention of adhesive wear.
- Chemical reactions can be suppressed by means of suitable coatings. For example, phosphating of a surface.
- In order to improve the sliding friction contacts, bearing cages are coated by electroplating means with silver or copper. This also makes it possible to prevent fretting corrosion.

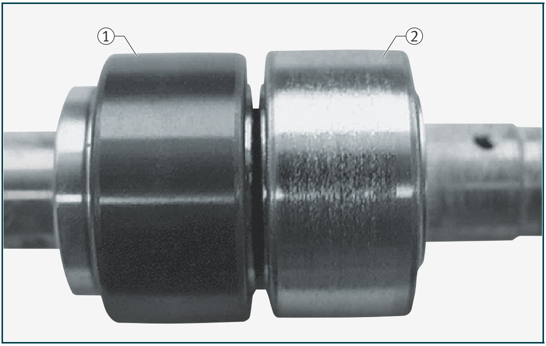


Figure 41
Comparison of coated and uncoated stud type track rollers

① Coated with Triondur C: no wear of any type
② Uncoated: adhesive wear on raceway and mating body

Principal function Wear protection and friction reduction

The following table shows Durotect coating systems for wear protection and friction reduction.

Designation of coating system	Description Suffix	Principal functions			Additional function	Main area of application Special feature
		Corrosion protection	Wear protection	Friction reduction		
Durotect B	Mixed iron oxide CT240			x	Improved running-in behaviour, reduced slippage damage, slight corrosion protection	Industrial, Automotive, bearing components, wind energy, full complement roller bearings
Durotect Z	Zinc phosphate CT250 – CT251			x	Temporary corrosion protection, protection against fretting corrosion, suitable for sliding seats	Industrial, Aerospace, linear guidance systems, bearings, bearing components
Durotect M	Manganese phosphate CT260 – CT261			x	Improved running-in behaviour, slight corrosion protection, emergency running lubrication	Industrial, Automotive, Aerospace, bearing components

Continuation of table, see Page 521.

Continuation of table Principal function Wear protection and friction reduction from Page 520.

Designation of coating system	Description Suffix	Principal functions			Additional function	Main area of application Special feature
		Corrosion protection	Wear protection	Friction reduction		
Durotect CK	Columnar thin dense chromium coating CT230		x		Corrosion protection, slight reduction in friction, reduced fretting corrosion	Industrial, Linear Technology, Aerospace, vibratory screen bearings, helicopter bearings, spindle bearings
Durotect CK^{+}	Columnar thin dense chromium coating plus mixed chromium oxide CT231	x	x	x	Additionally good corrosion protection	Industrial, bearing components, Linear Technology
Durotect CM	Microcracked thin dense chromium coating CT220 – CT224		x		Slight corrosion protection, slight reduction in friction	Industrial, needle roller bearings
Durotect NP	Chemical nickel CT200 – CT205	x	x		–	Industrial, drawn cups, guide ring segments
Durotect C	Copper CT270			x	Emergency running lubrication	Industrial, cages
Durotect S	Silver CT271			x	Emergency running lubrication	Industrial, Aerospace, Linear Technology, bearing components, cages
Durotect HA	Hard anodising (Al)	x	x		Current insulation	Automotive, sliding sleeves
Durotect P	Polymer-based coating CT700 – CT702			x	Protection against fretting corrosion, friction reduction	Industrial, bearing rings

Principal function Surfaces subjected to high tribomechanical loading

The following table shows Triondur coating systems for surfaces subjected to high tribomechanical loading.

Designation of coating system	Description Suffix	Principal functions			Additional function	Main area of application Special feature
		Corrosion protection	Wear protection	Friction reduction		
Triondur CN	CrN/Cr_2N CT400 – CT404		x	x	–	Automotive, valve train components
Triondur CNN	CrN/CrC CT405 – CT409		x	x	–	Automotive, valve train components
Triondur C	a-C:H:Me CT420		x	x	Reduced slippage damage	Industrial, Automotive, bearing components, engine components
Triondur C^+	a-C:H CT450 – CT479		x	x	–	Industrial, Automotive, bearing components, engine components
Triondur CX^+	a-C:H:X CT480 – CT509		x	x	Minimal friction in valve train	Automotive, valve train components, bearing components
Triondur TN	TiN CT415 – CT419		x		–	Aerospace, bearing components, rib surfaces
Triondur CH	ta-C CT520 – CT529		x		Very high abrasive wear resistance	Automotive, valve train components

Recommendations for corrosion-inhibiting coatings

Bearing parts with corrosion – as a result of contact with water or humidity – can in the case of standard bearings lead to malfunctions, lower efficiency and premature failure. Corrosion-resistant rolling bearing steels provide a remedy here but are expensive. The most economical variant under moderate corrosion conditions is therefore the combination of a standard rolling bearing steel with an appropriate coating.
The following coatings have proved effective:

- zinc phosphating with application of oil (for low requirements)
- extremely thin zinc alloy coatings, applied by electroplating
- columnar thin dense chromium coating as an anti-corrosion coating resistant to wear and overrolling
- nickel-phosphorus coatings (deposited by electroless methods) for highly corrosive media such as acids and alkalis.

Principal function Corrosion protection

The following table shows Corrotect coating systems for corrosion protection.

Designation of coating system	Description Suffix	Principal functions			Additional function	Main area of application Special feature
		Corrosion protection	Wear protection	Friction reduction		
Corrotect A	Zinc alloy CT001	x			–	Automotive, belt drives, selector shafts, bearings, bearing components, Cr(VI)-free
Corrotect N	CT004	x			–	Automotive, belt drives, detents, Cr(VI)-free
Corrotect ZK	Zinc CT010 – CT013	x			–	Simple corrosion protection
Corrotect ZI	Zinc-iron CT020 – CT023	x			–	Industrial, Automotive, belt drives, bearing components, screws
Corrotect ZN	Zinc-nickel CT020 – CT023	x			–	Industrial, Automotive, belt drives, bearing components, screws
Corrotect ZF	Zinc flakes CT100	x			–	Industrial, Automotive, chassis engineering, components, screws

Recommendations for electrically insulating coatings

In order to prevent rolling bearing failures as a result of current passage, the cylindrical surfaces and end faces of the bearing rings can be provided with ceramic insulating coatings, see Figure 42.

Current insulation is achieved by means of plasma spray coating of the outside diameter and the lateral faces on the outer ring or the bore and lateral faces of the inner ring. The insulation coating comprises aluminium oxide, in which the pores are sealed with resin to give protection against the ingress of moisture.

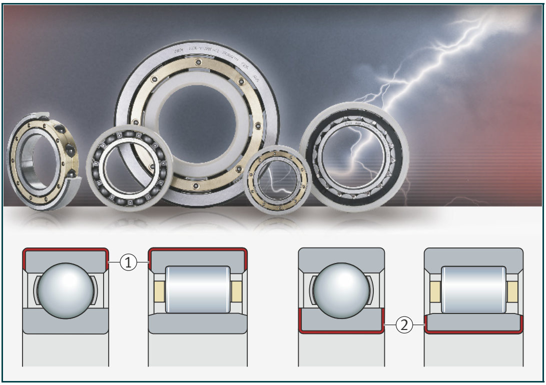
Principal function Current insulation

The following table shows Insutect coating systems for current insulation.

Designation of coating system	Description Suffix	Principal function	Main area of application Special feature
Insutect A	Aluminium oxide	Current insulation	Industrial, rail vehicles, electric motors, generators

Figure 42
Current-insulating bearings

① Coating of outer ring
② Coating of inner ring



Advantages of coated bearings:

- High level of insulation, even in a damp environment, due to a special sealing process
- The external dimensions of the bearing correspond to the dimensions in accordance with DIN 616 and are thus interchangeable with standard bearings
- The puncture strength of thin layers is up to 500 VDC, while the puncture strength of thick layers is guaranteed to be at least 1 000 VDC.

Coatings – Overview The following table gives an overview of coatings with their principal and additional functions.

Coating	Designation	Principal functions			Additional function
		Corrosion protection	Wear protection	Friction reduction	
Zinc	Corrotect ZK	x			–
Paint	Corrotect P	x			–
Tin	Corrotect T	x			Reduced contact corrosion
Zinc-iron	Corrotect N	x			–
Zinc-nickel	Corrotect ZN	x			–
Zinc flakes, Cr(VI)-free	Corrotect ZF	x			–
Chemical nickel	Durotect NP	x	x		–
Hard anodising (Al)	Durotect HA	x	x		Current insulation
Columnar thin dense chromium coating	Durotect CK		x		Corrosion protection, slight reduction in friction, reduced fretting corrosion
Columnar thin dense chromium coating plus mixed chromium oxide	Durotect CK^{+}	x	x	x	Additionally good corrosion protection
Microcracked thin dense chromium coating	Durotect CM		x		Slight corrosion protection, slight reduction in friction
Black oxide coating	Durotect B			x	Improved running-in behaviour, reduced slippage damage, slight corrosion protection
Manganese phosphate	Durotect M			x	Improved running-in behaviour, slight corrosion protection, emergency running lubrication
Zinc phosphate	Durotect Z			x	Temporary corrosion protection, protection against fretting corrosion, suitable for sliding seats
Copper	Durotect C			x	Emergency running lubrication
Silver	Durotect S			x	Emergency running lubrication
PTFE	Durotect P			x	Protection against fretting corrosion, friction reduction

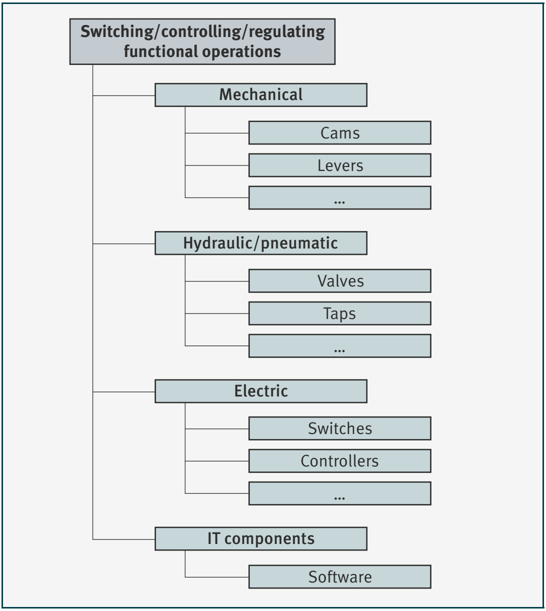
Switching/controlling/regulating functional operations

Overview

Technical systems must implement various functional operations in order to be able to achieve the required operational states. For this purpose, subsystems and the complete system must be switched, controlled and regulated in a suitable form.

In addition to the classical mechanical and hydraulic/pneumatic systems, a decisive role is now being played by electronic and IT-based systems and components. The latter are now represented, in modern mechatronic systems, by the information processing component of the complete system.

Figure 43
Switching/controlling/regulating functional operations



Guiding elements in a rotary direction – rolling bearings

Rotary bearings allow relative rotary motions. These can involve rotation in one direction or oscillation.

Support and guidance function

The task (function) of rotary rolling bearings is to guide parts that are movable in relation to each other and support them relative to the adjacent structure. They support forces and transmit these into the adjacent construction. In this way, they perform support and guidance tasks and thus form the connection between stationary and moving machine elements.

Support as a function

The function "Support" comprises the transmission of forces and moments between parts moving relative to each other.

Guidance as a function

The function "Guidance" principally comprises defining to an appropriate (normally high) accuracy the position of parts moving relative to each other.

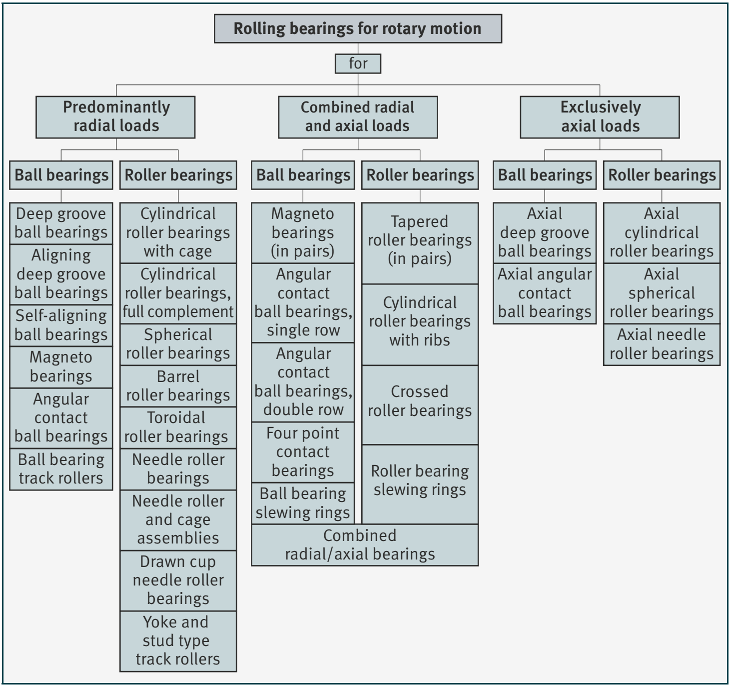
Principal requirements placed on bearings

Technical implementation is oriented to the two principal requirements:

- Function must be ensured and fulfilled for as along as possible
- The resistance to motion (bearing friction) should be as low as possible in order to reduce the energy required for motion (energy efficiency).

Overview of common rolling bearings

The following diagram shows an overview of common bearing types for rotary motion, which are then described in detail starting on Page 608.



Dimensioning and design of rolling bearing arrangements

The design of rolling bearing arrangements requires consideration of a large number of factors. This includes the following steps:

- selection of the bearing type and bearing arrangement
- calculation of the bearing size
- design of the bearing position
- definition of the lubrication
- considerations relating to mounting and dismounting.

Objectives and influencing factors

Long operating life, high reliability and cost-efficiency are essential objectives in the design of a rolling bearing arrangement. In order to achieve this, the designers must record in a design brief all the conditions and requirements that are an influence on the bearing arrangement.

At the draft stage, it is important to select not only the correct bearing type, bearing design and bearing arrangement. It is more important to match the adjacent parts, namely the shaft, housing and fasteners, the sealing and in particular the lubrication to the influences recorded in the design brief.

Design data

The following data should be available:

- Machine, device and mounting positions of the bearings (diagram)
- **Operating conditions**
  (load, speed, design envelope, temperature, ambient conditions, shaft arrangement, rigidity of the adjacent components)
- **Requirements**
  (rating life, accuracy, noise, friction and operating temperature, lubrication and maintenance, mounting and dismounting)
- **Commercial data**
  (delivery dates, quantities, costs).

Technical principles

The following chapters give an overview of the technical principles that must be applied in the design of a bearing arrangement:

- Dimensioning – load carrying capacity and life, Page 529
- Speeds, Page 557
- Noise, Page 562
- Lubrication, Page 564
- Bearing data, Page 572
- Design of bearing arrangements, Page 583.

In addition to the approximate calculation specifications in the printed catalogues, online software programs from the Schaeffler Group such as BEARINX and ***medias*** are available for more precise calculations of the bearing arrangement.

At the end of the chapter, various applications are presented to give examples of the design of bearing arrangements.

Dimensioning – load carrying capacity and life

The rating life calculation standardised in ISO 281 is based on Lundberg and Palmgren's fatigue theory which always gives a final rating life. However, modern, high quality bearings can exceed by a considerable margin the values calculated for the basic rating life under favourable operating conditions. Ioannides and Harris have developed a further model of fatigue in rolling contact that expands on the Lundberg and Palmgren theory and gives a better description of the performance capability of modern bearings.

The method "Expanded calculation of the adjusted rating life" takes account of the following influences:

- the bearing load
- the fatigue limit of the material
- the extent to which the surfaces are separated by the lubricant
- the cleanliness in the lubrication gap
- the additives in the lubricant
- the internal load distribution and friction conditions in the bearing.

The influencing factors, particularly those relating to contamination, are very complex. A great deal of experience is essential for an accurate assessment. The tables and diagrams can give only guide values.

Calculation of the bearing size

The required size of a rolling bearing is dependent on the demands made on its:

- rating life
- load carrying capacity
- operational reliability.

Dynamic load carrying capacity and operating life

The dynamic load carrying capacity of the rolling bearing is determined by the fatigue behaviour of the material. The dynamic load carrying capacity is described in terms of the basic dynamic load ratings, which are based on DIN ISO 281.

The dynamic load carrying capacity is described in terms of the basic dynamic load rating C and the basic rating life.

The fatigue life is dependent on:

- the load
- the operating speed
- the statistical probability of the initial appearance of failure.

The basic dynamic load rating C applies to rotating rolling bearings. It is:

- a constant radial load C_r for radial bearings
- a constant, concentrically acting axial load C_a for axial bearings.

The basic dynamic load rating C is that load of constant magnitude and direction which a sufficiently large number of apparently identical bearings can endure for a basic rating life of one million revolutions.

Calculation of the rating life

The methods for calculating the rating life are:

- the basic rating life L_{10} and L_{10h} in accordance with ISO 281
- the adjusted rating life L_{na} in accordance with DIN ISO 281:1990 (no longer a constituent part of ISO 281:2007)
- the expanded adjusted rating life L_{nm} in accordance with ISO 281, see Page 531.

Basic rating life

The basic rating life L_{10} and L_{10h} is determined as follows:

Equation 32

$$L_{10} = \left(\frac{C}{P}\right)^p$$

Equation 33

$$L_{10h} = \frac{16\,666}{n} \cdot \left(\frac{C}{P}\right)^p$$

Legend

L_{10} 10^6 revolutions
The basic rating life in millions of revolutions is the life reached or exceeded by 90% of a sufficiently large group of apparently identical bearings before the first evidence of material fatigue develops

L_{10h} h
The basic rating life in operating hours according to the definition for L_{10}

C N
Basic dynamic load rating

P N
Equivalent dynamic bearing load for radial and axial bearings

p –
Life exponent;
for roller bearings: p = 10/3
for ball bearings: p = 3

n min^{-1}
Operating speed (nominal speed).

Equivalent dynamic bearing load

The equivalent dynamic bearing load P is a calculated value. This value is constant in magnitude and direction; it is a radial load for radial bearings and an axial load for axial bearings. A load corresponding to P will give the same rating life as the combined load occurring in practice.

Equation 34

$$P = X \cdot F_r + Y \cdot F_a$$

Legend

P N
Equivalent dynamic bearing load

F_r N
Radial bearing load

F_a N
Axial bearing load

X –
Radial load factor
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings

Y –
Axial load factor
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings.

This calculation cannot be applied to radial needle roller bearings, axial needle roller bearings and axial cylindrical roller bearings. Combined loads are not permissible with these bearings.

Expanded adjusted rating life

Calculation of the expanded adjusted rating life L_{nm} was standardised in DIN ISO 281 Appendix 1. Since 2007, it has been standardised in the worldwide standard ISO 281. Computer-aided calculation to DIN ISO 281 Appendix 4 has been specified since 2008 in ISO/TS 16281 and standardised since 2010 in DIN 26281.

L_{nm} is calculated as follows:

Equation 35

$$L_{nm} = a_1 \cdot a_{ISO} \cdot L_{10}$$

Legend

L_{nm} — 10^6 revolutions
Expanded adjusted rating life in millions of revolutions in accordance with ISO 281:2007

a_1 — –
Life adjustment factor for a requisite reliability other than 90%, see following table

a_{ISO} — –
Life adjustment factor for operating conditions

L_{10} — 10^6 revolutions
Basic rating life, see Page 530.

Life adjustment factor a_1

The values for the life adjustment factor a_1 were redefined in ISO 281:2007 and differ from the previous data.

Requisite reliability %	Expanded adjusted rating life L_{nm}	Life adjustment factor a_1
90	L_{10m}	1
95	L_{5m}	0,64
96	L_{4m}	0,55
97	L_{3m}	0,47
98	L_{2m}	0,37
99	L_{1m}	0,25
99,2	$L_{0,8m}$	0,22
99,4	$L_{0,6m}$	0,19
99,6	$L_{0,4m}$	0,16
99,8	$L_{0,2m}$	0,12
99,9	$L_{0,1m}$	0,093
99,92	$L_{0,08m}$	0,087
99,94	$L_{0,06m}$	0,08
99,95	$L_{0,05m}$	0,077

Life adjustment factor a_{ISO}

The standardised method for calculating the life adjustment factor a_{ISO} essentially takes account of the following influences:

- the load on the bearing
- the lubrication conditions (viscosity and type of lubricant, speed, bearing size, additives)
- the fatigue limit of the material
- the type of bearing
- the residual stress in the material
- the ambient conditions
- contamination of the lubricant.

Equation 36

$$a_{ISO} = f\left[\frac{e_C \cdot C_u}{P}, \kappa\right]$$

Legend

a_{ISO} – Life adjustment factor for operating conditions, see Figure 44, Page 533 to Figure 47, Page 534	P N Equivalent dynamic bearing load
e_C – Life adjustment factor for contamination, see table Contamination factor e_C, Page 537	κ – Viscosity ratio, see Page 535 For $\kappa > 4$, calculation should be carried out sing $\kappa = 4$. This calculation method cannot be used for $\kappa < 0{,}1$.
C_u N Fatigue limit load	

In accordance with ISO 281, EP additives in the lubricant can be taken into consideration as follows:

- For a viscosity ratio $\kappa < 1$ and a contamination factor $e_C \geqq 0{,}2$, calculation of lubricants with EP additives that have been proven effective can be carried out using the value $\kappa = 1$. Under severe contamination (contamination factor $e_C < 0{,}2$), evidence must be obtained of the effectiveness of the additives under these contamination conditions. The effectiveness of the EP additives can be demonstrated in the actual application or on a rolling bearing test rig FE8 in accordance with DIN 51819-1.
- If calculation in the case of EP additives that have been proven effective is carried out using the value $\kappa = 1$, the life adjustment factor must be restricted to $a_{ISO} \leqq 3$. If the value a_{ISO} calculated for the actual κ is greater than 3, this value can be used in calculation.
- For practical purposes, the life adjustment factor should be restricted to $a_{ISO} \leqq 50$. This limit value also applies if $e_C \cdot C_u/P > 5$.

The following diagrams show the life adjustment factor a_{ISO} for various bearings.

Figure 44
Life adjustment factor a_{ISO} for radial roller bearings

a_{ISO} = life adjustment factor
C_u = fatigue limit load
e_C = contamination factor
P = equivalent dynamic bearing load
κ = viscosity ratio

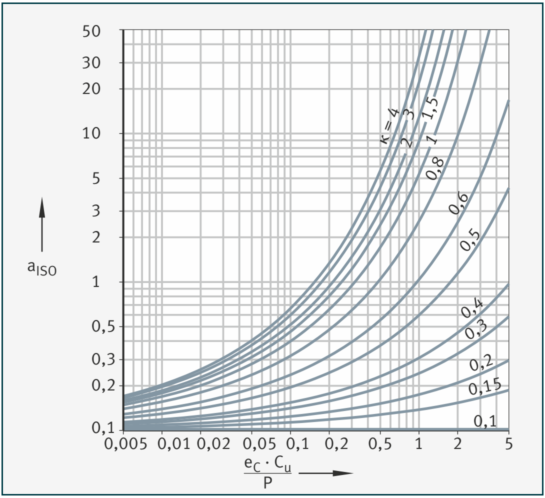
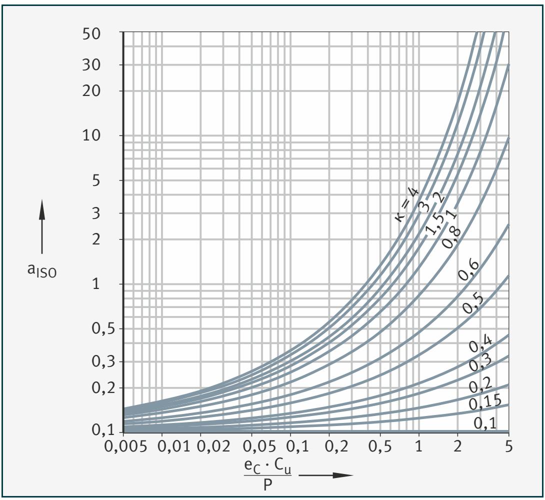


Figure 45
Life adjustment factor a_{ISO} for axial roller bearings

a_{ISO} = life adjustment factor
C_u = fatigue limit load
e_C = contamination factor
P = equivalent dynamic bearing load
κ = viscosity ratio



Continuation of diagrams showing the life adjustment factor a_{ISO} for various bearings.

Figure 46
Life adjustment factor a_{ISO} for radial ball bearings

a_{ISO} = life adjustment factor
C_u = fatigue limit load
e_C = contamination factor
P = equivalent dynamic bearing load
κ = viscosity ratio

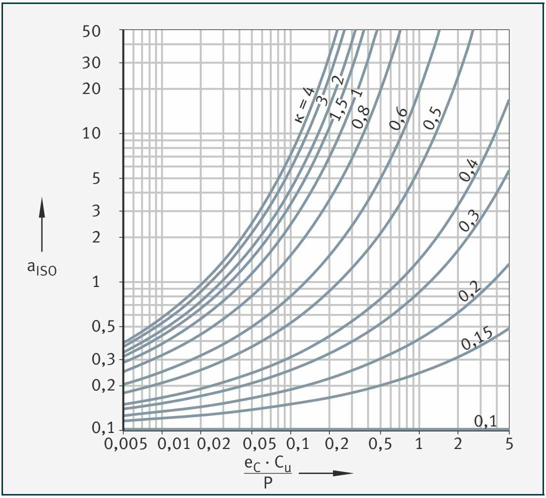
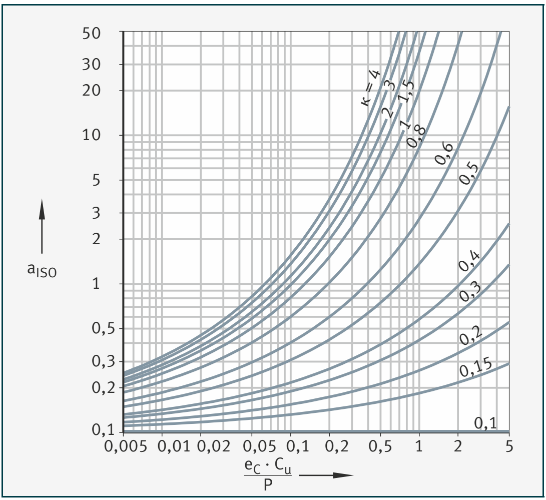


Figure 47
Life adjustment factor a_{ISO} for axial ball bearings

a_{ISO} = life adjustment factor
C_u = fatigue limit load
e_C = contamination factor
P = equivalent dynamic bearing load
κ = viscosity ratio



Fatigue limit load

The fatigue limit load C_u in accordance with ISO 281 is defined as the load below which, under laboratory conditions, no fatigue occurs in the material.

Viscosity ratio

The viscosity ratio κ is an indication of the quality of lubricant film formation:

Equation 37

$$\kappa = \frac{\nu}{\nu_1}$$

Legend

ν	mm^2/s	ν_1	mm^2/s
Kinematic viscosity of the lubricant at operating temperature		Reference viscosity of the lubricant at operating temperature.	

The reference viscosity ν_1 is determined from the mean bearing diameter $d_M = (D+d)/2$ and the operating speed n, see Figure 48, Page 536.

The nominal viscosity of the oil at +40°C is determined from the required operating viscosity ν and the operating temperature ϑ, see Figure 49, Page 536. In the case of greases, the operating viscosity ν of the base oil is the decisive factor.

In the case of heavily loaded bearings with a high proportion of sliding contact, the temperature in the contact area of the rolling elements may be up to 20 K higher than the temperature measured on the stationary ring (without the influence of any external heat sources).

For information on taking account of EP additives in calculation of the expanded adjusted rating life L_{nm}, see Page 532.

The following diagrams show methods for determining the reference viscosity ν_1 and the operating viscosity ν.

Figure 48
Determining the reference viscosity ν_1

ν_1 = reference viscosity
d_M = mean bearing diameter (d + D)/2
n = operating speed

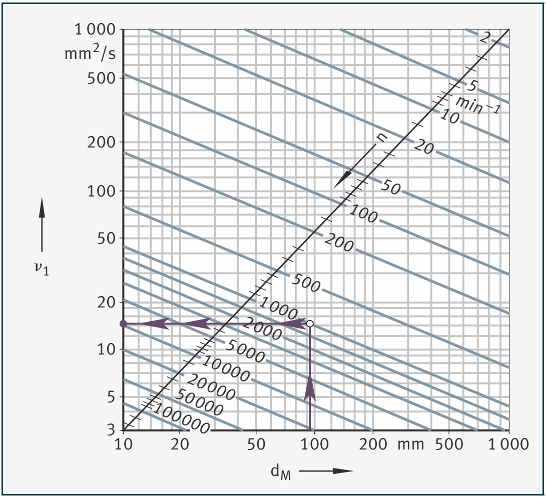
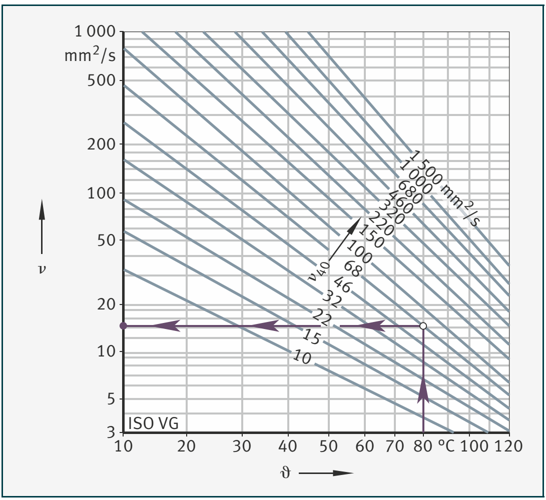


Figure 49
Determining the operating viscosity ν

ν = operating viscosity
ϑ = operating temperature
ν_{40} = viscosity at +40 °C



Life adjustment factor for contamination e_C
The life adjustment factor for contamination e_C takes account of the influence of contamination in the lubrication gap on the rating life.

The rating life is reduced by solid particles in the lubrication gap and is dependent on:

- the type, size, hardness and number of particles
- the relative lubricant film thickness
- the size of the bearing.

Due to the complex interactions between these influencing factors, it is only possible to give approximate guide values. The values in the tables are valid for contamination by solid particles (factor e_C). No account is taken of other contamination such as that caused by water or other fluids. Under severe contamination ($e_C \rightarrow 0$), the bearings may fail due to wear. In this case, the operating life is substantially less than the calculated life.

For contamination of various degrees of severity, the factors e_C are in accordance with the following table:

Contamination	**Contamination factor e_C**	
	$d_M < 100$ mm[1]	**$d_M \geqq 100$ mm[1]**
Extreme cleanliness ■ Particle size within the order of magnitude of the lubricant film thickness ■ Laboratory conditions	1	1
High cleanliness ■ Oil filtered through extremely fine filter ■ Sealed, greased bearings	0,8 to 0,6	0,9 to 0,8
Normal cleanliness ■ Oil filtered through fine filter	0,6 to 0,5	0,8 to 0,6
Slight contamination ■ Slight contamination of oil	0,5 to 0,3	0,6 to 0,4
Typical contamination ■ Bearing contaminated by wear debris from other machine elements	0,3 to 0,1	0,4 to 0,2
Heavy contamination ■ Bearing environment heavily contaminated ■ Bearing arrangement inadequately sealed	0,1 to 0	0,1 to 0
Very heavy contamination	0	0

[1] d_M = mean bearing diameter $(d + D)/2$.

Equivalent operating values

The rating life equations are based on the assumption that the bearing load P and bearing speed n are constant. If the load and speed are not constant, equivalent operating values can be determined that induce the same fatigue as the actual loading conditions.

The equivalent operating values calculated here already take account of the life adjustment factors a_{ISO}. This factor a_{ISO} must not be applied again in calculation of the expanded rating life. If only a basic rating life is to be calculated, the terms $1/a_{ISO}$ can be omitted from the equations.

Variable load and speed

If the load and speed vary over a time period T, the speed n and the equivalent bearing load P are calculated as follows (for an explanation of the designations used, see Page 540):

Equation 38

$$n = \frac{1}{T}\int_0^T n(t)\cdot dt$$

Equation 39

$$P = \sqrt[p]{\frac{\int_0^T \frac{1}{a_{ISO}(t)}\cdot n(t)\cdot F^p(t)\cdot dt}{\int_0^T n(t)\cdot dt}}$$

Variation in steps

If the load and speed vary in steps over a time period T, n and P are calculated as follows:

Equation 40

$$n = \frac{q_1\cdot n_1 + q_2\cdot n_2 + \ldots + q_z\cdot n_z}{100}$$

Equation 41

$$P = \sqrt[p]{\frac{\frac{1}{a_{ISO\,i}}\cdot q_i\cdot n_i\cdot F_i^p + \ldots + \frac{1}{a_{ISO\,z}}\cdot q_z\cdot n_z\cdot F_z^p}{q_i\cdot n_i + \ldots + q_z\cdot n_z}}$$

Variable load at constant speed

If the function F describes the variation in the load over a time period T and the speed is constant, P is calculated as follows:

Equation 42

$$P = \sqrt[p]{\frac{1}{T}\int_0^T \frac{1}{a_{ISO}(t)} \cdot F^p(t) \cdot dt}$$

Load varying in steps at constant speed

If the load varies in steps over a time period T and the speed is constant, P is calculated as follows:

Equation 43

$$P = \sqrt[p]{\frac{\frac{1}{a_{ISO\,i}} \cdot q_i \cdot F_i^p + \ldots + \frac{1}{a_{ISO\,z}} \cdot q_z \cdot F_z^p}{100}}$$

Constant load at variable speed

If the speed varies but the load remains constant, this gives:

Equation 44

$$n = \frac{1}{T}\int_0^T \frac{1}{a_{ISO}(t)} \cdot n(t) \cdot dt$$

Constant load at speed varying in steps

If the speed varies in steps, this gives:

Equation 45

$$n = \frac{\frac{1}{a_{ISO\,i}} \cdot q_i \cdot n_i + \ldots + \frac{1}{a_{ISO\,z}} \cdot q_z \cdot n_z}{100}$$

For oscillating bearing motion

The equivalent speed is calculated as follows:

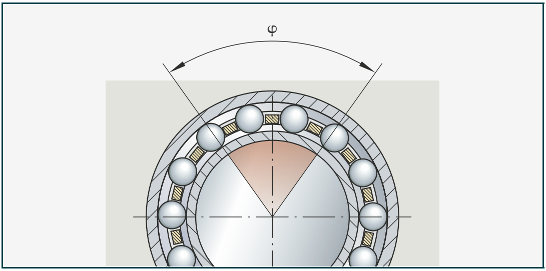
Equation 46

$$n = n_{osc} \cdot \frac{\varphi}{180°}$$

The equation is valid only if the swivel angle is larger than twice the pitch angle of the rolling elements. If the swivel angle is smaller, there is a risk of false brinelling.

The following diagram shows the swivel angle φ.

Figure 50
Swivel angle φ



Symbols, units and definitions
The following values are used in calculation of the equivalent operating values:

Legend

Symbol	Unit	Definition
n	min^{-1}	Mean speed
T	min	Time period under consideration
P	N	Equivalent bearing load
p	–	Life exponent; for roller bearings: $p = 10/3$ for ball bearings: $p = 3$
$a_{ISO\ i}$, $a_{ISO}(t)$	–	Life adjustment factor a_{ISO} for current operating condition, see Page 532
n_i, $n(t)$	min^{-1}	Bearing speed for a particular operating condition
q_i	%	Duration of an operating condition as a proportion of the total operating period; $q_i = (\Delta t_i/T) \cdot 100$
F_i, $F(t)$	N	Bearing load for a particular operating condition
n_{osc}	min^{-1}	Frequency of swivel motion
φ	°	Swivel angle, see Figure 50.

Requisite rating life

If no information is available on the rating life, guide values can be taken from tables or diagrams, such as Figure 51 and the following table. Guide values for dimensioning: see also Schaeffler catalogue HR 1, Rolling Bearings.

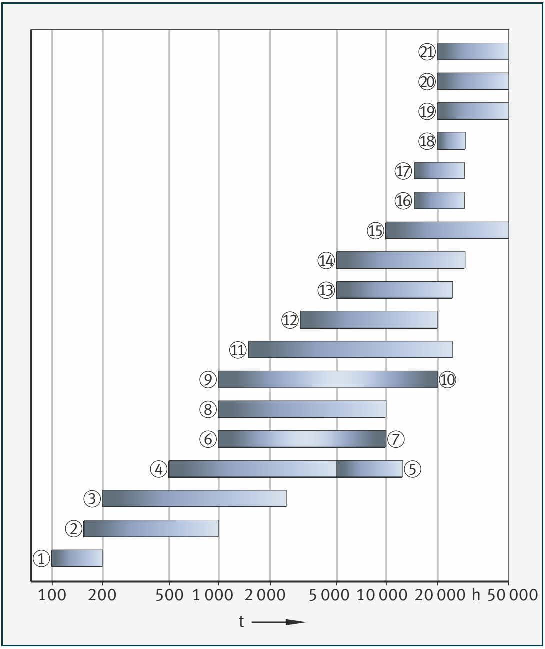
Bearings should not be overspecified, for the precise rating life see Schaeffler catalogue HR 1, Rolling Bearings.

Pay attention to the minimum load for the bearings, see Schaeffler catalogue HR 1, Rolling Bearings.

Figure 51
Guide values for rating life

t = operating hours

1 DIY tools
2 Power tools
3 Household appliances
4 Agricultural machinery
5 Tractors
6 Passenger cars
7 Commercial vehicles
8 Construction machinery
9 Hydraulic units, mobile
10 Hydraulic units, stationary
11 Office and computer equipment
12 Compressors
13 Handling equipment
14 Industrial gearboxes
15 Construction materials machinery
16 Crushers
17 Machine tools
18 Extruders
19 Rolling mills
20 Textile machinery
21 Printing machinery



Operating mode	Operating hours/year
Intermittent operation	≈ 500 to 1 000 h
Single shift operation	≈ 2 000 h
Double shift operation	≈ 4 000 h
Triple shift operation	≈ 6 000 h
Continuous operation	≈ 8 000 h

Static load carrying capacity

If high, static or shock loads occur, the raceways and rolling elements may undergo plastic deformation. These deformations limit the static load carrying capacity of the rolling bearing with respect to the permissible noise level during operation of the bearing.

If a rolling bearing operates without rotary motion or with only infrequent rotary motion, its size is determined in accordance with the basic static load rating C_0.

In accordance with DIN ISO 76, this is:

- a constant radial load C_{0r} for radial bearings
- a constant, concentrically acting axial load C_{0a} for axial bearings.

The basic static load rating C_0 is that load under which the Hertzian pressure at the most heavily loaded point between the rolling elements and raceways reaches the following values:

- for roller bearings, 4 000 N/mm^2
- for ball bearings, 4 200 N/mm^2
- for self-aligning ball bearings, 4 600 N/mm^2.

Under normal contact conditions, this load causes a permanent deformation at the contact points of approximately one tenth of a thousandth of the rolling element diameter.

Static load safety factor

In addition to dimensioning on the basis of the fatigue life, it is advisable to check the static load safety factor. The guide values for shock loads occurring in operation as given in the following table should be taken into consideration.

The static load safety factor S_0 is the ratio between the basic static load rating C_0 and the equivalent static bearing load P_0:

Equation 47

$$S_0 = \frac{C_0}{P_0}$$

Legend

S_0 –
Static load safety factor

C_0 (C_{0r}, C_{0a}) N
Basic static load rating

P_0 (P_{0r}, P_{0a}) N
Equivalent static load on the radial or axial bearing, see Page 543.

Guide values for static load safety factor

Guide values for axial spherical roller bearings and high precision bearings: see Schaeffler catalogue HR 1, Rolling Bearings. For drawn cup needle roller bearings, the value must be $S_0 \geqq 3$.

For the static load safety factor, the following guide values can be used:

Operating conditions and application	Static load safety factor S_0 min.	
	Roller bearings	Ball bearings
Low-noise, smooth running, free from vibrations, high rotational accuracy	2	3
Normal, smooth running, free from vibrations, normal rotational accuracy	1	1,5
Pronounced shock loading[1)]	1,5	3

1) If the order of magnitude of the shock loading is not known, the values used for S_0 should be at least 1,5. If the order of magnitude of the shock loading is known precisely, lower values are possible.

Equivalent static bearing load

The equivalent static bearing load P_0 is a calculated value. It corresponds to a radial load in radial bearings and a concentric axial load in axial bearings.

P_0 induces the same load at the centre point of the most heavily loaded contact point between the rolling element and raceway as the combined load occurring in practice.

The relationship is as follows:

Equation 48

$$P_0 = X_0 \cdot F_{0r} + Y_0 \cdot F_{0a}$$

Legend

P_0 N
Equivalent static bearing load

F_{0r} N
Largest radial load present

F_{0a} N
Largest axial load present

X_0 –
Radial load factor
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings

Y_0 –
Axial load factor
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings.

This calculation cannot be applied to radial needle roller bearings, axial needle roller bearings and axial cylindrical roller bearings. Combined loads are not permissible with these bearings.

In the case of radial needle roller bearings and all radial cylindrical roller bearings, $P_0 = F_{0r}$. For axial needle roller bearings and axial cylindrical roller bearings, $P_0 = F_{0a}$.

Operating life The operating life is defined as the life actually achieved by the bearing. It may differ significantly from the calculated value.

This may be due to wear or fatigue as a result of:

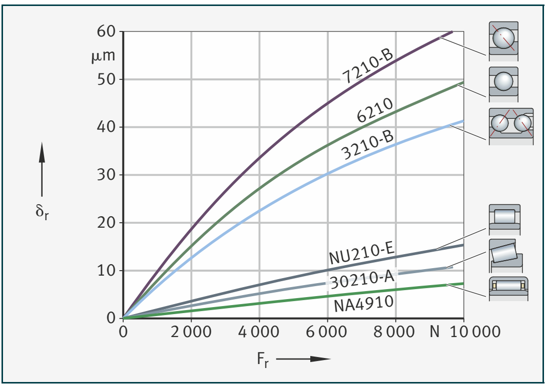
- deviations in the operating data
- misalignment between the shaft and housing
- insufficient or excessive operating clearance
- contamination
- inadequate lubrication
- excessive operating temperature
- oscillating bearing motion with very small swivel angles (false brinelling)
- high vibration loads and false brinelling
- very high shock loads (static overloading)
- prior damage during installation.

Due to the wide variety of possible mounting and operating conditions, it is not possible to precisely predetermine the operating life. The most reliable way of arriving at a close estimate is by comparison with similar applications.

Rigidity The rigidity is determined by the type, size and operating clearance of the bearing. It increases with the number of rolling elements supporting the load. Due to the line contact between the rolling elements and raceways, it is higher in roller bearings than in ball bearings. Figure 52 shows typical characteristic curves for the radial deflection of various bearings with the same bore diameter.

Figure 52
Radial deflection of various radial bearings of bore diameter d = 50 mm

δ_r = radial displacement
F_r = radial bearing load



Calculation of the radial or axial displacement

Rolling bearings have a progressive deflection rate. The displacement values for needle and cylindrical roller bearings can be determined using approximation equations. This simplified calculation cannot be applied to other bearing types. The displacement and rigidity at the operating point can be determined using the calculation program BEARINX-online.

The equations are valid for bearings without misalignment and with a rigid surrounding structure. In axial bearings, a concentrically acting load is assumed.

Equation 49

$$\delta_r = \frac{1}{c_s} \cdot F_r^{0,84} + \frac{s}{2}$$

Equation 50

$$\delta_a = \frac{1}{c_s} \cdot \left[\left(F_{aV} + F_a \right)^{0,84} - F_{aV}{}^{0,84} \right]$$

Equation 51

$$c_s = K_c \cdot d^{0,65}$$

Legend

c_s $N^{0,84}$ / µm
Rigidity parameter

d mm
Bearing bore diameter

δ_r µm
Radial displacement between shaft axis and bore centre, Figure 53, Page 546

δ_a µm
Axial displacement between shaft locating washer and housing locating washer, Figure 54, Page 546

s µm
Radial operating clearance of fitted, unloaded bearing

F_r N
Radial bearing load

F_a N
Axial bearing load

F_{aV} N
Axial preload force

K_c –
Factor for determining the rigidity parameter, see table Page 545.

Factor K_c

The factor K_c is given in the following table.

Bearing series	Factor K_c	Bearing series	Factor K_c	Bearing series	Factor K_c
NA48	24,9	NJ2..-E	11,1	SL1818	12,8
NA49	23,5	NJ3..-E	11,3	SL1829, SL1830, SL1923	16
NA69	37,3	NJ22..-E	15,4	SL1850, SL0148, SL0248, SL0249	29,2
NKIS	21,3	NJ23..-E	16,9	K811, 811, K812, 812	36,7
NKI	$4,4 \cdot B^{0,8}/d^{0,2}$	NU10	9,5		
HK, BK	$4,2 \cdot C^{0,8}/d^{0,2}$	NU19	11,3	K893, 893, K894, 894	59,7
		NN30..-AS-K	18,6		

The following diagrams show the radial and axial displacement.

Figure 53
Radial displacement – radial cylindrical roller bearings

δ_r = radial displacement
F_r = radial bearing load

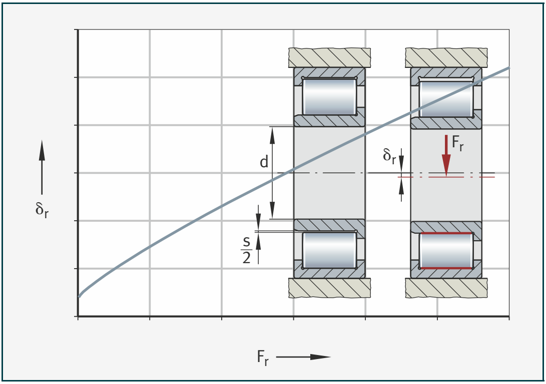
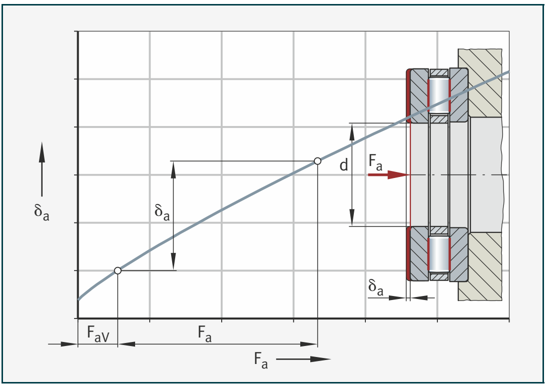


Figure 54
Axial displacement – axial cylindrical roller bearings

δ_a = axial displacement
F_a = axial bearing load
F_{aV} = axial preload force



Friction and increases in temperature

The friction in a rolling bearing is made up of several components. Due to the large number of influencing factors, such as dynamics in speed and load, tilting and skewing as a result of mounting, actual frictional torques and frictional power may deviate significantly from the calculated values.

If the frictional torque is an important design criterion, please consult Schaeffler. The calculation module BEARINX-online Easy Friction, which is available from Schaeffler free of charge, can be used to calculate and analyse the frictional torque.

The frictional component and the influencing factor are shown in the following table.

Frictional component	Influencing factor
Rolling friction	Magnitude of load
Sliding friction of rolling elements Sliding friction of cage	Magnitude and direction of load Speed and lubrication conditions, running-in condition
Fluid friction (flow resistance)	Type and speed Type, quantity and operating viscosity of lubricant
Seal friction	Type and preload of seal

The idling friction is dependent on the lubricant quantity, speed, operating viscosity of the lubricant, seals and the running-in condition of the bearing.

Heat dissipation

Friction is converted into heat. This must be dissipated from the bearing. The equilibrium between the frictional power and heat dissipation allows calculation of the thermally safe operating speed n_ϑ, see Page 558.

Heat dissipation via the lubricant
Lubricating oil dissipates a portion of the heat. Recirculating oil lubrication with additional cooling is particularly effective. Grease does not give dissipation of heat.

Heat dissipation via the shaft and housing
Heat dissipation via the shaft and housing is dependent on the temperature difference between the bearing and the surrounding structure.

Any additional adjacent sources of heat or thermal radiation must be taken into consideration.

Determining the friction values

For this process, the speed and load must be known. The type of lubrication, lubrication method and viscosity of lubricant at operating temperature are further important factors in calculation.

Total frictional torque M_R:
(calculation of axially loaded cylindrical roller bearings, see Page 555):

Equation 52

$$M_R = M_0 + M_1$$

Frictional power N_R:

Equation 53

$$N_R = M_R \cdot \frac{n}{9\,550}$$

Frictional torque as a function of speed for $\nu \cdot n \geqq 2\,000$:

Equation 54

$$M_0 = f_0 \cdot (\nu \cdot n)^{2/3} \cdot d_M{}^3 \cdot 10^{-7}$$

Frictional torque as a function of speed for $\nu \cdot n < 2\,000$:

Equation 55

$$M_0 = f_0 \cdot 160 \cdot d_M{}^3 \cdot 10^{-7}$$

Frictional torque as a function of load for needle roller and cylindrical roller bearings:

Equation 56

$$M_1 = f_1 \cdot F \cdot d_M$$

Frictional torque as a function of load for ball bearings, tapered roller bearings and spherical roller bearings:

Equation 57

$$M_1 = f_1 \cdot P_1 \cdot d_M$$

Legend

M_R Nmm
Total frictional torque

M_0 Nmm
Frictional torque as a function of speed

M_1 Nmm
Frictional torque as a function of load

N_R W
Frictional power

n min^{-1}
Operating speed

f_0 –
Bearing factor for frictional torque as a function of speed, see Figure 55, Page 549 and tables from Page 549 to Page 554

f_1 –
Bearing factor for frictional torque as a function of load, see tables from Page 549 to Page 554

ν mm^2s^{-1}
Kinematic viscosity of lubricant at operating temperature. In the case of grease, the decisive factor is the viscosity of the base oil at operating temperature

F_r, F_a N
Radial load for radial bearings, axial load for axial bearings

P_1 N
Decisive load for frictional torque. For ball bearings, tapered roller bearings and spherical roller bearings, see Page 554

d_M mm
Mean bearing diameter $(d + D)/2$.

Bearing factors

The bearing factors f_0 and f_1 are mean values derived from series of tests and correspond to the data according to ISO 15312.

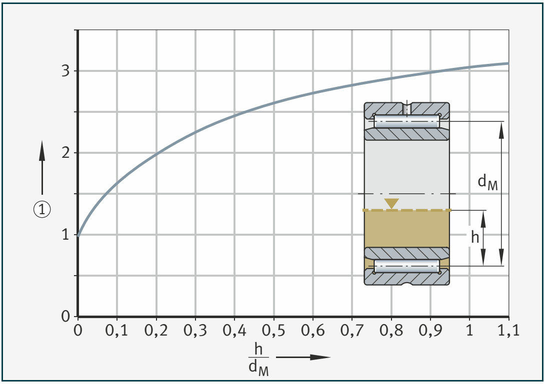
They are valid for bearings after running-in and with uniform distribution of lubricant. In the freshly greased state, the bearing factor f_0 can be two to five times higher.

If oil bath lubrication is used, the oil level must reach the centre of the lowest rolling element. If the oil level is higher, f_0 may be up to 3 times the value given in the table, Figure 55.

Figure 55
Increase in the bearing factor, as a function of the oil level

h = oil level
d_M = mean bearing diameter (d + D)/2

① Increase in the bearing factor f_0



The bearing factors for various rolling bearings are given in the following tables.

Bearing factors for needle roller bearings, drawn cup needle roller bearings, needle roller and cage assemblies:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
NA48	3	5	0,000 5
NA49	4	5,5	
RNA48	3	5	
RNA49	4	5,5	
NA69, RNA69	7	10	
NKI, NK, NKIS, NKS, NAO, RNO, RNAO, K	(12 · B)/(33 + d)	(18 · B)/(33 + d)	
NK..-TW, NKI..-TW, NK..-D	(10 · B)/(33 + d)	(15 · B)/(33 + d)	
HK, BK	(24 · B)/(33 + d)	(36 · B)/(33 + d)	
HN	(30 · B)/(33 + d)	(45 · B)/(33 + d)	

Bearing factors for cylindrical roller bearings, full complement:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
SL1818	3	5	0,000 55
SL1829	4	6	
SL1830	5	7	
SL1822	5	8	
SL0148, SL0248	6	9	
SL0149, SL0249	7	11	
SL1923	8	12	
SL1850	9	13	

Bearing factors for cylindrical roller bearings with cage:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
LSL1923	1	3,7	0,000 20
ZSL1923	1	3,8	0,000 25
NU2..-E, NNU41	1,3	2	0,000 30
NU3..-E			0,000 35
NU4			0,000 40
NU10, NU19			0,000 20
NU22..-E	2	3	0,000 40
NU23..-E	2,7	4	0,000 40
NU30..-E, NN30..-E	1,7	2,5	0,000 40

Bearing factors for axial roller bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
AXK, AXW	3	4	0,001 5
810, K810, 811, K811, 812, K812, 893, K893, 894, K894	2	3	

Bearing factors for combined bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
ZARN, ZARF	3	4	0,001 5
NKXR	2	3	
NX, NKX	2	3	$0{,}001 \cdot (P_0/C_0)^{0,33}$
ZKLN, ZKLF	4	6	0,000 5
NKIA, NKIB	3	5	

Bearing factors for tapered roller bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
302, 303, 329 320, 330, JK0S, T4CB, T4DB, T7FC	2	3	0,000 4
313, 322, 323, 331, 332, T2EE, T2ED, T5ED	3	4,5	

Bearing factors for axial and radial spherical roller bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
213..-E1	2,3	3,5	$0{,}000\,5 \cdot (P_0/C_0)^{0,33}$
222..-E1	2,7	4	
223	3	4,5	$0{,}000\,8 \cdot (P_0/C_0)^{0,33}$
238, 239, 230			$0{,}000\,75 \cdot (P_0/C_0)^{0,5}$
231	3,7	5,5	$0{,}001\,2 \cdot (P_0/C_0)^{0,5}$
232	4	6	$0{,}001\,6 \cdot (P_0/C_0)^{0,5}$
240	4,3	6,5	$0{,}001\,2 \cdot (P_0/C_0)^{0,5}$
248, 249, 241	4,7	7	$0{,}002\,2 \cdot (P_0/C_0)^{0,5}$
292..-E	1,7	2,5	0,000 23
293..-E	2	3	0,000 30
294..-E	2,2	3,3	0,000 33

Bearing factors for toroidal roller bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
C22..-K	3,7	5,5	$0{,}0012 \cdot (P_0/C_0)^{0,5}$
C22..-V	4	6	
C23..-K	3,8	5,7	$0{,}0014 \cdot (P_0/C_0)^{0,5}$
C23..-V	4,3	6,5	
C30..-K	3,3	5	
C30..-V, C31..-V	4	6	
C31..-K	3,7	5,5	
C32..-K	3,8	5,7	$0{,}0016 \cdot (P_0/C_0)^{0,5}$
C39..-K	3,3	5	$0{,}0014 \cdot (P_0/C_0)^{0,5}$
C40..-K, C41..-K	5	7,5	$0{,}0018 \cdot (P_0/C_0)^{0,5}$
C40..-V, C41..-V	6	9	

Bearing factors for deep groove ball bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
618	1,1	1,7	$0{,}0005 \cdot (P_0/C_0)^{0,5}$
160, 60, 619	1,1	1,7	$0{,}0007 \cdot (P_0/C_0)^{0,5}$
622, 623	1,1	1,7	$0{,}0009 \cdot (P_0/C_0)^{0,5}$
62	1,3	2	
63, 630, 64	1,5	2,3	
60..-C	1,1	1,5	$0{,}0006 \cdot (P_0/C_0)^{0,5}$
62..-C	1,3	1,7	$0{,}0007 \cdot (P_0/C_0)^{0,5}$
63..-C	1,5	2	
42..-B	2,3	3,5	$0{,}0010 \cdot (P_0/C_0)^{0,5}$
43..-B	4	6	

Bearing factors for angular contact ball bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
708, 719, 718..-B, 70..-B, 72..-B	1,3	2	$0{,}001 \cdot (P_0/C_0)^{0,33}$
73..-B	2	3	
74..-B	2,5	4	
30..-B, 32..-B, 38..-B	2,3	3,5	
33..-B	4	6	
32..-BD	2	3	
33..-BD	3,5	5	

Bearing factors for self-aligning ball bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
10, 112	1,7	2	$0{,}000\,3 \cdot (P_0/C_0)^{0,4}$
12	1,7	2,5	
13	2,3	3,5	
22	2	3	
23	2,7	4	

Bearing factors for four point contact bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
QJ2	1,3	2	$0{,}001 \cdot (P_0/C_0)^{0,33}$
QJ3	2	3	
QJ10	1,3	2	

Bearing factors for axial deep groove ball bearings:

Series	Bearing factor f_0		Bearing factor f_1
	Grease, oil mist	Oil bath, recirculating oil	
511, 512, 513, 514, 532, 533, 534	1	1,5	$0{,}0012 \cdot (F_a/C_0)^{0{,}33}$
522, 523, 524, 542, 543, 544	1,3	2	

The following table shows the decisive load P_1 for the frictional torque as a function of load M_1 for ball bearings, tapered roller bearings and spherical roller bearings:

Bearing type	Decisive load P_1	
	Single bearing	Bearing pair
Deep groove ball bearings	$3{,}3 \cdot F_a - 0{,}1 \cdot F_r$	–
Angular contact ball bearings, single row	$F_a - 0{,}1 \cdot F_r$	$1{,}4 \cdot F_a - 0{,}1 \cdot F_r$
Angular contact ball bearings, double row	$1{,}4 \cdot F_a - 0{,}1 \cdot F_r$	–
Four point contact bearings	$1{,}5 \cdot F_a + 3{,}6 \cdot F_r$	–
Tapered roller bearings	$2 \cdot Y \cdot F_a$ or F_r, use the greater value	$1{,}21 \cdot Y \cdot F_a$ or F_r, use the greater value
Spherical roller bearings	$1{,}6 \cdot F_a/e$, if $F_a/F_r > e$ $F_r \cdot \{1 + 0{,}6 \cdot [F_a/(e \cdot F_r)]^3\}$, if $F_a/F_r \leqq e$	
Cylindrical roller bearings	F_r, the frictional component of axial load F_a must be taken into account using M_2	

If $P_1 \leqq F_r$, then $P_1 = F_r$.

Cylindrical roller bearings under axial load

In radial cylindrical roller bearings under axial load, sliding friction between the end faces of the rolling elements and the ribs on the rings leads to an additional frictional torque M_2.

The total frictional torque M_R is therefore calculated as follows:

Equation 58

$$M_R = M_0 + M_1 + M_2$$

Equation 59

$$M_2 = f_2 \cdot F_a \cdot d_M$$

Equation 60

$$A = k_B \cdot 10^{-3} \cdot d_M{}^{2,1}$$

Legend

M_R Nmm
Total frictional torque

M_0 Nmm
Frictional torque as a function of speed

M_1 Nmm
Frictional torque as a function of radial load

M_2 Nmm
Frictional torque as a function of axial load

f_2 –
Factor as a function of the bearing series, Figure 56 and Figure 57, Page 556

A –
Bearing parameter

F_a N
Axial dynamic bearing load

k_B –
Bearing factor as a function of the bearing series, see table, Page 557

d_M mm
Mean bearing diameter (d + D)/2.

The bearing factors f_2 are subject to wide scatter. They are valid for recirculating oil lubrication with an adequate quantity of oil. The curves must not be extrapolated, Figure 56 and Figure 57, Page 556.

Bearings of TB design

In the case of bearings of TB design (bearings with a toroidal roller end), the axial load carrying capacity has been significantly improved through the use of new calculation and manufacturing methods.

Optimum contact conditions between the roller and rib are ensured by means of a special curvature of the roller end faces. As a result, axial surface pressures on the rib are significantly reduced and a lubricant film with improved load carrying capacity is achieved. Under normal operating conditions, wear and fatigue at the rib contact running and roller end faces is completely eliminated.

In addition, axial frictional torque is reduced by up to 50%. The bearing temperature during operation is therefore significantly lower.

Bearing factor k_B

The bearing factor k_B in the equations takes into consideration the size and thus the load carrying capacity of the hydrodynamic contacts at the bearing ribs; see table Page 557.

The following diagrams show the bearing factors for cylindrical roller bearings.

Figure 56
Bearing factor f_2, as a function of operating parameter

Cylindrical roller bearings of standard design

f_2 = bearing factor
ν = operating viscosity
n = operating speed
d_M = mean bearing diameter
$\nu \cdot n \cdot d_M$ = operating parameter
F_a = axial dynamic bearing load
A = bearing parameter

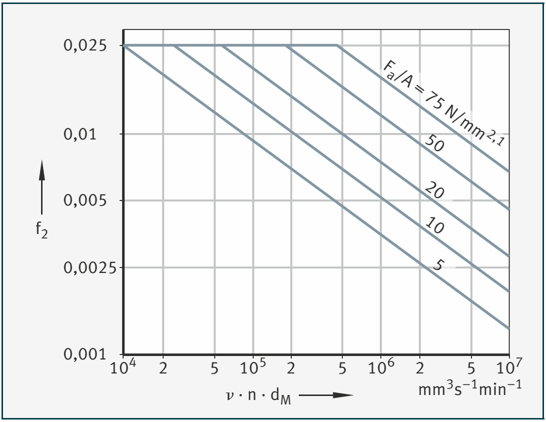
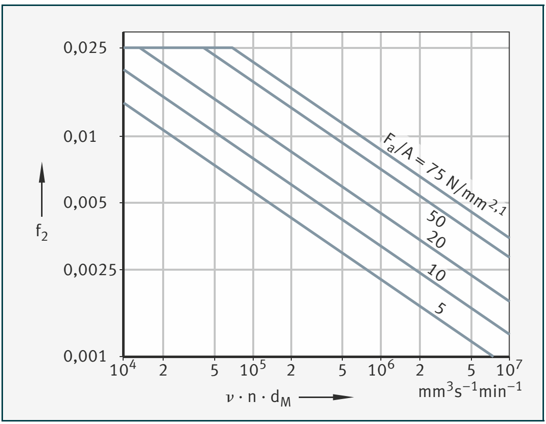


Figure 57
Bearing factor f_2, as a function of operating parameter

Cylindrical roller bearings of TB design

f_2 = bearing factor
ν = operating viscosity
n = operating speed
d_M = mean bearing diameter
$\nu \cdot n \cdot d_M$ = operating parameter
F_a = axial dynamic bearing load
A = bearing parameter



Bearing factor k_B

Bearing series	Bearing factor k_B	Bearing series	Bearing factor k_B
SL1818, SL0148	4,5	SL1923	30
SL1829, SL0149	11	NJ2..-E, NJ22..-E, NUP2..-E, NUP22..-E	15
SL1830, SL1850	17		
SL1822	20	NJ3..-E, NJ23..-E, NUP3..-E, NUP23..-E	20
LSL1923, ZSL1923	28	NJ4	22

Speeds

Calculation of the thermal speed rating $n_{\vartheta r}$ is standardised in ISO 15312:2003.

Limiting speed

The limiting speed n_G is based on practical experience and takes account of additional criteria such as smooth running, sealing function and centrifugal forces. The limiting speed must not be exceeded even under favourable operating and cooling conditions.

Thermal reference speed

The thermal speed rating $n_{\vartheta r}$ is used as an ancillary value in calculating the thermally safe operating speed n_{ϑ}. This is the speed at which, under defined reference conditions, a bearing operating temperature of +70 °C is achieved. The thermal speed rating is not a speed limit for the application of a bearing. It is primarily for the purpose of comparing the speed suitability of different bearing types under defined reference conditions.

A speed limit taking account of the thermal balance can be calculated using the thermally safe operating speed.

Reference conditions

The reference conditions are based on the normal operating conditions of the most significant bearing types and sizes and are defined in ISO 15312 as follows:

- mean ambient temperature ϑ_{Ar}= +20 °C
- mean bearing temperature at the outer ring ϑ_r = +70 °C
- load on radial bearings (pure, constant radial load) $P_1 = 0{,}05 \cdot C_{0r}$
- load on axial bearings (concentrically acting, constant axial load) $P_1 = 0{,}02 \cdot C_{0a}$
- oil bath lubrication with an oil level up to the centre of the lowest rolling element
- lubricant with a kinematic viscosity ν_r at ϑ_r = +70 °C
  - radial bearings: $\nu_r = 12\ mm^2/s$ (ISO VG 32)
  - axial bearings: $\nu_r = 24\ mm^2/s$ (ISO VG 68)
- heat dissipation via the bearing seating surfaces. As a simplification, calculation is based on the bearing seating surfaces dissipating heat under the reference conditions A_r. For the calculation of the bearing seating surfaces dissipating heat under the reference conditions, see Legend, Page 561. The relationships described below apply in this case.

For radial bearings with a heat-dissipating bearing seating surface under reference conditions, $A_r \leqq 50\,000\ \text{mm}^2$:

Equation 61

$$q_r = 0{,}016\ \text{W/mm}^2$$

For radial bearings with a heat-dissipating bearing seating surface under reference conditions, $A_r > 50\,000\ \text{mm}^2$:

Equation 62

$$q_r = 0{,}016 \cdot \left(\frac{A_r}{50\,000}\right)^{-0{,}34} \text{W/mm}^2$$

For axial bearings with a heat-dissipating bearing seating surface under reference conditions, $A_r \leqq 50\,000\ \text{mm}^2$:

Equation 63

$$q_r = 0{,}020\ \text{W / mm}^2$$

For axial bearings with a heat-dissipating bearing seating surface under reference conditions, $A_r > 50\,000\ \text{mm}^2$:

Equation 64

$$q_r = 0{,}020 \cdot \left(\frac{A_r}{50\,000}\right)^{-0{,}16} \text{W/mm}^2$$

Thermally safe operating speed

The thermally safe operating speed n_ϑ is calculated in accordance with DIN 732:2010. The basis for the calculation is the heat balance in the bearing, the equilibrium between the frictional energy as a function of speed and the heat dissipation as a function of temperature. When conditions are in equilibrium, the bearing temperature is constant.

The permissible operating temperature determines the thermally safe operating speed n_ϑ of the bearing. The preconditions for calculation are correct mounting, normal operating clearance and constant operating conditions.

The calculation method is not valid for:

- sealed bearings with contact seals, since the maximum speed is restricted by the permissible sliding velocity at the seal lip
- yoke and stud type track rollers
- aligning needle roller bearings
- axial deep groove ball bearings and axial angular contact ball bearings.

The limiting speed n_G must always be observed.

Calculation of the thermally safe operating speed

The thermally safe operating speed n_ϑ is a product of the thermal reference speed $n_{\vartheta r}$ and the speed ratio f_n:

Equation 65

$$n_\vartheta = n_{\vartheta r} \cdot f_n$$

The equation derived from the equilibrium of the frictional power and the heat dissipation can be expressed, through the introduction of a lubricant parameter K_L, a load parameter K_P and the speed ratio f_n in the following form:

Equation 66

$$K_L \cdot f_n^{5/3} + K_P \cdot f_n = 1$$

The speed ratio f_n can be determined, in the normal value range of $0{,}01 \leqq K_L \leqq 10$ and $0{,}01 \leqq K_P \leqq 10$, using an approximation equation, see also Figure 58, Page 560:

Equation 67

$$f_n = 490{,}77 \cdot \left(1 + 498{,}78 \cdot K_L^{0{,}599} + 852{,}88 \cdot K_P^{0{,}963} - 504{,}5 \cdot K_L^{0{,}055} \cdot K_P^{0{,}832}\right)^{-1}$$

Heat dissipation via the bearing seating surfaces $\dot{Q}_S$, Figure 59, Page 560:

Equation 68

$$\dot{Q}_S = k_q \cdot A_S \cdot \Delta\vartheta_A$$

Heat dissipation via the lubricant $\dot{Q}_L$:

Equation 69

$$\dot{Q}_L = 0{,}028\,6 \frac{\text{kW}}{\text{l/min} \cdot \text{K}} \cdot \dot{V}_L \cdot \Delta\vartheta_L$$

Total dissipated heat flow $\dot{Q}$:

Equation 70

$$\dot{Q} = \dot{Q}_S + \dot{Q}_L + \dot{Q}_E$$

Lubricant parameter K_L:

Equation 71

$$K_L = 10^{-6} \cdot \frac{\pi}{30} \cdot n_{\vartheta r} \cdot \frac{10^{-7} \cdot f_0 \cdot \left(\nu \cdot n_{\vartheta r}\right)^{\frac{2}{3}} \cdot d_M^{\,3}}{\dot{Q}}$$

Load parameter K_P:

Equation 72

$$K_P = 10^{-6} \cdot \frac{\pi}{30} \cdot n_{\vartheta r} \cdot \frac{f_1 \cdot P_1 \cdot d_M}{\dot{Q}}$$

The following diagrams show the speed ratio and the heat transition coefficient.

Figure 58
Speed ratio

f_n = speed ratio
K_L = lubricant parameter
K_P = load parameter

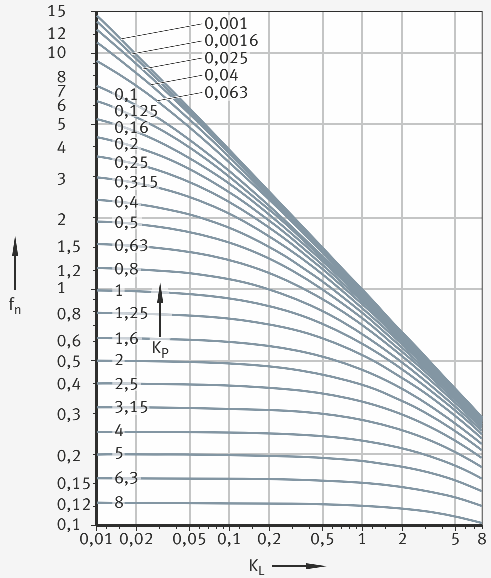
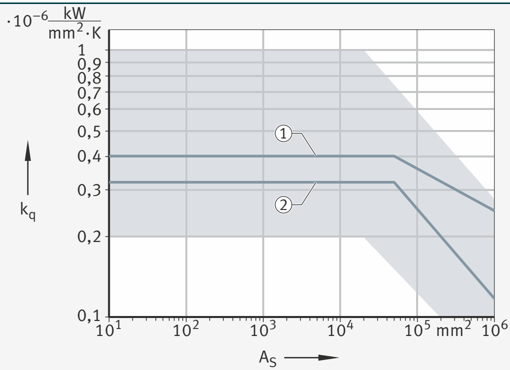


Figure 59
Heat transition coefficient, as a function of the bearing seating surface

k_q = heat transition coefficient
A_S = heat-dissipating bearing seating surface

① Reference condition for radial bearings
② Reference condition for axial bearings



Symbols, units and definitions

The following values are used in calculation of the thermally safe operating speed n_ϑ:

Legend

A_S mm^2
Heat-dissipating bearing seating surface: In general, $A_S = A_r$

A_r mm^2
Heat-dissipating bearing seating surface under reference conditions:
radial bearings: $A_r = \pi \cdot B \cdot (D + d)$
axial bearings: $A_r = (\pi/2) \cdot (D^2 - d^2)$
tapered roller bearings: $A_r = \pi \cdot T \cdot (D + d)$
axial spherical roller bearings:
$A_r = (\pi/4) \cdot (D^2 + d_1{}^2 - D_1{}^2 - d^2)$

B mm
Bearing width

d mm
Bearing bore diameter

D mm
Bearing outside diameter

d_1 mm
Outside diameter of shaft locating washer

D_1 mm
Inside diameter of housing locating washer

d_M mm
Mean bearing diameter $(D + d)/2$

f_0 –
Bearing factor for frictional torque as a function of speed, see section Friction and increases in temperature, Page 546

f_1 –
Bearing factor for frictional torque as a function of load, see section Friction and increases in temperature, Page 546

f_n –
Speed ratio, Figure 58, Page 560

K_L –
Lubricant parameter

K_P –
Load parameter

n_ϑ min^{-1}
Thermally safe operating speed

$n_{\vartheta r}$ min^{-1}
Thermal speed rating
Product tables: see Schaeffler catalogue HR 1, Rolling Bearings

k_q 10^{-6} kW/(mm$^2 \cdot$ K)
Heat transition coefficient of bearing seating surface, Figure 59, Page 560. This is dependent on the housing design and size, the housing material and the mounting situation. For normal applications, the heat transition coefficient for bearing seating surfaces up to 25,000 mm^2 is between 0,2 and $1{,}0 \cdot 10^{-6}$ kW/(mm$^2 \cdot$ K)

P_1 N
Radial load for radial bearings, axial load for axial bearings

q_r W/mm^2
Heat flow density

$\dot{Q}$ kW
Total dissipated heat flow

$\dot{Q}_E$ kW
Heat flow due to heating by external source

$\dot{Q}_L$ kW
Heat flow dissipated by the lubricant

$\dot{Q}_S$ kW
Heat flow dissipated via the bearing seating surfaces

T mm
Total width of tapered roller bearing

$\dot{V}_L$ l/min
Oil flow

$\Delta\vartheta_A$ K
Differential between mean bearing temperature and ambient temperature

$\Delta\vartheta_L$ K
Differential between oil inlet temperature and oil outlet temperature

ν mm^2/s
Kinematic viscosity of lubricant at operating temperature.

Noise Schaeffler Noise Index

The Schaeffler Noise Index (SGI) has been developed as a new feature for comparing the noise level of different bearing types. The SGI value is based on the maximum permissible noise level of a bearing in accordance with internal standards, which is calculated on the basis of ISO 15242. In order that different bearing types and series can be compared, the value is plotted against the basic static load rating C_0, allowing the direct comparison of bearings of the same load carrying capacity. The upper limit value is given in the diagrams. This means that the average noise level of the bearings is lower than illustrated in the diagram.

Until now, the Noise Index has only been available for the main series of radial deep groove ball bearings, radial angular contact ball bearings, tapered roller bearings and cylindrical roller bearings. Additional bearing types and series will be updated and introduced in subsequent publications.

The SGI is an additional performance characteristic in the selection of bearings for noise-sensitive applications. The specific suitability of a bearing for an application in terms of installation space, load carrying capacity or speed limit for example, must be checked independently of this.

The following section presents the diagrams for radial deep groove ball bearings and tapered roller bearings and an example assessment is shown using the radial deep groove ball bearings. Diagrams for other types of bearings can be found in the Schaeffler catalogue HR 1, Rolling Bearings, or in the electronic product catalogue ***medias:*** *http://medias.schaeffler.com*.

Figure 60
Example: Comparison of deep groove ball bearings using the Schaeffler Noise Index

SGI = Schaeffler Noise Index
C_0 = basic static load rating
① = standard series 62
② = series 62..-C (Generation C)

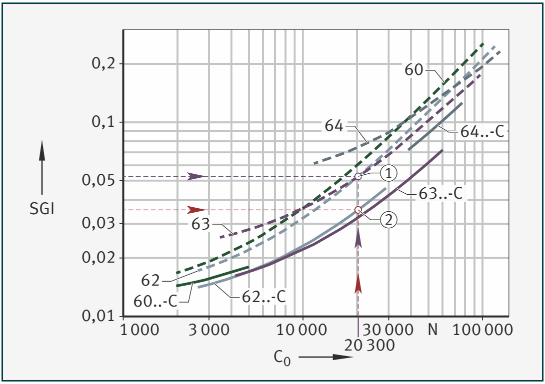
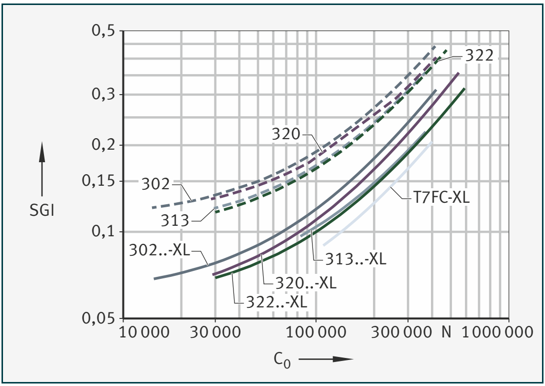


Figure 61
Schaeffler Noise Index for tapered roller bearings

SGI = Schaeffler Noise Index
C_0 = basic static load rating



Example of Noise Index calculation
If the requisite basic load rating is known for an application, the bearing arrangement can also be designed using the Noise Index as an additional performance characteristic. If the requisite basic static load rating is C_0 = 20 300 N, for example, various ball bearings are available with a different SGI value, see Figure 60, Page 563. As a result, the calculation can be carried out for the bearing application using the smallest SGI value. In particular, bearings of Generation C offer an advantage in this respect.

Lubrication

Lubrication and maintenance are important for the reliable operation and long operating life of rolling bearings.

Functions of the lubricant

The lubricant should:

- form a lubricant film sufficiently capable of supporting loads on the contact surfaces and thus prevent wear and premature fatigue
- dissipate heat in the case of oil lubrication
- provide additional sealing for the bearing against external solid and fluid contaminants in the case of grease lubrication
- give damping of running noise and protect the bearing against corrosion.

Selection of the type of lubrication

It should be determined as early as possible in the design process whether bearings should be lubricated using grease or oil. The following factors are decisive in determining the type of lubrication and quantity of lubricant: the operating conditions, the type and size of the bearing, the adjacent construction and the type of lubricant feed.

Characteristics of grease lubrication

In the case of grease lubrication, the following advantages and disadvantages must be considered:

- very little design work required
- the sealing action and reservoir action
- long operating life with little maintenance work (lifetime lubrication possible in certain circumstances)
- in the case of relubrication, the provision of collection areas for old grease and feed ducts
- no heat dissipation by the lubricant
- no rinsing out of wear debris and other particles.

Characteristics of oil lubrication

In the case of oil lubrication, the following criteria must be considered:

- good lubricant distribution and supply to contact areas
- the possibility for dissipation of heat from the bearing (significant principally at high speeds and loads)
- rinsing out of wear debris
- very low friction losses with minimal quantity lubrication
- more demanding requirements in terms of feed and sealing.

Grease lubrication

For grease lubrication, rolling bearing greases K in accordance with DIN 51825 are suitable.

Greases should be selected in accordance with the operating conditions of the bearing: temperature, pressure capacity, speed, water and moisture.

Pressure capacity

The viscosity at operating temperature must be sufficiently high for the formation of a lubricant film capable of supporting loads. At high loads, greases with EP (extreme pressure) characteristics and high base oil viscosity should be used (KP grease to DIN 51825). Such greases should also be used for bearings with a substantial sliding component and with line contact.

Silicone greases can only be used at low loads ($P \leqq 0,03 \cdot C$).

Greases with solid lubricants should preferably be used for applications with mixed or boundary friction conditions. The solid lubricant particle size should not exceed 5 μm.

Speed

Greases should be selected in accordance with the speed parameter $n \cdot d_M$ for grease, see table Lubricating greases, Page 566:

- For rolling bearings running at high speeds or with a low starting torque, greases with a high speed parameter should be selected
- For bearings running at low speeds, greases with a low speed parameter should be used.

Under centrifugal accelerations $> 500 \cdot g$, separation (of the thickener and base oil) may occur. In this case, please consult the lubricant manufacturer.

The consistency of polycarbamide greases can be altered by shear stresses to a greater extent than that of metal soap greases.

Water and moisture

Water in the grease has a highly detrimental effect on the operating life of bearings:

- The static behaviour of greases in the presence of water is assessed in accordance with DIN 51807
- The anti-corrosion characteristics can be tested in accordance with DIN 51802 (Emcor test) (information in the datasheets of grease manufacturers).

Lubricating greases The following greases have proved particularly suitable:

Designation[1]	Classification	Type of grease
GA01	Ball bearing grease for $\vartheta < +180$ °C	Polycarbamide Ester oil
GA02	Ball bearing grease for $\vartheta < +160$ °C	Polycarbamide SHC
GA13	Standard ball bearing and insert bearing grease for $D > 62$ mm	Lithium soap Mineral oil
GA14	Low noise ball bearing grease for $D \leqq 62$ mm	Lithium soap Mineral oil
GA15	Low noise ball bearing grease for high speeds	Lithium soap Ester oil/SHC
GA22	Free-running grease with low frictional torque	Lithium soap Ester oil, mineral oil
L069[4]	Insert bearing grease for wide temperature range	Polycarbamide Ester oil
GA08	Grease for line contact	Lithium complex soap Mineral oil
GA26	Standard grease for drawn cup roller clutches	Calcium/lithium soap Mineral oil
GA28	Screw drive bearing grease	Lithium soap Synthetic oil/mineral oil
GA11	Rolling bearing grease resistant to media for temperatures up to +250 °C	PTFE Alkoxyfluoroether
GA47	Rolling bearing grease resistant to media for temperatures up to +140 °C	Barium complex soap Mineral oil

[1] GA.. stands for **G**rease **A**pplication Group.., based on Grease Spec 00.

[2] The upper continuous limit temperature $\vartheta_{upperlimit}$ must not be exceeded if a reduction in the grease operating life due to temperature is to be avoided.

[3] Dependent on bearing type.

[4] Since January 2008, the grease L069 has been used in insert bearings instead of L014 and L086.

Operating temperature range °C	Upper continuous limit temperature $\vartheta_{upperlimit}$[2] °C	NLGI class	Speed parameter $n \cdot d_M$ $min^{-1} \cdot mm$	ISO VG class (base oil)[3]	Designation[1]	Recommended Arcanol grease for relubrication
−30 to +180	+125	2 to 3	600 000	68 to 220	**GA01**	–
−40 to +160	+90	2 to 3	500 000	68 to 220	**GA02**	–
−20 to +120	+75	3	500 000	68 to 150	**GA13**	**MULTI3**
−30 to +120	+75	2	500 000	68 to 150	**GA14**	**MULTI2**
−40 to +120	+75	2 to 3	1 000 000	22 to 32	**GA15**	–
−50 to +120	+70	2	1 500 000	10 to 22	**GA22**	–
−40 to +180	+120	2	700 000	68 to 220	**L069**[4]	–
−20 to +140	+95	2 to 3	500 000	150 to 320	**GA08**	**LOAD150**
−20 to +80	+60	2	500 000	10 to 22	**GA26**	–
−30 to +140	+80	2	800 000	15 to 100	**GA28**	**MULTITOP**
−30 to +260	+200	2	300 000	460 to 680	**GA11**	**TEMP200**
−20 to +130	+70	1 to 2	350 000	150 to 320	**GA47**	–

Oil lubrication

For the lubrication of rolling bearings, mineral oils and synthetic oils are essentially suitable. Oils with a mineral oil base are used most frequently. They must, as a minimum, fulfil the requirements in accordance with DIN 51517 or DIN 51524.

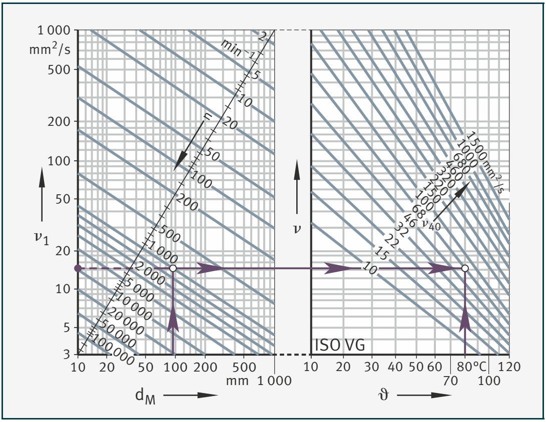
Reference viscosity for mineral oils

The guide value for ν_1 is dependent on the mean bearing diameter d_M and the speed n. It takes account of the EHD theory of lubricant film formation and practical experience. Depending on the operating speed, the oil at operating temperature must as a minimum have the reference viscosity ν_1, see Figure 62.

Figure 62

Reference viscosity and ν/ϑ diagram for mineral oils

n = operating speed
ν_1 = reference viscosity
ν = operating viscosity
d_M = mean bearing diameter (d + D)/2
ϑ = operating temperature
ν_{40} = viscosity at +40 °C



Calculation of reference viscosity

The reference viscosity ν_1 is determined as follows:

- Allocate ν_1 to an ISO VG nominal viscosity between 10 and 1500 (mid-point viscosity in accordance with DIN 51519)
- Round intermediate values to the nearest ISO VG (due to the steps between groups).

This method cannot be used for synthetic oils, since these have different V/P (viscosity/pressure) and V/T (viscosity/temperature) characteristics.

Influence of temperature on viscosity

As the temperature increases, the viscosity of the oil decreases. This temperature-dependent change in the viscosity is described using the viscosity index VI. In the case of mineral oils, the viscosity index should be at least 95.

When selecting the viscosity, the lower operating temperature must be taken into consideration, since the increasing viscosity will reduce the flowability of the lubricant. As a result, the level of power losses may increase.

A very long life can be achieved with a viscosity ratio $\kappa = \nu/\nu_1 = 3$ to 4. Highly viscous oils do not, however, bring only advantages. In addition to the power losses arising from lubricant friction, there may be problems with the feed and removal of oil at low or even at normal temperatures.

The oil selected must be sufficiently viscous that it gives the highest possible fatigue life. At the same time, it must be ensured that the bearings are always supplied with adequate quantities of oil.

Pressure capacity and anti-wear additives

If the bearings are subjected to high loads or the operating viscosity ν is less than the reference viscosity ν_1, oils with anti-wear additives (type P in accordance with DIN 51502) should be used. Such oils are also necessary for rolling bearings with a substantial proportion of sliding contact (for example, bearings with line contact).

These additives form boundary layers to reduce the harmful effects of metallic contact occurring at various points (wear). The suitability of these additives varies and is normally heavily dependent on temperature. Their effectiveness can only be assessed by means of testing in the rolling bearing (for example on a test rig FE8 in accordance with DIN 51819).

Silicone oils should only be used for low loads ($P \leqq 0{,}03 \cdot C$).

Lubrication methods

The essential lubrication methods are:

- drip feed oil lubrication
- pneumatic oil lubrication (also used, in order to protect the environment, as a substitute for oil mist lubrication)
- oil bath lubrication (immersion or sump lubrication)
- recirculating oil lubrication.

Drip feed oil lubrication

This is suitable for bearings running at high speeds. The oil quantity required is dependent on the type and size of bearing, the operating speed and the load. The guide value is between 3 drops/min and 50 drops/min for each rolling element raceway (one drop weighs approx. 0,025 g). Excess oil must be allowed to flow out of the bearing arrangement.

Pneumatic oil lubrication

This method is particularly suitable for radial bearings running at high speeds and under low loads ($n \cdot d_M$ = 800 000 to 3 000 000 $\text{min}^{-1} \cdot \text{mm}$). Oil is fed to the bearing by means of clean, compressed air. As a result, an excess pressure is generated. This prevents contaminants from entering the bearing.

With a pneumatic oil lubrication system designed for minimal quantity lubrication, low frictional torque and a low operating temperature can be achieved.

Oil bath lubrication

The oil level should reach the centre line of the lowest rolling element. If the oil level is higher than this, the bearing temperature may increase at high circumferential velocities (with losses due to splashing). Furthermore, foaming of the oil may occur.

The speed capacity is generally up to $n \cdot d_M = 300\,000\ \text{min}^{-1} \cdot \text{mm}$. At $n \cdot d_M < 150\,000\ \text{min}^{-1} \cdot \text{mm}$, the bearing may be completely immersed.

In bearings with an asymmetrical cross-section (such as angular contact ball bearings), oil return ducts must be provided due to the pumping effect so that recirculation can be achieved.

In axial bearings, the oil level must cover the inside diameter of the axial cage.

The oil quantity in the housing must be adequately proportioned, otherwise very short oil change intervals will be necessary.

Recirculating oil lubrication

During recirculating oil lubrication, the oil is cooled. The oil can therefore dissipate heat from the bearing. The quantity of oil required for heat dissipation is dependent on the cooling conditions, see section Speeds, Page 557.

For bearings with an asymmetrical cross-section (such as angular contact ball bearings, tapered roller bearings, axial spherical roller bearings), larger throughput quantities are permissible due to the pumping effect than for bearings with a symmetrical cross-section. Large quantities can be used to dissipate wear debris or heat.

Design of adjacent construction for oil lubrication

The lubrication holes in the housing and shaft must align with those in the rolling bearings.
Adequate cross-sections must be provided for annular slots, pockets, etc. The oil must be able to flow out without pressure (this prevents oil build-up and additional heating of the oil). In axial bearings, the oil must always be fed from the inside to the outside.

Oil injection lubrication

In bearings running at high speeds, the oil is injected into the gap between the cage and bearing ring. Injection lubrication using large recirculation quantities is associated with high power loss.

Heating of the bearings can only be held within limits with a considerable amount of effort. The appropriate upper limit for the speed parameter $n \cdot d_M = 1\,000\,000 \text{ min}^{-1} \cdot \text{mm}$ can, in the case of suitable bearings (for example spindle bearings), be exceeded to a considerable degree when using injection lubrication.

Heat dissipation via the lubricant

Oil can dissipate frictional heat from the bearing. It is possible to calculate the heat flow $\dot{Q}_L$ that is dissipated by the lubricant and the necessary lubricant volume flow $\dot{V}_L$, see Schaeffler catalogue HR 1, Rolling Bearings.

Oil change

At temperatures in the bearing of less than +50 °C and with only slight contamination, an oil change once per year is generally sufficient.
The precise oil change intervals should be agreed in consultation with the oil manufacturer.

Severe operating conditions

Under more severe conditions, the oil should be changed more frequently. This applies, for example, in the case of higher temperatures and low oil quantities with a high circulation index.

The circulation index indicates how often the entire oil volume available is circulated or pumped around the system per hour, see Schaeffler catalogue HR 1, Rolling Bearings.

Equation 73

$$\text{Circulation index} = \frac{\text{Pump displacement m}^3/\text{h}}{\text{Container volume m}^3}$$

Bearing data The difference between internal clearance and bearing clearance is explained below. In addition, bearing parts such as bearing materials, cages, guides etc. are described.

Radial internal clearance The radial internal clearance applies to bearings with an inner ring and is determined on the unmounted bearing. It is defined as the amount by which the inner ring can be moved relative to the outer ring in a radial direction from one extreme position to the other, see Figure 64, Page 573.

The following table gives the radial internal clearance groups in accordance with DIN 620-4 or ISO 5753-1 respectively.

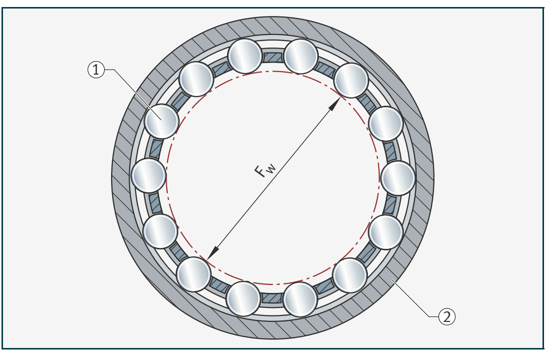
Internal clearance group in accordance with DIN 620-4	Internal clearance group in accordance with ISO 5753-1	Description	Area of application
C2	Group 2	Internal clearance < CN	For heavy alternating loads combined with swivel motion
CN	Group N	Normal internal clearance, CN is not normally included in bearing designations	For normal operating conditions with shaft and housing tolerances, see section Operating clearance, Page 573, and Design of bearing arrangements, Page 583
C3	Group 3	Internal clearance > CN	For bearing rings with press fits and large temperature differential between the inner and outer ring
C4	Group 4	Internal clearance > C3	
C5	Group 5	Internal clearance > C4	

Enveloping circle For bearings without an inner ring, the enveloping circle F_w is used. This is the inner inscribed circle of the rolling elements in clearance-free contact with the outer raceway, see Figure 63.

Figure 63
Enveloping circle

F_w = enveloping circle diameter

1 Rolling element
2 Outer raceway



Operating clearance The operating clearance is determined on a mounted bearing still warm from operation. It is defined as the amount by which the shaft can be moved in a radial direction from one extreme position to the other.
The operating clearance is derived from the radial internal clearance and the change in the radial internal clearance as a result of interference fit and thermal influences in the mounted condition.

Operating clearance value The operating clearance value is dependent on the operating and mounting conditions of the bearing, see also section Design of bearing arrangements, Page 583.

A larger operating clearance is, for example, necessary if heat is transferred via the shaft, the shaft undergoes deflection or if misalignment occurs.

An operating clearance smaller than CN should only be used in special cases, for example in high precision bearing arrangements. The normal operating clearance is achieved with an internal clearance of CN or, in the case of larger bearings, predominantly with C3 if the recommended shaft and housing tolerances are maintained, see section Design of bearing arrangements, Page 583.

The operating clearance can be calculated as follows:

Equation 74

$$s = s_r - \Delta s_P - \Delta s_T$$

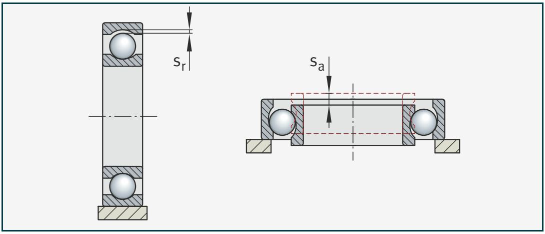
Legend

s µm Radial operating clearance of mounted bearing warm from operation	Δs_P µm Reduction in radial internal clearance due to fit
s_r µm Radial internal clearance	Δs_T µm Reduction in radial internal clearance due to temperature.

Axial internal clearance The axial internal clearance s_a is defined as the amount by which one bearing ring can be moved relative to the other, without load, along the bearing axis, see Figure 64.

Figure 64
Axial and radial internal clearance

s_a = axial internal clearance
s_r = radial internal clearance



With various bearing types, the radial internal clearance s_r and the axial internal clearance s_a are dependent on each other. The following table shows guide values for the relationship between the radial and axial internal clearance for some bearing types.

Bearing type		Ratio between axial and radial internal clearance s_a/s_r
Self-aligning ball bearings		$2,3 \cdot Y_0$[1)]
Spherical roller bearings		$2,3 \cdot Y_0$[1)]
Tapered roller bearings	Single row, arranged in pairs	$4,6 \cdot Y_0$[1)]
	Matched pairs (N11CA)	$2,3 \cdot Y_0$[1)]
Angular contact ball bearings	Double row, series 32 and 33	1,4
	Double row, series 32..-B and 33..-B	2
Four point contact bearings		1,4

[1)] Y_0 = axial load factor in accordance with product table.

Bearing materials

Schaeffler rolling bearings fulfil the requirements for fatigue strength, wear resistance, hardness, toughness and structural stability.

The material used for the rings and rolling elements is generally a low-alloy, through hardening chromium steel of high purity. For bearings subjected to considerable shock loads and reversed bending stresses, case hardening steel is also used.

In recent years, the improved quality of rolling bearing steels has been the principal factor in achieving considerable increases in basic load ratings.

The results of research as well as practical experience confirm that bearings made from the steel currently used as standard can achieve their endurance limit if loads are not excessively high and the lubrication and cleanliness conditions are favourable.

High Nitrogen Steel

Through the use of special bearings made from HNS (High Nitrogen Steel), it is possible to achieve adequate service life even under the most challenging conditions (high temperatures, moisture, contamination).

High performance steels Cronidur and Cronitect

For increased performance requirements, highly corrosion-resistant, nitrogen-alloyed martensitic HNS steels are available such as Cronidur and the newly developed steel Cronitect.

In contrast to Cronidur, the more economical alternative Cronitect has nitrogen introduced into the structure by means of a surface layer hardening process.

Both steels are clearly superior to conventional corrosion-resistant steels for rolling bearings in terms of corrosion resistance and fatigue strength.

Ceramic materials

Ceramic hybrid spindle bearings contain balls made from silicon nitride. These ceramic balls are significantly lighter than steel balls. The centrifugal forces and friction are considerably lower.

Hybrid bearings allow very high speeds, even with grease lubrication, as well as long operating life and low operating temperatures.

Materials for bearing components

The following table shows suitable materials and their applications in bearing technology.

Material	Bearing components (example)
Through hardening steel – rolling bearing steel in accordance with ISO 683-17	Outer and inner ring, axial washer, balls, rollers
HNS – High Nitrogen Steel	Outer and inner ring
Corrosion-resistant steel – rolling bearing steel in accordance with ISO 683-17	Outer and inner ring
Case hardening steel	For example, outer ring of yoke type track rollers
Flame or induction hardening steel	Roller stud of stud type track rollers
Steel strip in accordance with EN 10139, SAE J403	Outer ring for drawn cup needle roller bearings
Silicon nitride	Ceramic balls
Brass alloy	Cage
Aluminium alloy	Cage
Polyamide (thermoplastic)	Cage
NBR, FPM, TPU	Sealing ring

Cages The most important functions of the cage are:

- to separate the rolling elements from each other, in order to minimise friction and heat generation
- to maintain the rolling elements at the same distance from each other, in order to ensure uniform load distribution
- to prevent the rolling elements from falling out in bearings that can be dismantled or swivelled out
- to guide the rolling elements in the unloaded zone of the bearing.

Rolling bearing cages are subdivided into sheet metal and solid section cages.

Sheet metal cages These cages are predominantly made from steel and for some bearings from brass, see Figure 65, Page 577. In comparison with solid section cages made from metal, sheet metal cages are of lower mass.

Since a sheet metal cage only fills a small proportion of the gap between the inner and outer ring, lubricant can easily reach the interior of the bearing and is held on the cage.

In general, a sheet steel cage is only included in the bearing designation if it is not defined as a standard version of the bearing.

Solid section cages These cages are made from metal, laminated fabric or plastic, see Figure 66 and Figure 67, Page 577. They can be identified from the bearing designation.

Solid section cages made from metal or laminated fabric
Solid section cages made from metal are used where there are requirements for high cage strength and at high temperatures.

Solid section cages are also used if the cage must be guided on ribs. Rib-guided cages for bearings running at high speeds are made in many cases from light materials, such as light metal or laminated fabric, in order to achieve low inertia forces.

Solid section cages made from polyamide PA66
Solid section cages made from polyamide PA66 are produced using injection moulding, see Figure 67, Page 577. As a result, cage types can generally be realised that allow designs with particularly high load carrying capacity. The elasticity and low mass of polyamide are favourable under shock type bearing loads, high accelerations and decelerations and tilting of the bearing rings in relation to each other. Polyamide cages have very good sliding and emergency running characteristics.

Cages made from glass fibre reinforced polyamide PA66 are suitable for long term temperatures up to +120 °C. For higher operating temperatures, plastics such as PA46 or PEEK can be used.

When using oil lubrication, additives in the oil can impair the cage operating life. Aged oil can also impair the cage operating life at high temperatures, so attention must be paid to compliance with the oil change intervals.

Cage designs

The following images show some typical cage designs.

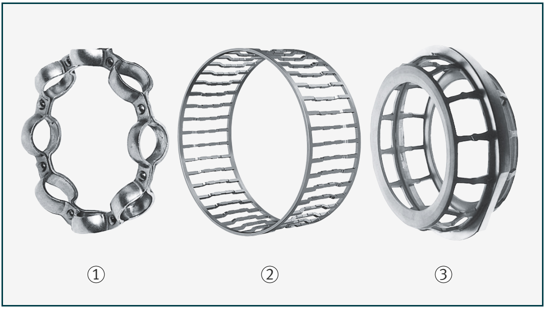


Figure 65
Sheet steel cages

① Riveted cage for deep groove ball bearings
② Window cage for needle roller bearings
③ Window cage for spherical roller bearings

Figure 66
Solid brass cages

① Riveted solid section cage for deep groove ball bearings
② Window cage for angular contact ball bearings
③ Riveted cage with crosspiece rivets for cylindrical roller cages

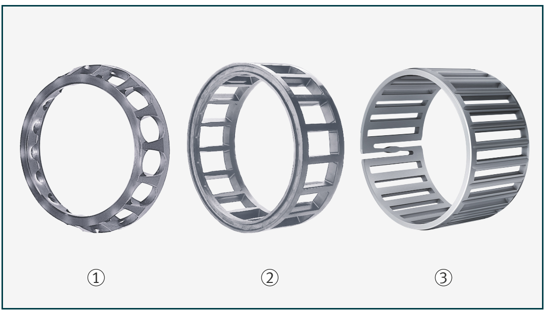


Figure 67
Solid section cages made from glass fibre reinforced polyamide

① Window cage for single row angular contact ball bearings
② Window cage for cylindrical roller bearings
③ Window cage for needle roller bearings

ISO dimension plans for bearing types

German and international standards (DIN, ISO) contain the external dimensions of common rolling bearings defined in the form of dimension plans. The principal basis for the formulation of dimension plans is several series of rolling bearings that were manufactured to coincident external dimensions as early as the start of the 20th Century. These series have been systematically expanded and additional series have been added. In order to prevent dimensions from multiplying and spreading in an uncontrolled manner, attention was paid not simply to the range to be considered at present but also to defining the dimensions of bearings that may be designed and manufactured in future.

ISO describes the dimension plans for the various types in separate documents as follows:

- radial bearings (excluding tapered roller bearings) in ISO 15
- tapered roller bearings in ISO 355
- axial bearings in ISO 104.

DIN 616 describes dimension plans for radial and axial bearings. An overview of ISO and DIN rolling bearing standards in given in DIN 611:2010-05.

Advantages of dimension plans

The dimension plans are valid for different bearing types. Rolling bearings of different types can be manufactured with the same external dimensions. As a result, a designer working on the same design envelope can make a selection between bearings of several types with the same external dimensions.

In the dimension plans, one bearing bore is allocated several outside diameters and width dimensions, see Figure 70, Page 580. In this way, it is possible to design several bearings of the same type that, for the same bore, exhibit different load carrying capacities. The development of new bearing series and individual new rolling bearings in accordance with the dimension plans has considerable advantages for users and manufacturers. The dimension plans should always be used as a basis for all future developments.

Width and diameter series

Width and diameter series are described using numbers. In the case of radial bearings in accordance with DIN 616 and ISO 15, these are as follows:

- for identification of the width series, the numbers 8, 9, 0, 1, 2, 3, 4, 5, 6, 7, see Figure 68, Page 579
- for identification of the diameter series, the numbers 7, 8, 9, 0, 1, 2, 3, 4, 5, see Figure 69, Page 579.

The dimension series, part series and diameter series are included in numerous standardised rolling bearing designations.

Figure 68
Width series

① Width series

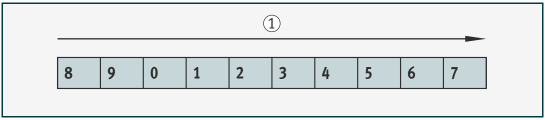
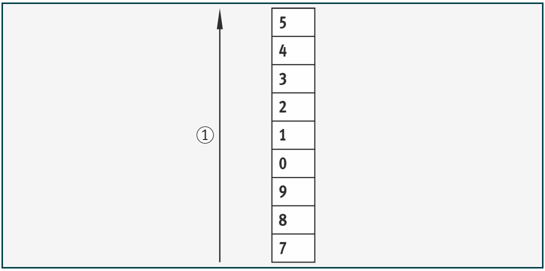


Figure 69
Diameter series

① Diameter series

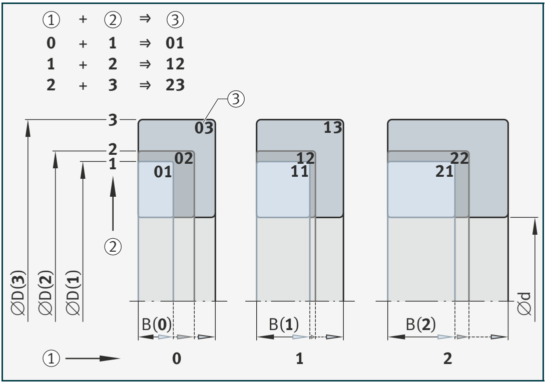


Dimension series
The specific number of the width and diameter series, when combined, identifies the dimension series, see following table. When this table is used, for example, for a radial bearing of the width series 2 and the diameter series 3, the result is the dimension series 23, see Figure 70, Page 580. If the bearing bore code is then added, the bearing size is completely defined.

Diameter series – increase in cross-sectional height	**Width series** – increase in cross-sectional width									
	8	9	0	1	2	3	4	5	6	7
5	–	–	–	–	–	–	–	–	–	–
4	–	–	04	–	24	–	–	–	–	–
3	83	–	03	12	23	33	–	–	–	–
2	82	–	02	12	22	32	42	52	62	–
1	–	–	01	11	21	31	41	51	61	–
0	–	–	00	10	20	30	40	50	60	–
9	–	–	09	19	29	39	49	59	69	–
8	–	–	08	18	28	38	48	58	68	–
7	–	–	–	17	27	37	47	–	–	–

Figure 70
Generation of the dimension series

① Width series
② Diameter series
③ Dimension series



Specification of the bearing bore

For certain bearing types, the bearing bores are stated directly or in an encoded form in accordance with DIN 623-1. Up to $d < 10$ mm, the bearing bore diameter is specified in the dimension-specific part of the designation (basic designation) directly as a number indicating the diameter.
Example: Deep groove ball bearing 623, bore diameter = 3 mm.

Bore code

For nominal dimensions $d \geqq 10$ mm to $d < 500$ mm, the diameter is described by means of a bore code.

For bores from 10 mm to 17 mm, this is as follows:
d = 10 mm, bore code 00

d = 12 mm, bore code 01

d = 15 mm, bore code 02

d = 17 mm, bore code 03.

For all rolling bearings in the range from d = 20 mm to d = 480 mm (with the exception of double direction axial bearings), the bore code is generated by dividing the bearing bore dimension by 5.
Example: Bearing bore d = 360 mm divided by 5 (360 : 5), bore code = 72.

At or above $d > 480$ mm, the unencoded bore diameter is indicated with an oblique after the bearing series, for example 618/500 with bore diameter d = 500 mm. The intermediate sizes such as bore diameter d = 22, 28 and d = 32 mm are also indicated with an oblique as /22, /28 and /32.

In the case of magneto bearings, the unencoded nominal bore dimension is given.

Bearing designations

Each rolling bearing has a designation that clearly indicates the type, dimensions, tolerances and internal clearance, if necessary with other important features. Bearings that have the same standardised designation are interchangeable with each other. In the case of separable bearings, it cannot always be ensured that individual parts with the same origin can be interchanged with each other.

In Germany, the bearing designations are standardised in DIN 623-1. These designations are also used in many other countries.

Rolling bearing designations

The designation for the bearing series comprises numbers and letters or letters and numbers. This indicates the type of bearing, the diameter series and, in many cases, the width series as well, see Figure 71.

Basic designation, prefix and suffix

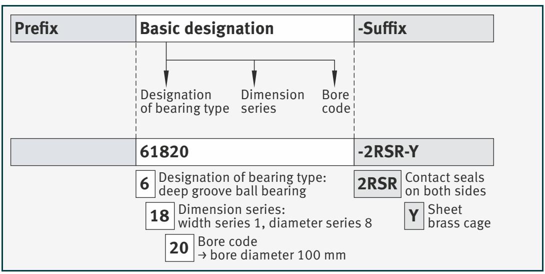
The basic designation contains the code for the bearing series and the bearing bore. Examples of basic designations: see section Bearing designations comprising the basic designations – examples, Page 582.

The prefix normally identifies individual bearing parts of complete bearings or material variants of a bearing (in certain cases, this may also be part of the basic designation). The suffix defines special designs and features.

The prefix and suffix describe other features of the bearing but are not standardised in all cases and may vary in use depending on the manufacturer.

Figure 71 shows an example of designations on the basis of their definition.

Figure 71
Example of bearing identification data for a deep groove ball bearing



Bearing designations comprising the basic designations – examples

6203
62 = bearing series 62, deep groove ball bearing,
width series 0 (not included in basic designation),
diameter series 2
03 = bore code, bore d = 17 mm

2201
22 = bearing series 22, self-aligning ball bearing, width series 2,
diameter series 2
01 = bore code, bore d = 12 mm

239/800
239 = bearing series 239, spherical roller bearing, width series 3,
diameter series 9
800 = bore code, bore d = 800 mm

3315
33 = bearing series 33, angular contact ball bearing, double row,
width series 3, diameter series 3
15 = bore code, bore d = 75 mm (15 · 5)

NU2314
NU23 = bearing series NU23, cylindrical roller bearing with two ribs
on outer ring, width series 2, diameter series 3
14 = bore code, bore d = 70 mm (14 · 5)

51268
512 = bearing series 512, axial deep groove ball bearing, height series 1,
diameter series 2
68 = bore code, bore d = 340 mm (68 · 5).

Bearing designations that comprise only the basic designation and do not include prefixes or suffixes, identify normal bearings with normal dimensional, geometrical and running accuracy as well as with normal radial internal clearance.

Deviations from the normal design are indicated in the prefixes and suffixes.

Design of bearing arrangements

The guidance and support of a rotating machine part generally requires at least two bearings arranged at a certain distance from each other (exceptions: four point contact, crossed roller and slewing bearings). Depending on the application, a decision is made between a locating/non-locating bearing arrangement, an adjusted bearing arrangement and a floating bearing arrangement.

Locating/non-locating bearing arrangement

On a shaft supported by two radial bearings, the distances between the bearing seats on the shaft and in the housing frequently do not coincide as a result of manufacturing tolerances. The distances may also change as a result of temperature increases during operation. These differences in distance are compensated in the non-locating bearing. Examples of locating/non-locating bearing arrangements: see Figure 72, Page 584 to Figure 75, Page 585.

Non-locating bearings

Ideal non-locating bearings are cylindrical roller bearings with cage N and NU or needle roller bearings, see Figure 72 ②, ④, Page 584. In these bearings, the roller and cage assembly can be displaced on the raceway of the bearing ring without ribs.

All other bearing types, for example deep groove ball bearings and spherical roller bearings, can only act as non-locating bearings if one bearing ring has a fit that allows displacement, see Figure 73, Page 585. The bearing ring subjected to point load therefore has a loose fit; this is normally the outer ring, see table Conditions of rotation, Page 592.

Locating bearings

The locating bearing guides the shaft in an axial direction and supports external axial forces. In order to prevent axial preload, shafts with more than two bearings have only one locating bearing. The type of bearing selected as a locating bearing depends on the magnitude of the axial forces and the accuracy with which the shafts must be axially guided.

A double row angular contact ball bearing, see Figure 74 ①, Page 585, will for example give closer axial guidance than a deep groove ball bearing or a spherical roller bearing. A pair of symmetrically arranged angular contact ball bearings or tapered roller bearings, see Figure 75, Page 585, used as a locating bearing will provide extremely close axial guidance.

There are particular advantages in using angular contact ball bearings of the universal design, see Figure 76, Page 586. The bearings can be fitted in pairs in any O or X arrangement without shims. Angular contact ball bearings of the universal design are matched so that, in an X or O arrangement, they have a low axial internal clearance (design UA), zero clearance (UO) or slight preload (UL).

Spindle bearings of the universal design UL, see Figure 77, Page 586 have slight preload when mounted in an X or O arrangement.

In gearboxes, a four point contact bearing is sometimes mounted directly adjacent to a cylindrical roller bearing to give a locating bearing arrangement, see Figure 74 ③, Page 585. The four point contact bearing, without radial support of the outer ring, can only support axial forces. The radial force is supported by the cylindrical roller bearing.

If a lower axial force is present, a cylindrical roller bearing with cage NUP can also be used as a locating bearing, see Figure 75 ③, Page 585.

No adjustment or setting work with matched pairs of tapered roller bearings

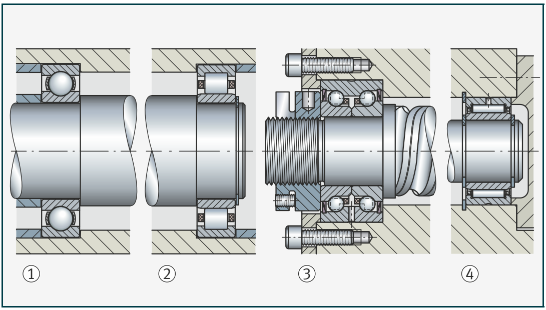
Mounting is also made easier with a matched pair of tapered roller bearings as a locating bearing (313..-N11CA), see Figure 78 ②, Page 586. They are matched with appropriate axial internal clearance such that no adjustment or setting work is required.

Examples of locating/non-locating bearing arrangements

The following figures show examples of locating/non-locating bearing arrangements.

Figure 72
Locating/non-locating bearing arrangements

① Locating bearing: deep groove ball bearing
② Non-locating bearing: cylindrical roller bearing NU
③ Locating bearing: axial angular contact ball bearing ZKLN
④ Non-locating bearing: needle roller bearing NKIS



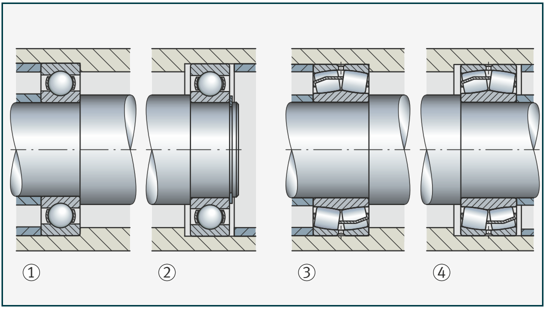


Figure 73
Locating/non-locating bearing arrangements

① Locating bearing: deep groove ball bearing
② Non-locating bearing: deep groove ball bearing
③ Locating bearing: spherical roller bearing
④ Non-locating bearing: spherical roller bearing

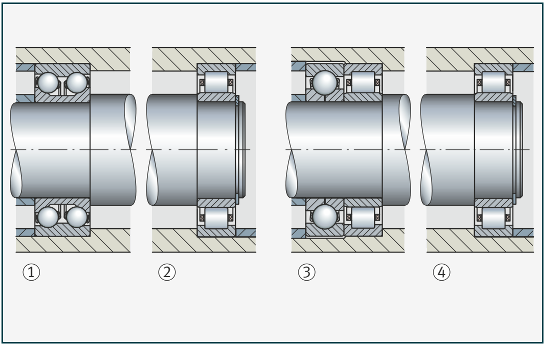


Figure 74
Locating/non-locating bearing arrangements

① Locating bearing: double row angular contact ball bearing
② Non-locating bearing: cylindrical roller bearing NU
③ Locating bearing: four point contact bearing and cylindrical roller bearing
④ Non-locating bearing: cylindrical roller bearing NU

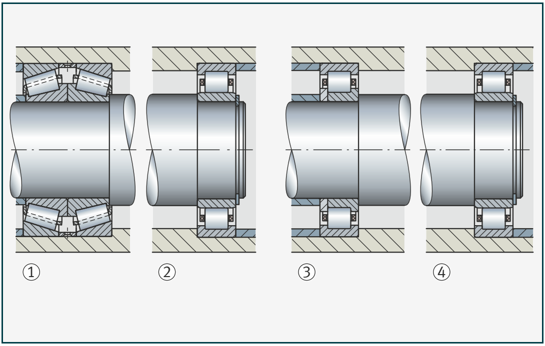


Figure 75
Locating/non-locating bearing arrangements

① Locating bearing: two tapered roller bearings
② Non-locating bearing: cylindrical roller bearing NU
③ Locating bearing: cylindrical roller bearing NUP
④ Non-locating bearing: cylindrical roller bearing NU

Figure 76
Locating bearing arrangements

Pair of angular contact ball bearings
of universal design

① O arrangement
② X arrangement

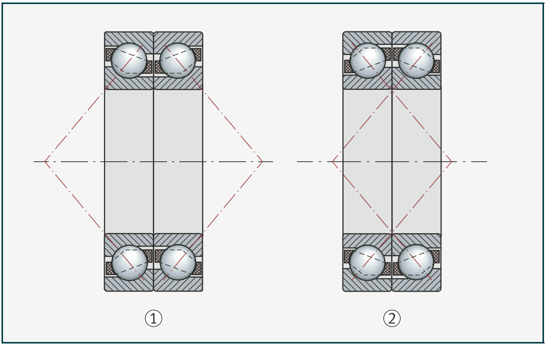


Figure 77
Locating bearing arrangements

Spindle bearings
of universal design

① O arrangement
② X arrangement
③ Tandem O arrangement

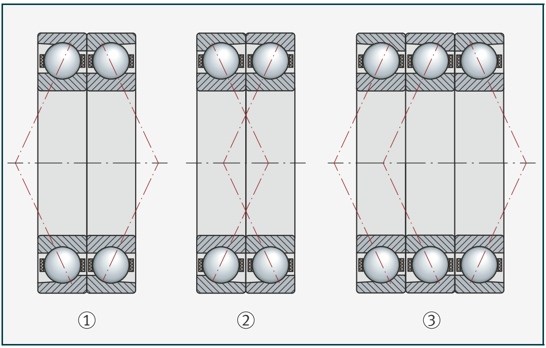
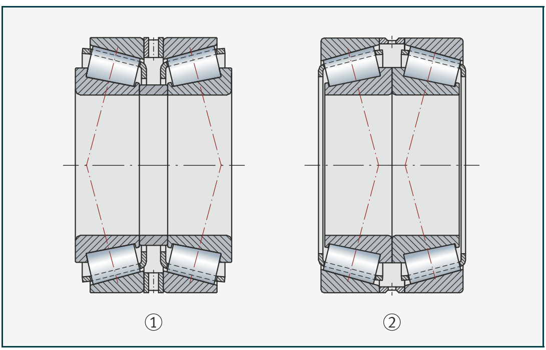


Figure 78
Locating bearing arrangements

Pair of tapered roller bearings

① O arrangement
② X arrangement



Adjusted bearing arrangement

These bearing arrangements normally consist of two symmetrically arranged angular contact bearings (angular contact ball bearings, tapered roller bearings), see Figure 79. The inner and outer rings of the bearings are displaced relative to each other until the required clearance or the required preload is achieved. This process is known as “adjustment”.

Angular contact bearings and deep groove ball bearings suitable for adjusted bearing arrangements

Angular contact bearings support forces comprising a radial and an axial component. These are thus a combination of a radial and an axial bearing. Depending on the size of the nominal contact angle α, angular contact bearings are classified as radial or axial bearings. Deep groove ball bearings can also be used for an adjusted bearing arrangement; these are then angular contact ball bearings with a small nominal contact angle. Due to the possibility of regulating the clearance, adjusted bearing arrangements are particularly suitable if close guidance is necessary.

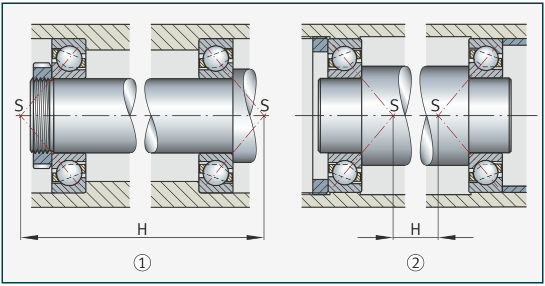
O or X arrangement

In an adjusted bearing arrangement, an O or X arrangement of the bearings is essentially possible, see Figure 79. In the O arrangement, the cones and their apexes S formed by the contact lines point outwards; in the X arrangement, the cones point inwards. The support base H, the distance between the apexes of the contact cones, is wider in the O arrangement than in an X arrangement. An O arrangement should be used in preference if the component with small bearing spacing must be guided with the smallest possible tilting clearance or tilting forces must be supported.

Figure 79
Adjusted bearing arrangement

S = contact cone apex
H = support distance
Angular contact ball bearings

① O arrangement
② X arrangement



Influence of thermal expansion in O and X arrangements

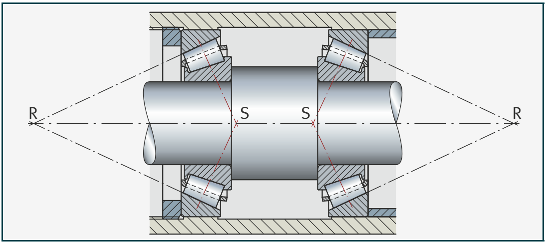
When deciding between an O and X arrangement, attention must also be paid to the temperature conditions and thermal expansions. This is based on the position of the roller cone apexes R. The roller cone apex R represents the intersection point of the extended outer ring raceway with the bearing axis, see Figure 80, Page 588.

If the shaft is warmer than the housing ($T_W > T_G$), the shaft expands more than the housing in an axial and radial direction. As a result, the clearance set in an X arrangement decreases in every case (assuming the following precondition: shaft and housing of same material).

Figure 80
Adjusted bearing arrangement

S = contact cone apex
R = roller cone apex
Tapered roller bearings

X arrangement



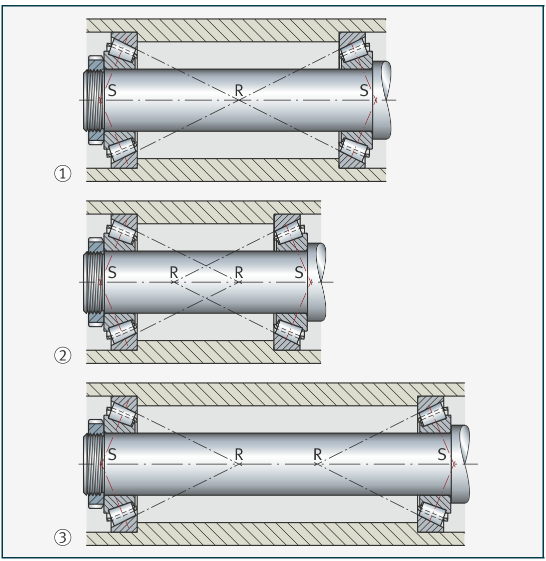
The behaviour is different in an O arrangement. A distinction is made here between three cases:

- If the roller cone apexes R coincide at a point, the axial and radial thermal expansion cancel each other out and the clearance set is maintained, see Figure 81 ①
- If there is a small distance between the bearings and the roller cones overlap, see Figure 81 ②, the radial expansion has a stronger effect than the axial expansion on the bearing clearance: the axial clearance is reduced. This must be taken into consideration in the adjustment of the bearings
- If there is a large distance between the bearings and the roller cones do not overlap, see Figure 81 ③, the radial expansion has a lesser effect than the axial expansion on the bearing clearance: the axial clearance is increased.

Figure 81
Tapered roller bearings, O arrangement

S = contact cone apex
R = roller cone apex

① Roller cone apexes coincide
② Roller cone apexes overlap
③ Roller cone apexes do not overlap



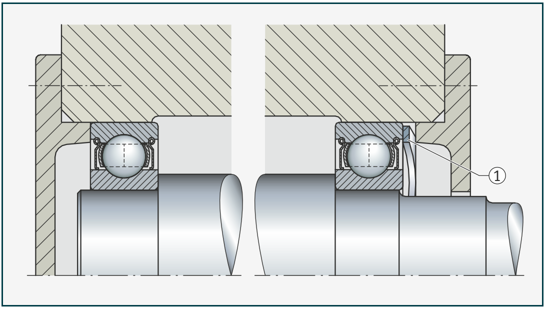
Elastic adjustment

Adjusted bearing arrangements can also be achieved by preloading using springs, see Figure 82 ①. This elastic adjustment method compensates for thermal expansion. It can also be used where bearing arrangements are at risk of vibration while stationary.

Figure 82
Adjusted bearing arrangement

Deep groove ball bearing preloaded by means of spring washer

① Spring washer



Floating bearing arrangement

The floating bearing arrangement is essentially similar in its arrangement to the adjusted bearing arrangement. While freedom from clearance or even preload is desirable when warm from operation in the latter case, floating bearing arrangements always have an axial clearance s of several tenths of a millimetre depending on the bearing size; see Figure 83, Page 590. The value s is defined as a function of the required guidance accuracy such that the bearings are not axially stressed even under unfavourable thermal conditions.

Suitable bearing types

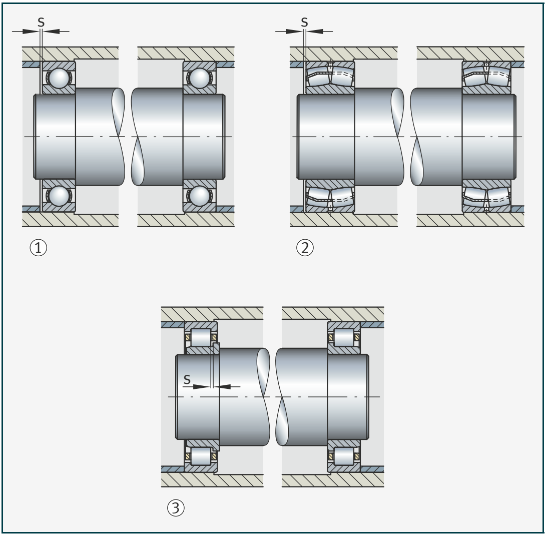
For a floating bearing arrangement, almost all bearing types that cannot be adjusted may be considered; examples: see Figure 83, Page 590. Floating arrangements are thus possible with, for example, deep groove ball bearings, self-aligning ball bearings and spherical roller bearings; one ring of each of the two bearings (usually the outer ring) then has a sliding seat. In the floating bearing arrangement with cylindrical roller bearings NJ, length compensation is possible within the bearing.

Tapered roller bearings and angular contact ball bearings are not suitable for a floating bearing arrangement, since they must be adjusted in order to run correctly.

Figure 83
Floating bearing arrangements

s = axial displacement (axial clearance)

① Two deep groove ball bearings
② Two spherical roller bearings
③ Two cylindrical roller bearings NJ



Fits

Rolling bearings are located on the shaft and in the housing in a radial, axial and tangential direction in accordance with their function. In a radial and tangential direction, this occurs by means of a tight fit. However, this is only possible under certain conditions in an axial direction, therefore rolling bearings are generally axially located by means of form fit.

Criteria for selection of fits

The following must be taken into consideration in determining the fit:

- The bearing rings must be well supported on their circumference in order to allow full utilisation of the load carrying capacity of the bearing
- The rings must not creep on their mating parts, otherwise the seats will be damaged
- The non-locating bearing must compensate changes in the length of the shaft and housing, so one ring must be axially adjustable
- The bearings must be easy to mount and dismount.

Good support of the bearing rings on their circumference requires a tight fit. The requirement that rings must not creep on their mating parts also requires a tight fit. If non-separable bearings must be mounted and dismounted, a tight fit can only be achieved for one bearing ring.
In the case of cylindrical roller bearings N, NU and needle roller bearings, both rings can have tight fits, since the length compensation takes place within the bearing and since the rings can be fitted separately.

As a result of tight fits and a temperature differential between the inner and outer ring, the radial internal clearance of the bearing is reduced. This must be taken into consideration when selecting the radial internal clearance.

If a material other than cast iron or steel is used for the adjacent construction, the modulus of elasticity and the differing coefficients of thermal expansion of the materials must also be taken into consideration to achieve a tight fit.

For aluminium housings, thin-walled housings and hollow shafts, a closer fit should be selected if necessary in order to achieve the same force locking as with cast iron, steel or solid shafts.

Higher loads, especially shocks, require a larger interference fit and narrower geometrical tolerances.

Bearing seat for axial bearings

Axial bearings, which support axial loads only, must not be guided radially – with the exception of axial cylindrical roller bearings which have a degree of freedom in the radial direction due to flat raceways. In the case of groove-shaped raceways this is not present and must be achieved by a loose seat for the stationary washer. A rigid seat is normally selected for the rotating washer.

Where axial bearings also support radial forces, such as in axial spherical roller bearings, fits should be selected in the same way as for radial bearings.

The contact surfaces of the mating parts must be perpendicular to the axis of rotation (total axial run-out tolerance to IT5 or better), in order to ensure uniform load distribution over all the rolling elements.

Conditions of rotation

The condition of rotation indicates the motion of one bearing ring with respect to the load direction and is present as either circumferential load or point load, see table Conditions of rotation, Page 592.

Point load

If the ring remains stationary relative to the load direction, there are no forces that displace the ring relative to its seating surface. This type of loading is described as point load.

There is no risk that the seating surface will be damaged. In this case, a loose fit is possible.

Circumferential load

If forces are present that displace the ring relative to its seating surface, every point on the raceway is subjected to load over the course of one revolution of the bearing. This type of loading is described as circumferential load.

There is a risk that the seating surface will be damaged. A tight fit should therefore be provided.

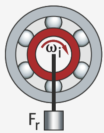
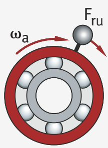
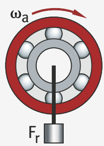
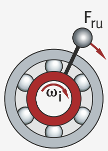
Indeterminate load direction

If the load changes in direction irregularly or by swivelling, or if shocks or vibrations occur, the raceway is also subjected to load. This type of loading is described as an indeterminate load direction.

There is a risk that the seating surface will be damaged. A tight fit should therefore be provided.

Guidelines for the selection of shaft and hole tolerances for the various bearing designs under specific mounting and loading conditions can be found in the Schaeffler catalogues for rolling bearings.

The following table shows different conditions of rotation (conditions of motion):

Conditions of rotation	Example	Schematic	Load case	Fit
Rotating inner ring Stationary outer ring Constant load direction	Shaft with weight load		Circumferential load on inner ring and Point load on outer ring	Inner ring: tight fit necessary and Outer ring: loose fit permissible
Stationary inner ring Rotating outer ring Load direction rotates with outer ring	Hub bearing arrangement with significant imbalance			
Stationary inner ring Rotating outer ring Constant load direction	Passenger car front wheel, track roller, (hub bearing arrangement)		Point load on inner ring and Circumferential load on outer ring	Inner ring: loose fit permissible and Outer ring: tight fit necessary
Rotating inner ring Stationary outer ring Load direction rotates with inner ring	Centrifuge, vibrating screen			

Shaft and housing tolerances

The fit is determined by the ISO tolerances for shafts and housings (ISO 286), in conjunction with the tolerances $t_{\Delta dmp}$ for the bore and $t_{\Delta Dmp}$ for the outside diameter of the bearings (ISO 492 for radial bearings, ISO 199 for axial bearings).

ISO tolerance classes

The shaft and housing tolerances are defined in the form of ISO tolerance classes to ISO 286-1 and ISO 286-2. The designation of the tolerance classes, for example "E8", comprises one or two upper case letters for housings or lower case letters for shafts (= fundamental deviation identifier, which defines the tolerance position relative to the zero line, such as "E") and the grade number of the standard tolerance grade (this defines the tolerance quality, for example "8").

Reference to tables of shaft and housing tolerances

The tables on Page 593 to Page 596 contain recommendations for the selection of shaft and housing tolerances that are valid for normal mounting and operating conditions.

Deviations are possible if particular requirements apply, for example in relation to running accuracy, smooth running or operating temperature. Increased running accuracies thus require closer tolerances such as standard tolerance grade 5 instead of 6. If the inner ring is warmer than the shaft during operation, the seating may loosen to an impermissible extent. A tighter fit must then be selected, for example m6 instead of k6.

In such cases, the question of fits can only be resolved by a compromise. The individual requirements must be weighed against each other and those selected that give the best overall solution.

Shaft tolerances for radial bearings with cylindrical bore

For radial bearings with a cylindrical bore, the shaft tolerances are as follows:

Condition of rotation	Bearing type	Shaft diameter in mm	Displacement facility Load	Tolerance class[1]
Point load on inner ring	Ball bearings, roller bearings	All sizes	Inner ring easily displaced	g6 (g5)
			Inner ring not easily displaced Angular contact ball bearings and tapered roller bearings with adjusted inner ring	h6 (j6)
	Needle roller bearings	All sizes	Non-locating bearings	h6 (g6)[2]

Continuation of table, see Page 594.

[1] The envelope requirement Ⓔ applies.

[2] For easier mounting.

Continuation of table, Shaft tolerances for radial bearings with cylindrical bore, from Page 593.

Condition of rotation	Bearing type	Shaft diameter in mm	Displacement facility Load	Tolerance class[1]
Circumferential load on inner ring or indeterminate load direction	Ball bearings	incl. 50	Normal loads[2]	j6 (j5)
		50 to 100	Low loads[3]	j6 (j5)
			Normal and high loads[4]	k6 (k5)
		100 to 200	Low loads[2]	k6 (m6)
			Normal and high loads[5]	m6 (m5)
		over 200	Low loads	m6 (m5)
			Normal and high loads	n6 (n5)
	Roller bearings	incl. 60	Low loads	j6 (j5)
			Normal and high loads	k6 (k5)
		60 to 200	Low loads	k6 (k5)
			Normal loads	m6 (m5)
			High loads	n6 (n5)
		200 to 500	Normal loads	m6 (n6)
			High loads, shocks	p6
		over 500	Normal loads	n6 (p6)
			High loads	p6
Circumferential load on inner ring or indeterminate load direction	Needle roller bearings	incl. 50	Low loads	k6
			Normal and high loads	m6
		50 to 120	Low loads	m6
			Normal and high loads	n6
		120 to 250	Low loads	n6
			Normal and high loads	p6
		250 to 400	Low loads	p6
			Normal and high loads	r6
		400 to 500	Low loads	r6
			Normal and high loads	s6
		over 500	Low loads	r6
			Normal and high loads	s6

1) The envelope requirement Ⓔ applies.
2) $C_0/P_0 > 10$.
3) $C_0/P_0 > 12$.
4) $C_0/P_0 < 12$.
5) $C_0/P_0 < 10$.

Shaft tolerances for axial bearings

For axial bearings, the shaft tolerances are as follows:

Load	Bearing type	Shaft diameter in mm	Operating conditions	Tolerance class[1]
Axial load	Axial deep groove ball bearing	All sizes	–	j6
	Axial deep groove ball bearing, double direction		–	k6
	Axial cylindrical roller bearing with shaft locating washer		–	h8
	Axial cylindrical roller and cage assembly		–	h8
Combined load	Axial spherical roller bearing	All sizes	Point load on shaft locating washer	j6
		incl. 200	Circumferential load on shaft locating washer	j6 (k6)
		over 200		k6 (m6)

1) The envelope requirement Ⓔ applies.

Housing tolerances for radial bearings

For radial bearings with a cylindrical bore, the housing tolerances are as follows:

Condition of rotation	Displacement facility Load	Operating conditions	Tolerance class[1]
Point load on outer ring	Outer ring easily displaced, housing unsplit	The tolerance grade is determined by the running accuracy required	H7 (H6)[2]
	Outer ring easily displaced, housing split		H8 (H7)
	Outer ring not easily displaced, housing unsplit	High running accuracy required	H6 (J6)
	Outer ring not easily displaced, angular contact ball bearings and tapered roller bearings with adjusted outer ring, housing split	Normal running accuracy	H7 (J7)
	Outer ring easily displaced	Heat input via shaft	G7[3]

Continuation of table, see Page 596.

1) The envelope requirement Ⓔ applies.

2) G7 for housings made from flake graphite cast iron if bearing outside diameter D > 250 mm and temperature differential between outer ring and housing > 10 K.

3) F7 for housings made from flake graphite cast iron if bearing outside diameter D > 250 mm and temperature differential between outer ring and housing > 10 K.

Continuation of table, Housing tolerances for radial bearings, from Page 595.

Condition of rotation	Displacement facility Load	Operating conditions	Tolerance class[1]
Circumferential load on outer ring or indeterminate load direction	Low loads, outer ring cannot be displaced	For high running accuracy requirements: K6, M6, N6 and P6	K7 (K6)
	Normal loads, shocks, outer ring cannot be displaced		M7 (M6)
	High loads, shocks ($C_0/P_0 < 6$), outer ring cannot be displaced		N7 (N6)
	High loads, severe shocks, thin-walled housing, outer ring cannot be displaced		P7 (P6)

[1] The envelope requirement Ⓔ applies.

Housing tolerances for axial bearings

For axial bearings with a cylindrical bore, the housing tolerances are as follows:

Load, condition of rotation	Bearing type	Operating conditions	Tolerance class[1]
Axial load	Axial deep groove ball bearing	Normal running accuracy	E8
		High running accuracy	H6
	Axial cylindrical roller bearing with housing locating washer	–	H9
	Axial cylindrical roller and cage assembly	–	H10
	Axial spherical roller bearing	Normal loads	E8
		High loads	G7
Combined load Point load on housing locating washer	Axial spherical roller bearing	–	H7
Combined load Circumferential load on housing locating washer	Axial spherical roller bearing	–	K7

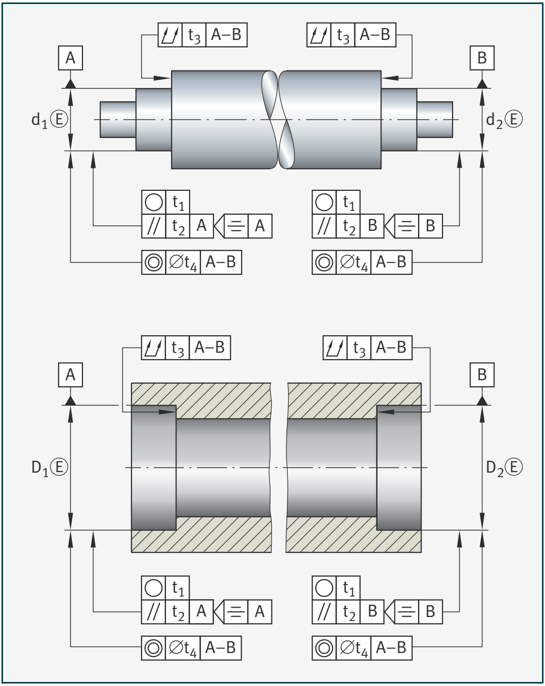
[1] The envelope requirement Ⓔ applies.

Geometrical and positional tolerances of bearing seating surfaces

In order to achieve the required fit, the bearing seats and fit surfaces of the shaft and housing bore must conform to certain tolerances, see Figure 84 and table, Page 598.

Figure 84
Geometrical and positional tolerances

t_1 = roundness tolerance
t_2 = parallelism tolerance
t_3 = total axial run-out tolerance of abutment shoulders
t_4 = coaxiality tolerance



For bearing seating surfaces, the geometrical and positional tolerances are as follows:

Tolerance class of bearings		Bearing seating surface	Fundamental tolerance grades[1]				
ISO 1492	DIN 620		Diameter tolerance	Load case	Roundness tolerance t_1	Parallelism tolerance t_2	Total axial run-out tolerance of abutment shoulder t_3
Normal 6X	PN (P0) P6X	Shaft	IT6 (IT5)	Circumferential load	IT4/2	IT4/2	IT4
				Point load	IT5/2	IT5/2	
		Housing	IT7 (IT6)	Circumferential load	IT5/2	IT5/2	IT5
				Point load	IT6/2	IT6/2	
6	P6	Shaft	IT5	Circumferential load	IT3/2	IT3/2	IT3
				Point load	IT4/2	IT4/2	
		Housing	IT6	Circumferential load	IT4/2	IT4/2	IT4
				Point load	IT5/2	IT5/2	
5	P5	Shaft	IT5	Circumferential load	IT2/2	IT2/2	IT2
				Point load	IT3/2	IT3/2	
		Housing	IT6	Circumferential load	IT3/2	IT3/2	IT3
				Point load	IT4/2	IT4/2	
4	P4 P4S[2] SP[2]	Shaft	IT4	Circumferential load	IT1/2	IT1/2	IT1
				Point load	IT2/2	IT2/2	
		Housing	IT5	Circumferential load	IT2/2	IT2/2	IT2
				Point load	IT3/2	IT3/2	
	UP[2]	Shaft	IT3	Circumferential load	IT0/2	IT0/2	IT0
				Point load	IT1/2	IT1/2	
		Housing	IT4	Circumferential load	IT1/2	IT1/2	IT1
				Point load	IT2/2	IT2/2	

[1] ISO fundamental tolerances (IT grades) in accordance with DIN ISO 286.

[2] Not included in DIN 620.

Accuracy of bearing seating surfaces

The degree of accuracy of the bearing seat tolerances on the shaft and in the housing is given in the table Geometrical and positional tolerances of bearing seating surfaces, Page 598.

Second bearing seat

The positional tolerances t_4 for a second bearing seat on the shaft (d_2) or in the housing (D_2) are dependent on the types of bearings used and the operating conditions. Values for the tolerances t_4 can be requested from Schaeffler.

Housings

In split housings, the joints must be free from burrs. The accuracy of the bearing seats is determined as a function of the accuracy of the bearing selected.

Roughness of bearing seats

The roughness of the bearing seats must be matched to the tolerance class of the bearings. The mean roughness value Ra must not be too high, in order to maintain the interference loss within limits. Shafts must be ground, while bores must be precision turned.

For bearing seating surfaces, the guide values for roughness are as follows:

Nominal diameter of bearing seat d (D) mm		Recommended mean roughness value for ground bearing seats Ramax µm			
		Diameter tolerance (IT grade)			
over	incl.	IT7	IT6	IT5	IT4
–	80	1,6	0,8	0,4	0,2
80	500	1,6	1,6	0,8	0,4
500	1 250	3,2[1]	1,6	1,6	0,8

[1] For the mounting of bearings using the hydraulic method, do not exceed the value Ra = 1,6 µm.

Axial location of bearings

Axial location of the bearing rings is matched to the specific bearing arrangement (locating bearings, non-locating bearings, bearings in adjusted and floating arrangements).

Design guidelines

The bearing rings must be located by force locking or form fit in order to prevent lateral creep. The bearing rings must only be in contact with the shaft or housing shoulder, but not with the fillet.

The shoulders on the mating parts must be large enough to provide a sufficiently wide contact surface even with the largest chamfer dimension of the bearing (DIN 5418).

Locating bearings
Locating bearings support axial forces. The retaining element must be matched to these axial forces. Shoulders on the shaft and housing, retaining rings, snap rings, housing covers, shaft covers, nuts and spacer rings are suitable.

Non-locating bearings
Non-locating bearings must support low axial forces occurring in thermal expansion. The means of axial location therefore only needs to prevent creep of the rings. A tight fit is often sufficient for this purpose.

Self-retaining bearings
In non-separable bearings, one bearing ring must have a tight fit, while the other ring is retained by the rolling elements.

For further information, see the Schaeffler catalogues.

Seals

The sealing arrangement has a considerable influence on the operating life of a bearing arrangement. Its function is to retain the lubricant in the bearing and prevent the ingress of contaminants into the bearing.

Contaminants may have various effects:

- A large quantity of very small, abrasive particles causes wear in the bearing. The increase in clearance or noise brings the operating life of the bearing to an end
- Large, overrolled hard particles reduce the fatigue life since pittings occur at the indentation points under high bearing loads.

A basic distinction is made between contact and non-contact seals in the adjacent construction and the bearing.

Non-contact seals in the adjacent construction

With non-contact seals, only lubricant friction occurs in the lubrication gap. The seals do not undergo wear and remain capable of operation for a long period. Since they generate no heat, non-contact seals are also suitable for very high speeds.

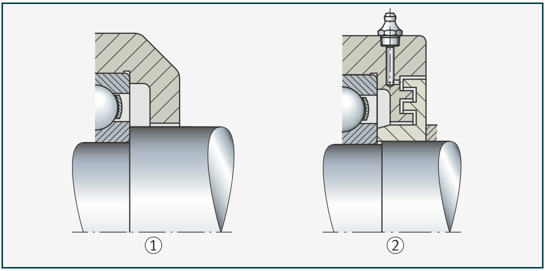
Gap seals
A simple design, although adequate in many cases, is a narrow seal gap between the shaft and housing.

Labyrinth seals

A considerably greater sealing action than with gap seals is achieved by labyrinths incorporating gaps filled with grease. In contaminated environments, grease should be pressed from the interior into the seal gap at short intervals.

Figure 85
Gap seals and labyrinth seals

① Simple gap seal
② Labyrinth seal



Ring with runoff edges

Where oil lubrication is used with a horizontal shaft, rings with a runoff edge are suitable for preventing the escape of oil. The oil outlet hole on the underside of the seal location must be sufficiently large that it cannot be clogged by contamination.

Baffle plates

Stationary (rigid) baffle plates ensure that grease remains in the area around the bearing. The grease collar that forms at the seal gap protects the bearing against contaminants.

Flinger shields

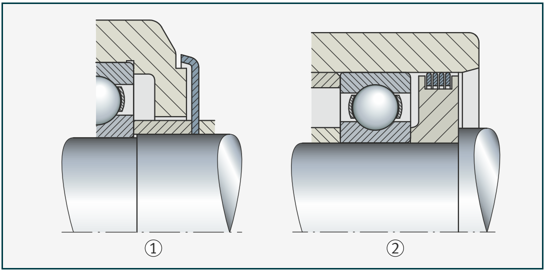
Co-rotating flinger shields have the effect of shielding the seal gap from heavy contamination.

Lamellar rings

Lamellar rings made from steel and radially sprung either outwards or inwards require little mounting space. They give protection against loss of grease and ingress of dust and are also used as an outer seal against spray water.

Figure 86
Flinger shields and lamellar rings

① Flinger shield
② Lamellar rings



Non-contact seals in the bearing

Non-contact seals can be fitted in the bearing instead of the adjacent construction.

Sealing shields

Sealing shields are compact sealing elements fitted on one or both sides of the bearing. Bearings with sealing shields on both sides are supplied with a grease filling.

BRS seals (labyrinth seals)

The friction in this case is as low as that in bearings with sealing shields. They have the advantage over these, however, that the outer rubber-elastic rim gives good sealing when fitted in the slot in the outer ring. This is important with a rotating outer ring since the base oil is separated from the soap suspension by centrifugal force and would escape through the unsealed metallic seat in the outer ring if sealing shields were fitted.

Contact seals in the adjacent construction

Contact seals are normally in contact with the running surface under radial contact force. The contact force should be kept small in order to avoid an excessive increase in frictional torque and temperature. The frictional torque and temperature as well as the wear of the seal are also affected by the lubrication condition at the running surface, the roughness of the running surface and the sliding velocity.

Felt rings

Felt rings and felt strips are sealing elements that have proved very effective with grease lubrication. They are impregnated with oil before mounting and give particularly good sealing against dust. They are suitable for circumferential velocities at the running surface of up to 4 m/s. In unfavourable environmental conditions, two felt rings are arranged adjacent to each other. Felt rings and annular slots are standardised in accordance with DIN 5419.

Rotary shaft seals

For the oil sealing of rotating shafts, rotary shaft seals (RWDR) in accordance with DIN 3760 and DIN 3761 and with spring preload are suitable. The sealing rings are designed for applications with slight pressure differentials. Depending on the seal material and the surface structure of the shaft, the geometry of the seal lips generates a pumping action in the sealing gap towards the steep flank of the seal lip. The sealing ring is therefore mounted with the steep flank facing in the direction of the medium against which sealing is required.

In the case of grease lubrication, the steep flank of the rotary shaft seal is often placed in the direction of grease egress. As a result, some grease passes under the seal lip for lubrication of the sealing edge.

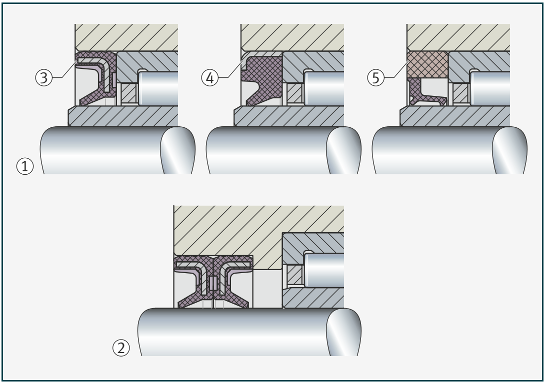
Springless Schaeffler sealing rings for needle roller bearings

Low-friction sealing of bearing positions with a small radial design envelope, such as bearing positions with needle roller bearings, can be effectively achieved using sealing rings G, GR and SD, see Figure 87. These sealing rings can be used individually or in a double arrangement.

In the double arrangement, one seal lip faces inwards to seal the lubrication medium, while the second seal lip faces outwards to give protection against contamination. In order to improve the protective function, the space between the seals can be filled with grease.
With an extended inner ring, a sealing ring with the same outside diameter as the outer ring can be used, where the seal lip runs on the extended inner ring. Sealing rings give good protection against contamination and spray water as well as against the egress of oil and grease under slight pressure differentials. In order to reduce friction and protect the seal lip against damage, the sealing edge must be lubricated.

Figure 87
Schaeffler sealing rings

① Single arrangement, bearing with extended inner ring
② Double arrangement, bearing with inner ring
③ G sealing ring
④ GR sealing ring
⑤ SD sealing ring



Lip seal with axial sealing action
Lip seals are seals with one or more seal lips that give axial or radial sealing. These seals are predominantly elastomer seals.

The V-ring is a lip seal with axial sealing action. The ring is made from elastic rubber NBR. During mounting, it is stretched and slid onto the shaft so that the seal lip is in contact with the housing wall. At circumferential velocities over 12 m/s, experience shows that the V-ring must be radially located so that it does not become detached due to centrifugal force. Precise circumferential velocities for specific applications must always be agreed in consultation with the sealing ring manufacturer.

Metallic sealing washers
When using grease lubrication, effective sealing can also be achieved by means of axially sprung sealing shields. The sheet metal shields are clamped to the end face of the inner ring or outer ring and are axially sprung against the other bearing ring.

Contact seals in the bearing

Contact seals can be fitted in the bearing instead of the adjacent construction.

Sealing washers

Bearings fitted with one or two sealing washers allow simple designs. The washers are suitable for giving protection against dust, contamination, damp atmospheres and slight pressure differentials. Sealing washers are used, for example, in maintenance-free bearings with grease filling. The sealing washer design RSR made from acrylonitrile butadiene rubber (NBR), normally used in deep groove ball bearings, is located under slight contact pressure against a cylindrically ground inner ring rib.

Multi-lip seals are also used. Double lip seals made from NBR are used, for example, in plummer block housings, while single lip and multi-lip seals are used in a sandwich arrangement in radial insert ball bearings. Further information: see Schaeffler catalogue HR 1, Rolling Bearings.

Products – overview

This section shows an excerpt from the Schaeffler product range of rotary rolling bearings in the Industrial Division.

The products are also used to a significant extent – once designed and matched in accordance with the specific requirements – in applications in the Automotive Division.

The product descriptions are, in accordance with the philosophy of this publication, presented in a condensed form and are essentially intended as an overview. While they highlight important characteristics of the products and will in many cases allow preliminary assessment of a particular bearing for its suitability in a bearing arrangement, they cannot be used directly for the design of bearing arrangements. The specific application is always the decisive factor in the use of products. In addition, the information in the specific product descriptions must always be taken into consideration.

Product information documents

The comprehensive technical product information documents (catalogues, Technical Product Information, datasheets, mounting manuals, operating manuals etc.) can be requested from Schaeffler.

The Schaeffler engineering service can, upon request, provide support in the selection of bearings and the design of bearing arrangements.

Electronic product catalogue *medias professional*

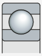
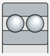
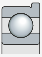
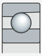
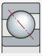
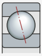
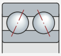
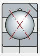
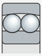
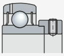
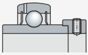
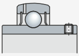
The standard range is described in detail in the online version ***medias*** *professional*. In addition, this tool for the design of bearing arrangements provides means of assistance such as a lubricant database, a calculation tool for determining rating life and other features.

Link to electronic product catalogue

The following link will take you to the Schaeffler electronic product catalogue: *http://medias.schaeffler.com*.

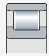
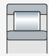
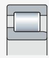
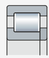
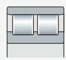
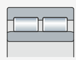
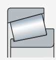
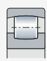
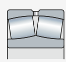
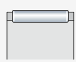
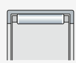
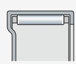
Compilation of standardised bearings

The following table shows a compilation of standardised bearings with their names and standard numbers (excerpt from DIN 623-1).

Name/design	Image	Standard number
Radial deep groove ball bearing, single row, without filling slot		DIN 625-1
Radial deep groove ball bearing, double row, with or without filling slot		DIN 625-3
Radial deep groove ball bearing, with flanged outer ring		DIN 625-4
Radial magneto bearing		DIN 615
Radial angular contact ball bearing, single row, without filling slot, non-separable		DIN 628-1
Radial angular contact ball bearing, single row, contact angle 15° and 25°		DIN 628-6
Radial angular contact ball bearing, double row, with or without filling slot		DIN 628-3
Angular contact ball bearing, four point contact bearing with split inner ring		DIN 628-4
Radial self-aligning ball bearing, double row		DIN 630
Radial insert ball bearing with inner ring extended on one side, curved outer ring outside surface and eccentric locking collar		DIN 626-1
Radial insert ball bearing with inner ring extended on both sides, curved outer ring outside surface and eccentric locking collar		DIN 626-1
Radial insert ball bearing with inner ring extended on both sides, curved outer ring outside surface and grub screw		DIN 626-1

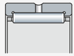
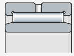
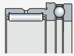
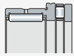
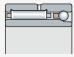
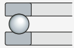
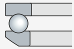
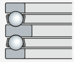
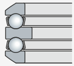
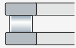
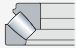
Continuation of table, see Page 606.

Continuation of table, Compilation of standardised bearings, from Page 605.

Name/design	Image	Standard number
Radial cylindrical roller bearing, single row, two rigid ribs on inner ring, ribless outer ring		DIN 5412-1
Radial cylindrical roller bearing, single row, two rigid ribs on outer ring, ribless inner ring		DIN 5412-1
Radial cylindrical roller bearing, single row, two rigid ribs on outer ring, one rigid rib on inner ring		DIN 5412-1
Radial cylindrical roller bearing, single row, two rigid ribs on outer ring, one rigid rib and one loose rib washer on inner ring		DIN 5412-1
L-section ring for cylindrical roller bearings		DIN 5412-1
Radial cylindrical roller bearing, double row, three rigid ribs on inner ring, ribless outer ring		DIN 5412-4
Radial cylindrical roller bearing, double row, three rigid ribs on outer ring, ribless inner ring		DIN 5412-4
Tapered roller bearing, single row		DIN 720
Radial spherical roller bearing, single row, barrel roller bearing		DIN 635-1
Radial spherical roller bearing, double row		DIN 635-2
Radial needle roller and cage assembly, single row		DIN 5405-1
Drawn cup needle roller bearing with open ends, single row		DIN 618
Drawn cup needle roller bearing with closed end, single row		DIN 618

Continuation of table, see Page 607.

Continuation of table, Compilation of standardised bearings, from Page 606.

Name/design	Image	Standard number
Radial needle roller bearing, single row, without inner ring		DIN 617
Radial needle roller bearing, single row, two rigid ribs on outer ring, ribless inner ring		DIN 617
Combined radial needle roller bearing/ axial deep groove ball bearing		DIN 5429-1
Combined radial needle roller bearing/ axial cylindrical roller bearing		DIN 5429-1
Combined radial needle roller bearing/ angular contact ball bearing		DIN 5429-2
Axial deep groove ball bearing, single direction, with flat housing locating washer		DIN 711
Axial deep groove ball bearing, single direction, with curved housing locating washer		DIN 711
Support washer for axial ball bearings		DIN 711
Axial deep groove ball bearing, double direction, with flat housing locating washer		DIN 715
Axial deep groove ball bearing, double direction, with curved housing locating washer		DIN 715
Axial needle roller and cage assembly, single row		DIN 5405-2
Axial bearing washer		DIN 5405-3
Axial cylindrical roller bearing, single direction		DIN 722
Axial spherical roller bearing, asymmetrical rollers		DIN 728

Basic structure of rotary rolling bearings

Rotary rolling bearings generally consist of the components in accordance with Figure 88:

- rolling elements (balls, ball rollers, rollers); see table Rolling element type
- inner and outer ring with rolling element raceways
- cage, see Page 576
- seals or sealing shields on one side or both sides of the bearing
- lubricant in the case of grease lubrication.

Figure 88 shows standard parts of ball and roller bearings.

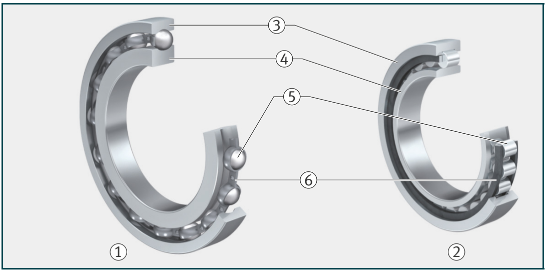
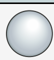
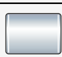
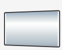


Figure 88
Structure of standard ball and roller bearings

① Ball bearing
② Roller bearing
③ Outer ring
④ Inner ring
⑤ Rolling elements (ball, cylindrical roller)
⑥ Cage (metal, polyamide)

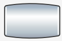
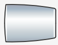
Rolling elements

Rolling elements are the connecting elements and contact elements between the stationary and the moving part of the rolling bearing. As the basis of the rolling element type, they essentially determine the characteristics of the bearing. Types of rolling elements, and the type of rolling bearing as derived from them, are shown in the following table.

Rolling element type		Bearing type
	Ball	Ball bearing
	Cylindrical roller	Cylindrical roller bearing
	Needle roller	Needle roller bearing, drawn cup needle roller bearing with open ends, drawn cup needle roller bearing with closed end, needle roller and cage assembly
	Tapered roller	Tapered roller bearing

Continuation of table, see Page 609.

Continuation of table Rolling element type from Page 608.

Rolling element type		Bearing type
	Symmetrical barrel roller	Barrel roller bearing, spherical roller bearing
	Long, slightly curved barrel roller	Toroidal roller bearing
	Asymmetrical barrel roller	Axial spherical roller bearing

Radial ball bearings

Radial ball bearings have balls as rolling elements, an operating contact angle between 0° and 45° and can support axial loads as well as radial loads.

Deep groove ball bearings

Deep groove ball bearings are versatile, self-retaining bearings with solid outer rings, inner rings and ball and cage assemblies, see Figure 89. These products, which are of simple design, robust in operation and easy to maintain, are available in single and double row designs and in open and sealed variants. Due to their very low frictional torque, deep groove ball bearings are suitable for high and very high speeds.

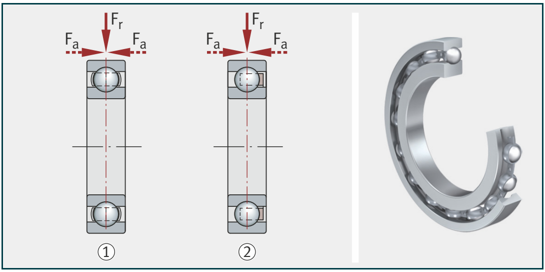


Figure 89
Deep groove ball bearings, single row

F_r = radial load
F_a = axial load

① Bearing with sheet metal cage
② Bearing with plastic cage

Combined load (axial and radial)
Due to the raceway geometry and the balls, the bearings can support axial loads in both directions as well as radial loads.

Single row deep groove ball bearings

Single row deep groove ball bearings are the most frequently used type of rolling bearing, see Figure 89. They are produced in numerous sizes and designs and are particularly economical. Open bearings are suitable for high to very high speeds, designs with gap seals are suitable for high speeds.

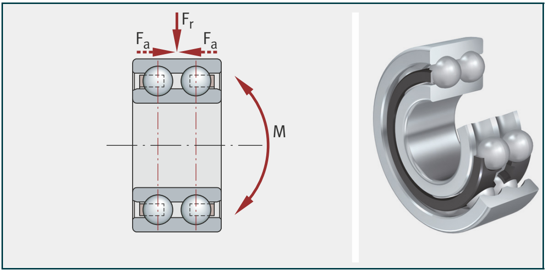
Double row deep groove ball bearings

Double row deep groove ball bearings correspond in their structure to a pair of single row deep groove ball bearings but have two raceway grooves each with one row of balls, see Figure 90. Due to the larger number of rolling elements, they can be subjected to higher loads than single row deep groove ball bearings.

Double row bearings are suitable for high to very high speeds and are used where the load carrying capacity of single row deep groove ball bearings is not sufficient.

Figure 90
Deep groove ball bearing, double row

M = tilting moment load

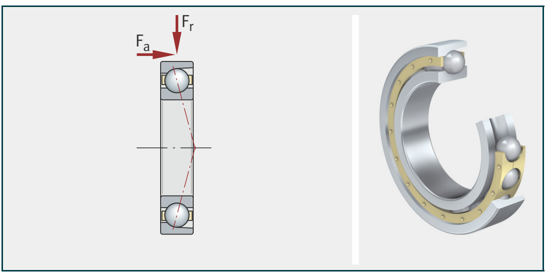


Magneto bearings

Magneto bearings correspond substantially to deep groove ball bearings without a slot (the slot is used to fill the bearing with the rolling elements). They have a shoulder on the outer ring and can therefore support axial forces in one direction only, see Figure 91. For axial guidance of the shaft, two bearings in a mirror image arrangement are always necessary. Magneto bearings are normally mounted with a small axial clearance; in this way, changes in length of the shaft and housing can be compensated.

Since the bearings are separable, it is possible to mount the outer ring separately from the inner ring with the ball and cage assembly.

Figure 91
Magneto bearing, single row



Angular contact ball bearings

The optimum load direction and the best flow of force can only be ensured using deep groove ball bearings if the active force acts in a vertical direction through the radial plane. In practice, it is frequently the case that the loads acting in a bearing arrangement are not purely radial but combined, where the loads comprise both radial and axial forces.
In contrast to deep groove ball bearings, the raceways on the inner and outer rings of angular contact ball bearings are offset from each other along the bearing axis, see Figure 92.

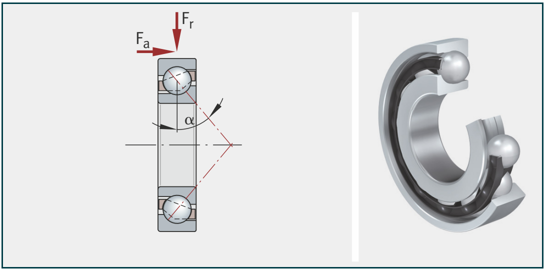
As a result, the forces are transmitted from one raceway to the other at a specific contact angle (oblique to the radial plane). They are therefore suitable for radial and axial loads acting simultaneously.

Contact angle

The contact angle is normally 15°, 25°, 30° or 40°. The axial load carrying capacity of the bearing increases with the contact angle. As a result, angular contact ball bearings are more suitable than deep groove ball bearings for supporting higher axial forces.

Figure 92
Angular contact ball bearing, single row

α = contact angle



Spindle bearings

Spindle bearings are self-retaining, single row angular contact ball bearings of a high precision design with contact angles of 15°, 20° or 25°. Bearings with a contact angle of 15° are particularly suitable for high radial loads, while those with a contact angle of 25° are more suitable for combined loads.

At high speeds, large centrifugal forces occur in the balls which are superimposed on the operating loads and thus have a significant influence on the speed capacity of the bearings. In order to reduce the centrifugal forces and increase the speeds, bearings can be used that have smaller balls while retaining the normal external dimensions.

Due to special contact conditions between the balls and raceways, the friction and operating temperature of the bearing remain low.
For even higher speeds, hybrid bearings (bearings with ceramic balls) can be used.

Double row angular contact ball bearings

Double row angular contact ball bearings correspond in their structure to a pair of single row angular contact ball bearings in an O arrangement, see Figure 93. They can support high radial and axial loads and are therefore particularly suitable for the rigid axial guidance of shafts.

Smaller double row angular contact ball bearings do not have a filling slot and therefore have the same axial load carrying capacity on both sides. In the case of larger double row bearings, the position of the filling slot determines the load direction. The axial force should always be supported by the ball row without a filling slot.

Where high, alternating axial forces must be supported, double row angular contact ball bearings with a split inner ring can be used. These bearings have a larger contact angle and do not have a filling slot.

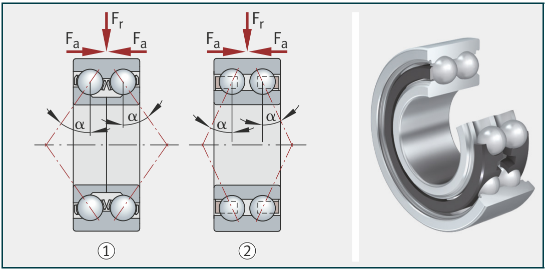


Figure 93
Angular contact ball bearings, double row

α = contact angle

1 Bearing with split inner ring, large contact angle
2 Bearing with unsplit inner ring, small contact angle

Four point contact bearings

Four point contact bearings are similar in their structure to double row angular contact ball bearings, see Figure 94. Since the centres of curvature of the arc-shaped raceways on the inner ring and outer ring are offset relative to each other, however, the balls are in contact with the bearing rings at four points under radial load. For this reason, four point contact bearings are only used under predominantly axial load. One of the two bearing rings, normally the inner ring, is split in order to allow filling with the balls.

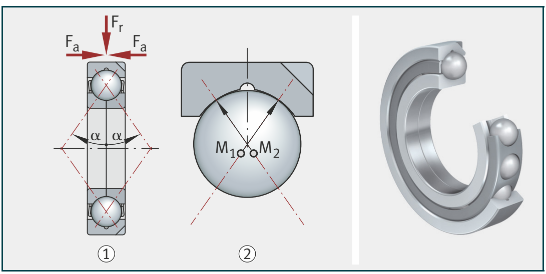


Figure 94
Four point contact bearing

α = contact angle
M_1, M_2 = centres of curvature of outer ring raceway

1 Four point contact bearing with retaining slot and split inner ring
2 Raceway geometry

For axial loads

The bearings can support alternating, pure axial or predominantly axial loads. Due to the large contact angle (usually 35°), four point contact bearings are suitable for high axial forces in alternating directions.

If predominantly radial load is present, four point contact bearings should not be used due to the higher friction in the four point contact.

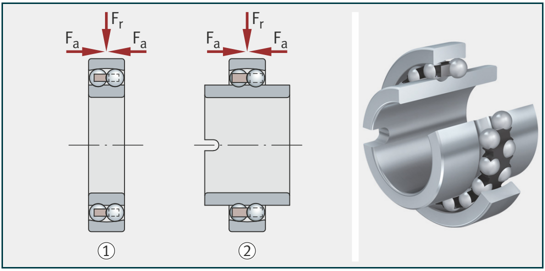
Self-aligning ball bearings

In the self-aligning ball bearing, two rows of rolling elements are held in two raceway grooves on the inner ring, see Figure 95. The raceway on the outer ring has a curved form. The cage combines the two rows of balls and the inner ring to form a unit that can align itself by swivelling relative to the outer ring.

Figure 95
Self-aligning ball bearings

Curved raceway of outer ring

① Bearing with cylindrical bore
② Bearing with extended inner ring and locating slot



Tolerant of misalignment

Due to the curved raceway on the outer ring, self-aligning ball bearings are tolerant of misalignment between the shaft and housing and deflections of the shaft. They can thus compensate static and dynamic angular defects within certain angular limits in a rotating shaft system.

They are used particularly in sectors such as agricultural machinery, conveying equipment, simple woodworking machinery and ventilators.

Thin section bearings

Thin section bearings have very small cross-sections relative to their diameters, see Figure 96. This allows designs with smaller design envelope and lower mass, while achieving high rigidity and running accuracy.

In contrast to the rolling bearing series standardised in accordance with DIN ISO, in which the cross-section increases with the bearing diameter, all sizes of bearing in one series have the same cross-section.

Deep groove ball bearings, four point contact bearings and angular contact ball bearings

Thin section bearings are available as deep groove ball bearings, four point contact bearings and angular contact ball bearings. The preferred diameter ranges are between 25 mm and 1000 mm.

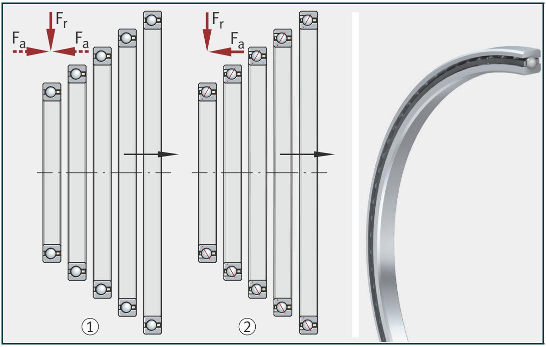


Figure 96
Thin section bearings

Curved raceway of outer ring

① Deep groove ball bearing type
② Angular contact ball bearing type

Radial insert ball bearings

Radial insert ball bearings are based on deep groove ball bearings of series 60, 62 and 63. The numerals identify the rolling element set and therefore the load carrying capacity. The inner ring is extended on one or both sides.

In conjunction with the corresponding housing, radial insert ball bearings allow the use of basic adjacent constructions. They are particularly easy to fit and are suitable for drawn shafts of Ⓔ to h9 Ⓔ. For non-locating bearings, shafts of tolerance classes h5 Ⓔ to h7 Ⓔ are recommended.

Location and designs

Axial location can be carried out using eccentric locking collars, grub screws or integrated adapter sleeves, see Figure 97. Certain series can be located by means of a fit on the shaft.

The extended inner ring on both sides provides a seal running surface and prevents significant tilting of the inner ring.

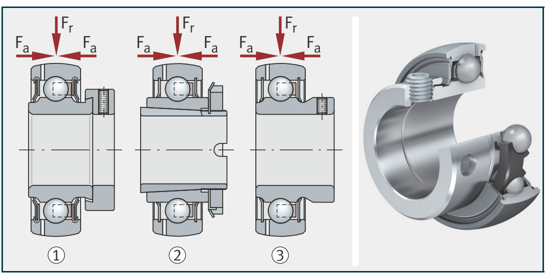


Figure 97
Location methods for radial insert ball bearings

① Location by means of eccentric locking collar
② Location by means of integral adapter sleeve
③ Location by means of grub screws in inner ring

Seals matched to application, numerous series with relubrication facility
A further, characteristic feature of radial insert ball bearings is the type of sealing used. The bearing is protected against contamination and the loss of lubricant by means of contact seals of a multi-piece design on both sides of the bearing. Additional protection of the seals can be provided by means of sheet steel flinger shields mounted in front of the lip seals.

The bearings can normally be relubricated via holes in the outer ring.

Compensation of static angular misalignments
Radial insert ball bearings are available in numerous designs with a curved or cylindrical outside surface of the outer ring. Bearings with a curved outside surface can, through adjusting motion of the outer ring in a curved bore, support angular misalignments (±2,5° with relubrication, ±5° without relubrication). Aligning rings with a curved bearing locating bore allow mounting of the bearings in housings with a cylindrical bore. The angular adjustment facility is thus maintained.

Radial insert ball bearings cannot be used for the compensation of dynamic angular misalignments.

Housing units

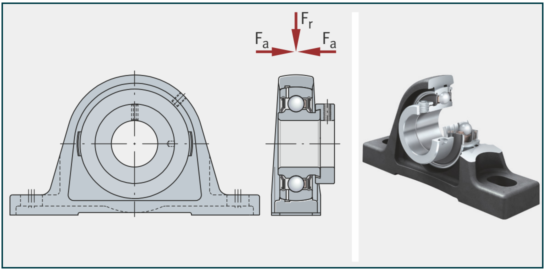
For radial insert ball bearings, Schaeffler can supply suitable plummer block and flanged housings that are made from flake graphite cast iron or sheet steel, see Figure 98. The housings can – like the radial insert ball bearings themselves – also be provided in a corrosion-resistant design. Cast iron housings are always one-piece units and can support high loads. Sheet steel housings are always two-piece units and are used where the priority is not the load carrying capacity of the housing but the low mass of the unit.

The housing units comprise radial insert ball bearings with a curved outer ring and a housing with a curved bore to form ready-to-fit units. The user is thus spared the need for costly production of the mounting environment required for these bearings. The areas of application correspond to those of the radial insert ball bearings. Bearings with a curved outside surface of the outer ring, when mounted in housings with a curved bore, can compensate static misalignments of the shaft, see section Compensation of static angular misalignments, Page 615.

The units are not suitable for supporting swivel or wobble motion.

Figure 98
Plummer block housing unit

Flake graphite cast iron housing with integrated radial insert ball bearing



Radial roller bearings

Radial roller bearings are rolling bearings with cylindrical, needle, tapered or barrel rollers as rolling elements.

Cylindrical roller bearings

Cylindrical roller bearings are used as non-locating, semi-locating and locating bearings. They are available with a full complement of rollers, with a cage or with spacers.

Axial load carrying capacity

The axial load carrying capacity of a cylindrical roller bearing is dependent on:

- the size of the sliding surfaces between the ribs and the end faces of the rolling elements
- the sliding velocity at the ribs
- the lubrication of the contact surfaces
- tilting of the bearing.

Ribs subjected to load must be supported across their entire height.

Cylindrical roller bearings must not be continuously subjected to pure axial loads. In order to prevent impermissibly high edge stresses on the rolling elements and raceways due to insufficient contact between the rolling elements and the raceways, a minimum radial load must always be present. The ratio F_a/F_r should not exceed 0,4. Cylindrical roller bearings with an optimised contact geometry allow a ratio F_a/F_r up to a value of 0,6.

Further information on the minimum radial load is given in the Schaeffler rolling bearing catalogues.

Cylindrical roller bearings with cage

Cylindrical roller bearings with cage are available in numerous designs, sizes and dimension series. In all standard designs, however, the cylindrical rollers are guided between rigid ribs by at least one bearing ring. Together with the cage and rollers, this forms a ready-to-fit unit. The other bearing ring can be removed. As a result, the inner ring and outer ring can be mounted separately. Tight fits can thus be achieved on both rings.

Tight fits increase the rigidity of the bearing arrangement and give precise radial guidance of the shaft.

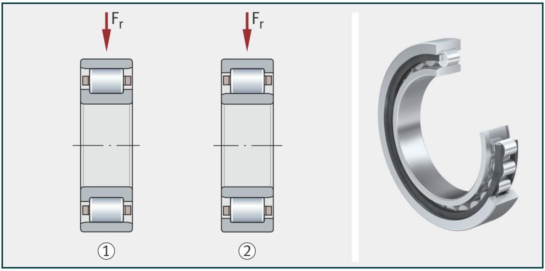
Non-locating bearings

Cylindrical roller bearings N and NU as well as NN and NNU are non-locating bearings and can support radial forces only, see Figure 99. In the case of series NU and NNU, the outer ring has ribs while the inner ring has no ribs. Bearings of the designs N and NN have ribs on the inner ring and an outer ring without ribs. The ring without ribs serves to compensate variations in the length of the shaft, for example as a result of temperature differentials.

Non-locating bearings NU can be combined with an L-section ring HJ to form a semi-locating bearing unit.

Figure 99
Non-locating bearings

① Cylindrical roller bearing N
② Cylindrical roller bearing NU



Semi-locating bearings

Cylindrical roller bearings NJ are semi-locating bearings, see Figure 100. They can support not only high radial forces but also axial forces in one direction and can therefore guide shafts axially in one direction. In the opposite direction, they act as non-locating bearings.

The outer ring with its two ribs provides axial guidance of the rollers. The L-shaped inner ring supports the rolling elements on one side against axial forces.

Semi-locating bearings NJ can be combined with an L-section ring HJ to form a locating bearing unit.

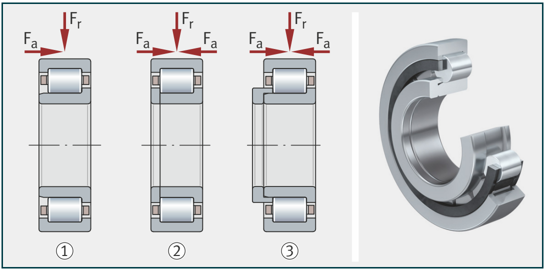
Locating bearings

Cylindrical roller bearings NUP are locating bearings, see Figure 100. They can support not only high radial forces but also axial forces in both directions and can therefore guide shafts axially in both directions.

The outer ring has two ribs, while the L-shaped inner ring has one rigid rib and one loose rib washer.

Figure 100
Semi-locating and locating bearings

① Cylindrical roller bearing NJ (semi-locating bearing)
② Cylindrical roller bearing NUP with rib washer (locating bearing)
③ Cylindrical roller bearing NJ with L-section ring HJ (locating bearing)



Disc cage or spacers

Figure 101 shows a semi-locating bearing of dimension series 23 with a space-saving disc cage. The flat brass disc cage and the plastic spacers prevent the cylindrical rollers from coming into contact with each other during rolling.

In the case of the disc cage, the rolling elements are guided between the ribs on the outer ring. Due to its low mass, the cage is subjected to only minimal load under acceleration.

The spacers are guided axially between the two outer ring ribs, see Figure 101. They are designed such that the rolling element set is self-retaining, so the bearing and inner ring can be mounted separately from each other. This gives easier mounting of the cylindrical roller bearings.

In comparison with conventional cage type bearings with roller retention, these cage designs allow a larger number of rolling elements to be accommodated in the bearing. Due to the lower frictional torques in comparison with full complement cylindrical roller bearings, less heat is generated in the bearing and higher speeds can thus be achieved.

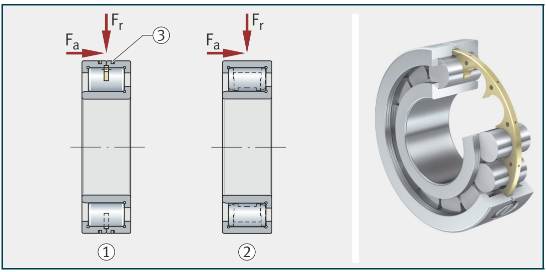


Figure 101
Cylindrical roller bearings with disc cage or spacers

1 Cylindrical roller bearing with disc cage
2 Cylindrical roller bearing with spacers
3 Retaining ring

Full complement cylindrical roller bearings

Full complement roller bearings are available as non-locating, semi-locating and locating bearings. They are of a single row or double row design and do not have a cage, see Figure 102.

Due to the lack of a cage, the bearing can accommodate more rolling elements. Since these bearings have the largest possible number of rolling elements, they have extremely high radial load carrying capacity, high rigidity and are suitable for particularly compact designs. Due to the kinematic conditions, however, they do not achieve the high speeds that are possible when using cylindrical roller bearings with cage.

Multi-row bearings have particularly high tilting rigidity but do not permit any skewing between the inner ring and outer ring.

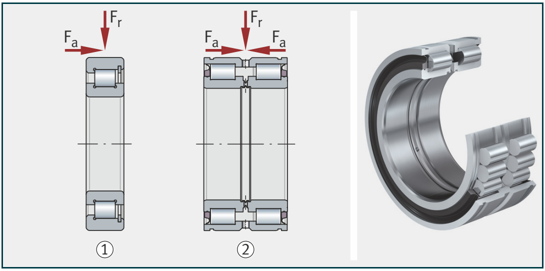


Figure 102
Full complement cylindrical roller bearings

① Single row
② Double row, with annular slots in outer ring, sealed (rope sheave bearings)

Rope sheave bearings
Special bearing types are available for special applications. For example, double row full complement cylindrical roller bearings with annular slots in the outer ring are used in bearing arrangements for the support of rope sheaves, see Figure 102. These locating bearings have high rigidity and can support moderate axial forces in both directions as well as high radial forces.

Rope sheave bearings comprise solid outer and inner rings with ribs and rib-guided cylindrical rollers. Axial location of the bearings is achieved by means of retaining rings in the annular slots in the outer ring.

Sealing rings on both sides protect the bearing against contamination, moisture and the loss of grease.

Super precision cylindrical roller bearings

The single and double row bearings are used when the very highest precision is required under very high radial load. Typical areas of application include machine tools and printing machinery. In these cases, they facilitate bearing arrangements with very high precision, high radial rigidity and very high load carrying capacity. In machine tool building, they provide radial support for the main spindle, see Figure 123, Page 633. Super precision cylindrical roller bearings N, NN and NNU are in the accuracy classes SP and UP.

Since variations in length during rotary motion can be compensated between the rollers and the ribless raceway without constraining forces, the cylindrical roller bearings are highly suitable as non-locating bearings. Axial forces are supported by axial bearings, such as double direction axial angular contact ball bearings, see Figure 123, Page 633.

Tapered roller bearings

Tapered roller bearings are single row or multi-row units comprising a ribless outer ring, an inner ring with two ribs of different heights and a cage, see Figure 103. The cage contains truncated conical rollers.

Cages, ribs, roller profile, projected lines of contact

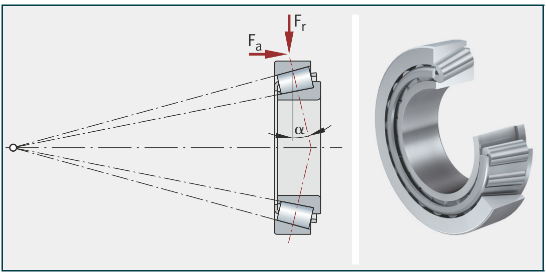
Pressed and stamped sheet steel cages are used as the standard cage. Cages made from glass fibre reinforced polyamide are also available. The lower rib retains – in conjunction with the cage – the rollers on the inner ring raceway. The high rib supports the axial force component arising from the tapered form of the rollers.

The sliding surfaces on the large rib and on the large roller end face are designed such that a lubricant film capable of supporting load is formed at the contact points between the roller and rib. The logarithmic profile induces the optimum distribution of stress at the contact between the rolling element and raceway and prevents stress peaks. The projected lines of contact of the rolling elements intersect the projected raceways of the inner and outer ring at a point on the bearing axis, see Figure 103. This prevents any kinematic forced slippage at the rolling contact.

Open bearings are not self-retaining, so the outer ring can be removed in the case of these bearings. The outer ring and the inner ring with the roller and cage assembly can thus be mounted separately from each other.

Figure 103
Single row tapered roller bearing, projected lines of contact of tapered rollers

α = contact angle



Load carrying capacity, adjustment, setting of clearance, preload

Single row tapered roller bearings can support radial loads, axial loads in one direction and combined loads (simultaneous radial and axial loads), while tapered roller bearings in an O or X arrangement can support high radial loads, axial loads in both directions and combined loads.

Due to the oblique position of the raceways, a single tapered roller bearing must not be subjected to pure radial load; an axial load or axial abutment must always be applied simultaneously. This is normally achieved by means of a second bearing mounted in a mirror image O or X arrangement.

In order to set the bearing clearance or preload, the bearings in bearing arrangements with two single row tapered roller bearings are adjusted against each other until the required value is achieved.

Nominal contact angle and axial load carrying capacity

The axial load carrying capacity is dependent on the nominal contact angle, i.e. the larger the angle, the higher the axial load to which the tapered roller bearing can be subjected. The size of the contact angle – and thus the magnitude of the load carrying capacity – is indicated by the bearing-specific value e in the product tables included in the rolling bearing catalogues.

Barrel roller bearings

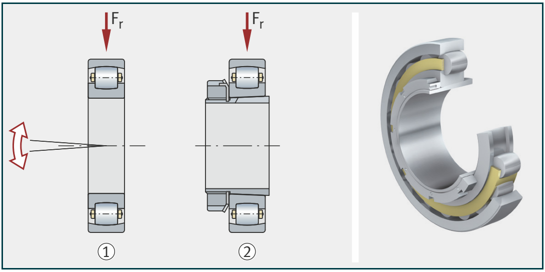
Barrel roller bearings are included in the group of self-aligning bearings, see Figure 104.

Barrel roller bearings have a concave outer ring raceway in the same way as self-aligning ball bearings and spherical roller bearings. As a result, the roller and cage assembly can align itself by swivelling (in either a static or dynamic manner), such that the bearing can respond to misalignment and deflections of the shaft without problems.

The inner ring has a cylindrical or tapered bore and the bearings are not separable. They have only a low axial load carrying capacity.

Figure 104
Barrel roller bearings

① Bearing with cylindrical bore
② Bearing with tapered bore, adapter sleeve, tab washer and locknut

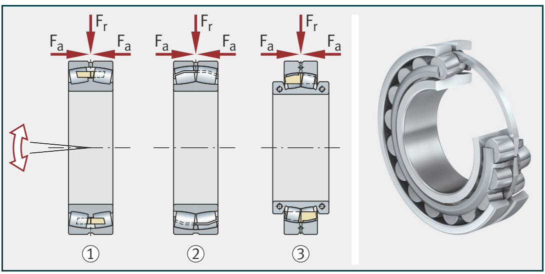


Spherical roller bearings

Spherical roller bearings have two rows of barrel rollers whose axes are inclined towards the rotational axis of the bearing, see Figure 105. The outer ring has a curved raceway in the same way as barrel roller bearings. The profile of the raceways is closely matched to the profile of the barrel rollers.

Figure 105
Spherical roller bearings

① Bearing with three rigid ribs on inner ring
② Spherical roller bearing E (increased capacity design without rigid ribs)
③ Axially split spherical roller bearing



For the supply of lubricant, the outer ring normally has a circumferential groove with radial holes between the rows of rollers. This gives improved lubrication of the bearing.

Load carrying capacity, dynamic compensation of angular misalignments
Spherical roller bearings have a high radial load carrying capacity and a higher axial load carrying capacity than barrel roller bearings. Due to the concave design of the outer ring, the barrel rollers align themselves by swivelling on the outer ring raceway in the case of misalignments and shaft deflection.

In order to ensure problem-free operation, the bearings must be subjected to a minimum load. Values for this load are given in the Schaeffler rolling bearing catalogues.

Spherical roller bearings are used where high, shock type radial loads must be dynamically supported and misalignments or more pronounced deflections of the shaft are anticipated.

Toroidal roller bearings

Toroidal bearings have elongated barrel rollers and raceways that are matched to the lines of contact of the rolling elements, see Figure 106. The centre point of the outer ring raceway radius lies under the central axis of the bearing. The design of the bearing rings allows skewing between the rings within certain limits. At the same time, changes in the length of the shaft relative to the housing can be compensated.

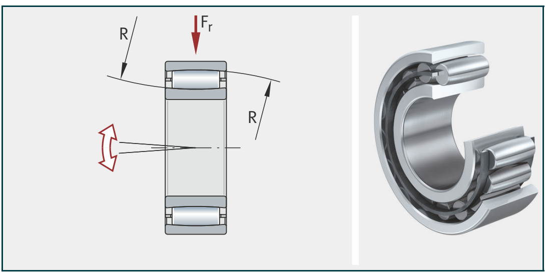
This design combines the dynamic self-alignment capacity of the spherical roller bearing with the axial displacement facility of cylindrical roller bearings. It is thus equally suitable for the dynamic and static compensation of misalignments.

Toroidal bearings are exclusively non-locating bearings and can support radial forces only.

The permissible skewing and axial displacement facility are restricted.

Figure 106
Toroidal roller bearing

R = raceway radius



Radial needle roller bearings

Radial needle roller bearings have a range of common characteristics, which are described in the following sections.

Needle roller bearings have needle rollers as rolling elements. In terms of rolling bearing technology, cylindrical rolling elements are classified as needle rollers if the rolling element diameter is $\leqq$ 6 mm and the ratio of the rolling element diameter to the rolling element length is $<$ 1:3.

Due to the small diameter of the rolling elements, all needle roller bearings have the common feature of a low radial section height. Due to the line contact, they are particularly suitable for bearing arrangements with high radial load carrying capacity and rigidity in a restricted radial design envelope.

Radial needle roller bearings can only be used as non-locating bearings. The defined axial displacement value in the case of bearings with inner rings allows axial motion between the shaft and housing. Where necessary, wider inner rings are available for larger displacement values.

In order to ensure slippage-free operation, a minimum radial load is necessary. This applies in particular to bearings running at high speeds since, if the radial load is not present, damaging sliding motion may occur between the rolling elements and raceways. Values for the minimum radial load are given in the Schaeffler rolling bearing catalogues.

Needle roller and cage assemblies

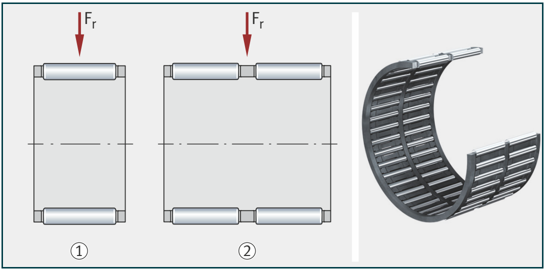
Apart from full complement needle roller sets, the needle roller and cage assembly is the simplest type of needle roller bearing, see Figure 107, Page 623. Needle roller and cage assemblies are of single or double row design and comprise a cage and needle rollers. Since they do not have an outer ring or inner ring, they run directly on the shaft and in the housing. The raceways must therefore be hardened and ground.

Since their radial section height corresponds directly to the diameter of the needle rollers, needle roller and cage assemblies allow bearing arrangements requiring only a very small radial design envelope. If the raceways are produced to high geometrical accuracy, high radial run-out accuracy can be achieved. The radial internal clearance is influenced by the shaft and housing tolerances as well as the grade of the needle rollers.

Needle roller and cage assemblies must be located axially by means of snap rings or an appropriate design of the adjacent construction with abutment shoulders.

Figure 107
Needle roller and cage assemblies

① Single row
② Double row



Drawn cup needle roller bearings with open ends and with closed end

Drawn cup needle roller bearings with open ends and with closed end are needle roller bearings with a very small radial section height. They comprise thin-walled, drawn cup outer rings and needle roller and cage assemblies which together form a complete unit, see Figure 108.

The thin-walled outer rings adopt the dimensional and geometrical accuracy of the housing bore.

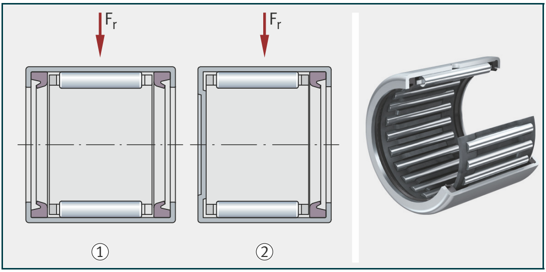
Drawn cup needle roller bearings are available either with both ends open or with one end closed. Drawn cup needle roller bearings with closed end are thus suitable for closing off the end of shafts in bearing positions. Where a rotating shaft is present, they give protection against injury, prevent the escape of lubricant and protect the rolling element system against contamination and moisture.

If the shaft cannot be produced as a raceway, the bearings can be combined with inner rings.

If axial locating elements such as shoulders and snap rings are not used, the housing bore can be produced easily and particularly economically. This also gives simpler mounting and dismounting of the bearings.

Figure 108
Drawn cup needle roller bearings with open ends and with closed end

① Drawn cup needle roller bearing with open ends, lip seal
② Drawn cup needle roller bearing with closed end, lip seal



Needle roller bearings

In comparison with drawn cup needle roller bearings with open ends and with closed end, the bearing rings of needle roller bearings are thicker, more rigid and are produced by machining methods. They place lower requirements on the dimensional accuracy, geometrical accuracy and hardness of the adjacent construction.

Needle roller bearings are subdivided into:

- needle roller bearings with ribs, with or without inner ring
- needle roller bearings without ribs, with or without inner ring
- aligning needle roller bearings
- combined needle roller bearings.

Needle roller bearings with ribs

In these needle roller bearings, the outer ring and the needle roller and cage assembly form self-retaining, single or double row complete units, see Figure 109. The cage assembly is guided axially by ribs on the outer ring.

The bearings are available with and without a removable inner ring and in both sealed and open versions.

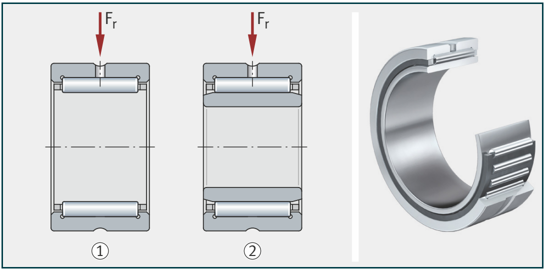


Figure 109
Needle roller bearings with ribs, single row

1 Without inner ring
2 With inner ring

Needle roller bearings without ribs

These complete units comprise outer rings without ribs, needle roller and cage assemblies and removable inner rings, see Figure 110.
Since the bearings are not self-retaining, the outer ring, needle roller and cage assembly and inner ring can be mounted independently of each other.

The bearings are available with and without an inner ring and in single and double row designs. The cage assembly is guided axially by thrust washers.

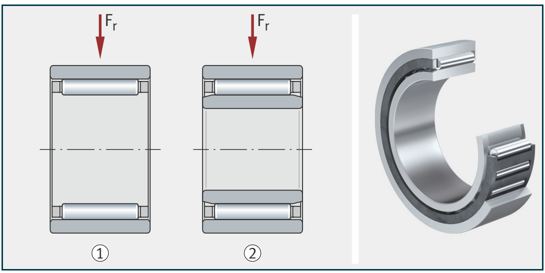


Figure 110
Needle roller bearings without ribs

1 Without inner ring
2 With inner ring

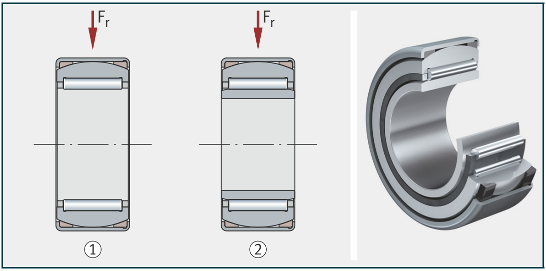
Aligning needle roller bearings

Due to their internal construction, needle roller bearings of the types described previously only allow slight misalignment of the bearing axis. Aligning needle roller bearings have a raceway ring with a curved outside surface and a curved support ring and can compensate static misalignments of the bearing axis of up to 3°.

The bearings cannot support swivel or wobble type motion.

Figure 111
Aligning needle roller bearings

① Without inner ring
② With inner ring



Combined needle roller bearings

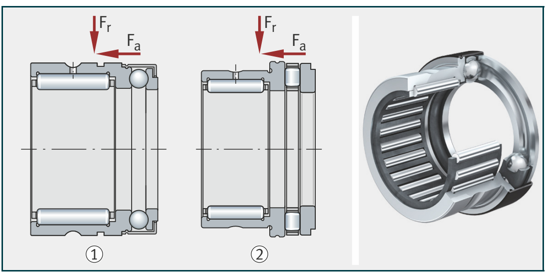
Radial needle roller bearings can support radial forces only. Combined needle roller bearings (radial needle roller bearings combined with a rolling element system capable of supporting axial loads) can additionally support axial forces.

Needle roller/axial deep groove ball bearings, needle roller/axial cylindrical roller bearings

Needle roller/axial ball bearings and needle roller/axial cylindrical roller bearings can support axial forces in one direction as well as high radial forces, see Figure 112. In order to support axial forces, the axial bearing component must be preloaded to 1% of the basic static axial load rating. Misalignments between the shaft and housing are not possible.

Figure 112
Needle roller/axial ball bearing, needle roller/axial cylindrical roller bearing

① Needle roller/axial deep groove ball bearing with end cap, without inner ring
② Needle roller/axial cylindrical roller bearing, without inner ring



Needle roller/angular contact ball bearings

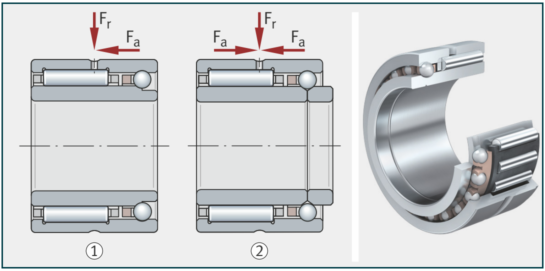
Needle roller/angular contact ball bearings can support axial forces in one or both directions, see Figure 113. Needle roller/angular contact ball bearings capable of supporting axial loads in one direction can be used as semi-locating bearings, while bearings capable of supporting axial loads in both directions can be used as semi-locating or locating bearings.

If the design capable of supporting axial loads in one direction is to support axial forces from alternating directions, two bearings must be adjusted against each other.

Where axial forces and misalignment between the shaft and housing are to be supported, the information in the section Needle roller/axial deep groove ball bearings, needle roller/axial cylindrical roller bearings must be observed.

Figure 113
Needle roller/angular contact ball bearings

① Capable of supporting axial loads in one direction, unsplit inner ring
② Capable of supporting axial loads in both directions, split inner ring



Bearings for screw drives

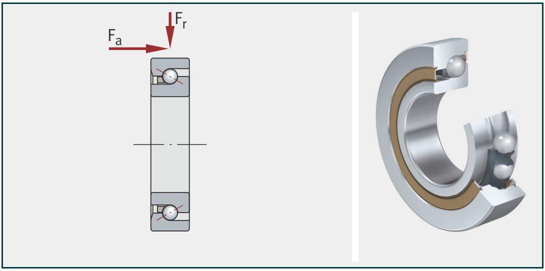
The essential requirements placed on bearings for screw drives are high axial run-out accuracy, axial and tilting rigidity as well as high load carrying capacity. The bearings that have proven effective in this area of application are single row and multi-row axial angular contact ball bearings and double direction needle roller/axial cylindrical roller bearings.

Single row axial angular contact ball bearings

The bearings can be universally combined in various bearing sets. Since the bearings are supplied already matched, a defined rolling bearing preload is achieved after mounting of the bearings that ensures clearance-free operation with high axial run-out accuracy and high axial rigidity. This bearing design can be used for a wide range of load requirements and is available in most cases in a greased and sealed design.

Single row axial angular contact ball bearings are particularly suitable for bearing arrangements for ball screw drives in machine tools, see Figure 114.

Figure 114
Axial angular contact ball bearing, single row



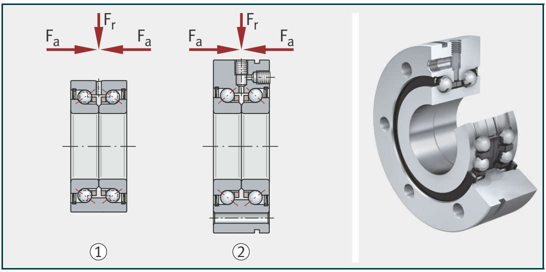
Double row axial angular contact ball bearings

Double row axial angular contact ball bearings are ready-to-fit, self-retaining, greased and sealed precision bearings in an O arrangement with a contact angle of 60°, see Figure 115. The bearing rings are matched to each other such that a defined preload is achieved when the rings are clamped in place using a precision locknut. The rolling element system is protected against contamination by means of contact seals on both sides of the bearing. For high speeds, minimal gap seals are available. Double row bearings are also available in matched pairs, so four-row bearing assemblies can be created. This gives an additional increase in the load carrying capacity and rigidity of the bearing arrangement.

The bearings are available with and without fixing holes in the outer ring, see Figure 115. Bearings with holes are screw mounted directly on the adjacent construction. This solution is particularly economical since there is then no need for the locating bore that would otherwise be required or for the bearing cover with the associated matching work.

Figure 115
Axial angular contact ball bearings, double row

① Double row, for mounting in locating bore
② Double row, for screw mounting to adjacent construction



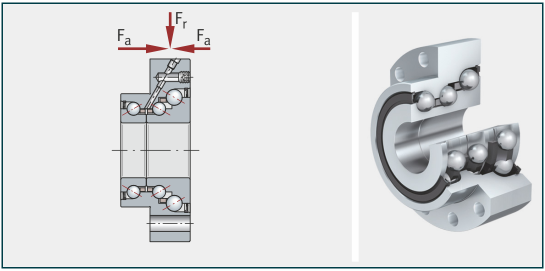
Triple row axial angular contact ball bearings

Triple row designs have, in addition to two rows of balls with a 60° contact angle in an O arrangement, a third row of balls, see Figure 116, Page 629. This additional row allows higher loads in one direction. Due to the stepped outer ring, the bearings can be easily flanged mounted on the adjacent construction. In order to reduce the radial design envelope required, the flange is flattened on two sides.

In order to make full use of the load carrying capacity, the bearings must be subjected continuously to load in the main load direction. They are therefore used mainly in screw drives with a locating/locating bearing arrangement and tensioned spindles or in vertically arranged screw drive bearing arrangements.

Figure 116
Axial angular contact ball bearing, triple row

For screw mounting to adjacent construction



Needle roller/axial cylindrical roller bearings

These bearings are double direction, precision cylindrical roller bearings with a radial bearing component and are not self-retaining. They comprise an outer ring with radial and axial raceways, a shaft locating washer, an inner ring, a radial needle roller and cage assembly and two axial cylindrical roller and cage assemblies, see Figure 117. The bearings are available in versions for screw mounting on the adjacent construction and for location in a housing bore. Since the outer ring can be screw mounted, there is no need for the cover that would otherwise be required and the associated matching work.

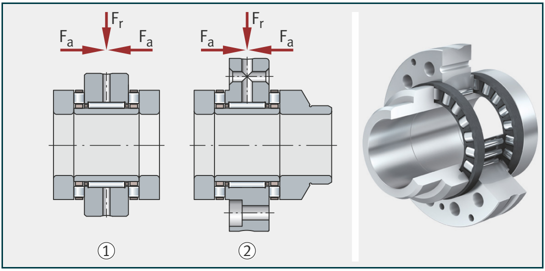
Needle roller/axial cylindrical roller bearings can support not only high radial forces but also axial forces in both directions and tilting moments. The outer ring, inner ring and axial cages are matched to each other such that the bearings are axially clearance-free after preloading by means of a precision locknut.

In comparison with axial angular contact ball bearings, they have higher load carrying capacity, rigidity and accuracy. For higher loads, bearings are available with a larger cross-section – and thus higher basic load ratings – as a heavy series.

If the axial abutment of the shaft locating washer is not sufficient or a seal raceway is required, bearings with a stepped shaft locating washer extended on one side are suitable, see Figure 117.

Figure 117
Needle roller/axial cylindrical roller bearings

① For mounting in housing bores
② For flange mounting, with stepped extended shaft locating washer



Yoke type, stud type and ball bearing track rollers

Rotary rolling bearings are normally mounted in a housing bore. In this case, the outer ring supports the loads originating from the shaft and transmits these into the surrounding housing. In the case of yoke type, stud type and ball bearing track rollers, the outer ring runs freely on a flat or curved mating track (such as a rail, guideway or cam plate).

The characteristic feature of this rolling bearing type is the particularly thick-walled outer rings. These rings replace the housing and support deflections and stresses.

In practice, yoke type, stud type and ball bearing track rollers with a crowned outside surface are predominantly used, since tilting relative to the mating track often occurs and edge stresses must be avoided. The load carrying capacity of the bearing is increased by optimised profiles on the outer ring.

Yoke type track rollers

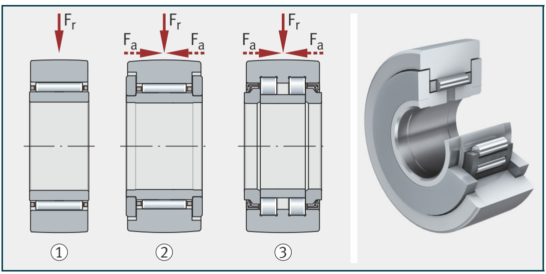
Yoke type track rollers are ready-to-fit needle or cylindrical roller bearings with a particularly thick-walled outer ring, see Figure 118. Yoke type track rollers are available with and without axial guidance of the outer ring and in both sealed and open versions. The outer rings have a crowned or cylindrical outside surface. They are mounted on shafts or studs and are supplied with or without an inner ring. They can support high radial loads. Yoke type track rollers with axial guidance tolerate axial loads which are due to slight misalignments, skewed running or temporary contact running impacts.

Bearings without an inner ring require a raceway on the shaft or stud corresponding to the quality of a rolling bearing raceway.

The bearings are used in applications including cam gears, conveying equipment and linear guidance systems.

Figure 118
Yoke type track rollers

① Without axial guidance, with cage, open design
② With axial guidance, with cage, gap seals
③ With axial guidance, full complement roller set, lip seals



Stud type track rollers

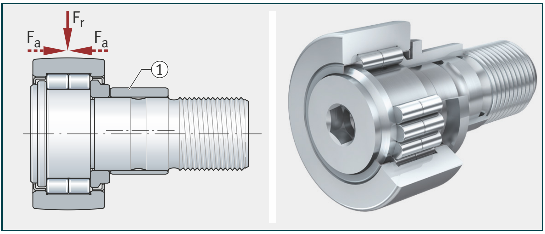
Stud type track rollers correspond in their design to yoke type track rollers with axial guidance but, in place of the inner ring, they have a heavy-section roller stud, see Figure 119. The stud has a fixing thread and, in many cases, a hexagonal socket on both sides for mounting of the stud type track rollers and adjustment in designs with an eccentric collar. The eccentric collar allows adjustment of the outer ring outside surface to match the mating track on the adjacent construction.

Stud type track rollers are available with various seals (such as labyrinth, gap or contact seals). The outer ring has a crowned or cylindrical outside surface.

Figure 119
Stud type track roller, double row, full complement roller set, with eccentric collar

Labyrinth seal

① Eccentric collar



Ball bearing track rollers

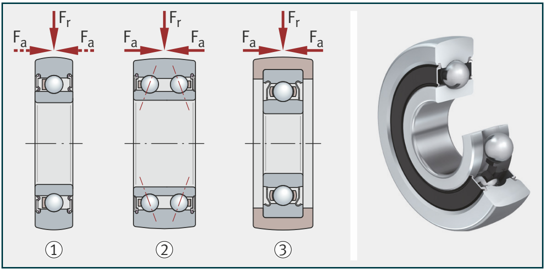
Ball bearing track rollers correspond in their design to sealed deep groove or angular contact ball bearings but have thick-walled outer rings with a crowned or cylindrical outside surface. They can support axial forces in both directions as well as high radial forces. Designs with a crowned outside surface are used where misalignments occur relative to the mating track.

Ball bearing track rollers are available with and without a stud, with a plastic tyre and with a profiled outer ring, see Figure 120.

Bearings without a stud are mounted on shafts or studs. Ball bearing track rollers with a plastic tyre can be used if particularly quiet running is required. Bearings with a profiled concave outer raceway are suitable for the design of robust shaft guidance systems, see Figure 146, Page 658.

Figure 120
Ball bearing track rollers without stud

① Single row, sealed ② Double row, sealed ③ With plastic tyre, sealed



Axial ball bearings

Axial ball bearings are pure axial bearings, which means that they may only be subjected to axial load.

Axial deep groove ball bearings

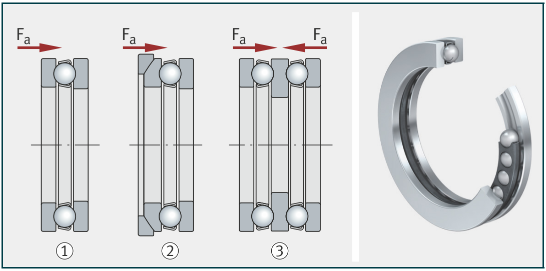
Axial deep groove ball bearings are single direction or double direction units that can support high axial forces and are not self-retaining.

Single direction bearings comprise a shaft locating washer and a housing locating washer between which a ball and cage assembly is arranged, see Figure 121, Page 632. They can support axial forces in one direction and support the shaft on one side.

Double direction bearings comprise a shaft locating washer, two housing locating washers and two ball and cage assemblies, see Figure 121. This design can support axial forces in both directions and can therefore guide the shaft on both sides.

Figure 121
Axial deep groove ball bearings

① Single direction
② Single direction, curved housing locating washer and support washer
③ Double direction



Angular adjustment facility
In addition to the series with flat housing locating washers, axial bearings with curved locating surfaces are also available for the compensation of static angular defects. These designs are normally used in conjunction with support washers and tolerate static misalignments of the shaft relative to the housing. They are not suitable, however, for wobble type motions of the shaft, since the friction on the curved locating surfaces is too high.

Axial angular contact ball bearings

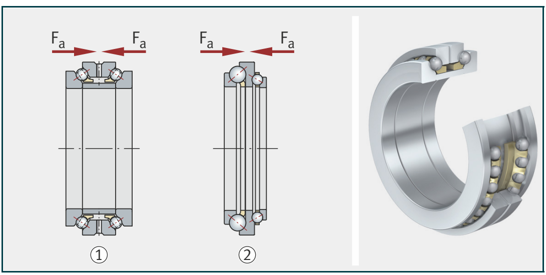
For axially rigid spindle bearing arrangements in machine tools, single row and double row axial angular contact ball bearings with increased accuracy are used, see Figure 122. The bearings can support axial forces in both directions.

Axial angular contact ball bearings transmit the forces from one raceway to the other by means of a defined contact angle. The ribs are sufficiently high that the contact ellipse does not contact or extend beyond the edge of the raceway even under the centrifugal force effect of high speeds and in axially preloaded bearings at high loads.

Figure 122
Axial angular contact ball bearings

Contact angle normally 60°

① Double direction, with spacer ring
② Double direction



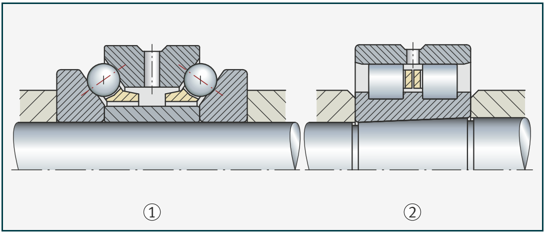
Double row axial angular contact ball bearings

Double row axial angular contact ball bearings have a ball-guided solid brass cage for each row of balls. A spacer ring is inserted between the shaft locating washers with a fit such that the bearing has the necessary preload once it is mounted and axially located.

In bearing arrangements for main spindles in machine tools, the radial forces are generally supported by a cylindrical roller bearing arranged adjacent to the axial angular contact ball bearing, see Figure 123.

Figure 123
Cylindrical roller bearing for support of radial forces

① Double direction axial angular contact ball bearing 2344
② Super precision cylindrical roller bearing NN30, double row



Axial roller bearings

Axial bearings of this type are single direction or double direction axial bearings based on cylindrical rollers or needle rollers.

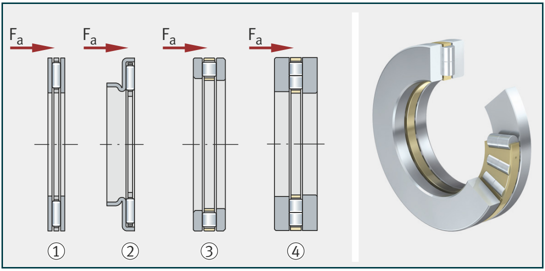
Axial cylindrical roller bearings, axial needle roller bearings

These axial bearings comprise flat, ribless washers between which axial needle roller and cage assemblies or axial cylindrical roller and cage assemblies are arranged, see Figure 124. Their axial section height corresponds to the diameter of the rolling elements plus the washer thickness values. They have high axial load carrying capacity, are extremely rigid and can support axial forces in one direction. The bearings can support axial forces only. Radial forces must be supported by an additional, suitable bearing.

For particularly low axial section heights, the cage assemblies can be integrated directly in the adjacent construction. The running surfaces for the rolling elements must therefore be produced as rolling bearing raceways.

Figure 124
Axial needle and cylindrical roller bearings

① Axial needle roller bearing
② Axial needle roller bearing with centring spigot
③ Axial cylindrical roller bearing, single row
④ Axial cylindrical roller bearing, double row



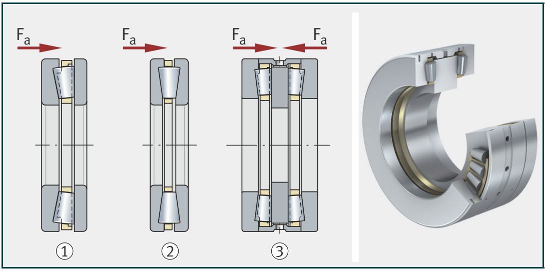
Axial cylindrical roller bearings and axial needle roller bearings are used where the load carrying capacity of axial deep groove ball bearings is not sufficient. Axial needle roller bearings are suitable for particularly low axial section heights, while axial cylindrical roller bearings can support higher loads. Misalignments between the shaft and housing are not permissible, but the bearings can align themselves radially.

Axial tapered roller bearings

The most frequently used type of axial tapered roller bearings has a shaft locating washer with a tapered raceway and a housing locating washer with a flat raceway, see Figure 125.

Figure 125
Axial tapered roller bearings

1 Single direction, with flat housing locating washer
2 Single direction, with two tapered raceways
3 Double direction



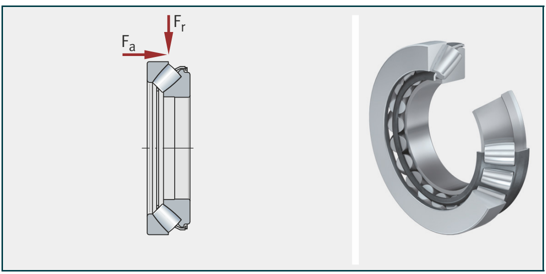
The rollers are guided by the rib of the shaft locating washer and are normally guided by a solid brass cage.

The bearings are used, for example, in rolling mill construction.

Axial spherical roller bearings

Axial spherical roller bearings comprise a housing locating washer, a shaft locating washer and asymmetrical barrel rollers, see Figure 126.

Figure 126
Axial spherical roller bearing



The barrel rollers abut the high rib of the inner ring and can align themselves on the curved raceway of the outer ring. As a result, the bearing is similar to other self-aligning bearings in being unaffected by misalignments and shaft deflections. The roller and cage assembly and the shaft locating washer are held together by solid brass cages or sheet steel cages.

Since the loads are transmitted from one raceway to the other at an angle inclined to the bearing axis, the bearings are also suitable for supporting additional radial loads ($F_{r\,max} = 0{,}55 \cdot F_a$) while an axial load is present.

In order to ensure problem-free operation, a minimum axial load is necessary. This applies particularly in the case of bearings running at high speeds with high accelerations and rapid load reversals. Due to the inertia forces of the rolling elements and cages, and the increasing frictional power, impermissible wear may occur between the rolling elements and raceways. In many cases (for example in vertical bearing arrangements) the inherent mass of the bearing arrangement is sufficient, however, to apply the necessary minimum axial load.

Crossed roller bearings, dimension series 18

Crossed roller bearings of this type are open bearings for high precision applications, see Figure 127. The spacing between the cylindrical rollers is maintained by plastic spacers. The outer ring is split and is held together by retaining rings. The highly rigid bearings have high running accuracy and are supplied with normal clearance, low clearance or preload. The outer rings are fixed in the adjacent construction by means of clamping rings. Sealing of the bearing position can be freely designed as necessary.

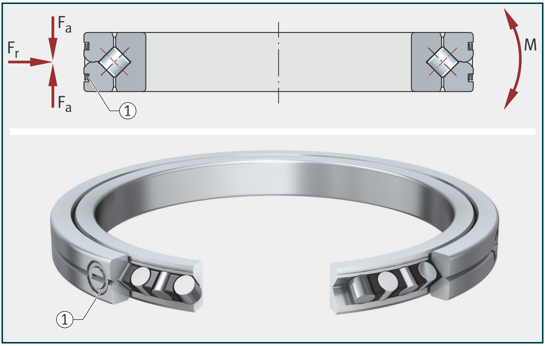
The dimensions of the crossed roller bearings conform to ISO dimension series 18 in accordance with DIN 616.

Due to the X arrangement of the cylindrical rollers, these bearings can support axial forces in both directions, radial forces and any combination of loads as well as tilting moments by means of a single bearing position. As a result, designs with two bearing positions can in many cases be reduced to a single bearing position.

Figure 127
Crossed roller bearing

F_r = radial load
F_a = axial load
M = tilting moment load

① Retaining ring



Swivel bearings (slewing rings)

Slewing rings are large size bearings with high load carrying capacity for oscillating and slow rotary motions, see Figure 128. The bearing rings are available without teeth, with internal teeth or with external teeth and are generally screw mounted directly to the immediate parts of the adjacent construction.

Swivel bearings are predominantly mounted in a horizontal position and are used to support axial forces and large tilting moments. In the applications, radial loads only occur to a subordinate extent.

Slewing rings are normally subjected to load while undergoing rotary motion only infrequently, slow swivel motion, slow rotation or while stationary and are preferably dimensioned on the basis of their static load carrying capacity.

Four point contact bearings and crossed roller bearings

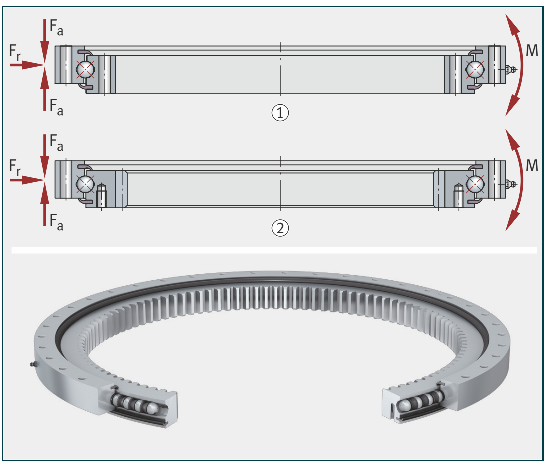
Swivel bearings can be designed in the form of ball or roller bearings. The cage comprises segments or spacers. These maintain the spacing between the rolling elements. In swivel bearings with a diameter of several metres, the rings are often split into segments for transport and mounting reasons. The raceways are subjected to induction or flame hardening. Figure 128 shows sealed four point contact bearings (designs without teeth and with internal teeth).

Crossed roller bearings can be used in joints for industrial robots and in bearing positions that make demands beyond the load carrying capacity, rigidity and accuracy of ball bearings. The rolling elements on the raceways comprise rollers arranged with a roller axis that is alternately offset by 90°. Crossed roller bearings have segment cages or a full complement of rollers. The inner or outer ring can be provided with teeth to facilitate drive solutions.

Figure 128
Four point contact bearings

M = tilting moment load

① Without teeth, sealed
② With internal teeth, sealed



Rotary table bearings (bearings for combined loads)

Rotary table bearings can support axial loads in both directions and high radial loads as well as tilting moments without clearance. Due to the fixing holes in the bearing rings, mounting of the units is very simple. After mounting, the bearings are radially and axially preloaded. They are particularly suitable for bearing arrangements requiring high running accuracy, such as rotary tables, reversible clamps, face plates and milling heads.

Axial angular contact ball bearings

Axial angular contact ball bearings are ready-to-fit and greased bearing units with particularly low friction with high accuracy for very high speeds, high axial and radial loads and high demands on tilting rigidity. They comprise a one-piece outer ring, a two-piece inner ring and two ball and cage assemblies, see Figure 129. The contact angle is 60°. The outer ring and inner ring have fixing holes for screw mounting of the bearing on the adjacent construction. The unit is secured by means of retaining screws for transport and safe handling.

Axial/radial bearings

Axial/radial bearings are double direction axial bearings with a radial guidance bearing that are suitable for screw mounting, see Figure 129. These ready-to-fit, greased units are very rigid, have high load carrying capacity and run with particularly high accuracy. Designs with an integrated angular measuring system can measure the angular position of the rotary axis to an accuracy of a few angular seconds.

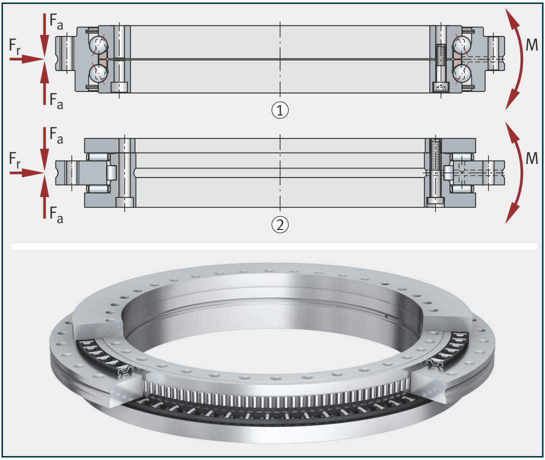


Figure 129
High precision bearings for combined loads

M = tilting moment load

1 Axial angular contact ball bearing
2 Axial/radial bearing

Guiding elements in a rotary direction – plain bearings

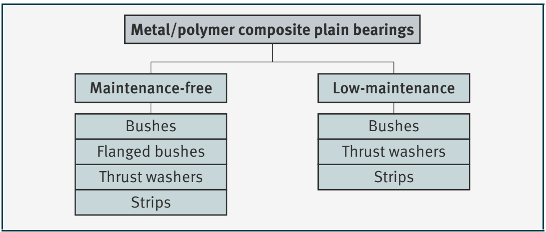
In contrast to rolling bearings, where low friction is achieved through the rolling of rolling elements between the raceways, plain bearings exhibit pure sliding motion between the shaft and bearing cup. Friction is minimised by the sliding contact surface, surface modification or suitable lubrication. While there is a wide range of different plain bearings, only dry running plain bearings are covered here in brief.

In addition to the metal/polymer composite plain bearings described below, the medium performance segment can be served using plain bearings with ELGOTEX and the upper performance segment using plain bearings with ELGOGLIDE.

Metal/polymer composite plain bearings

Plain bearings are bearings for very small radial and axial design envelopes. They have a steel or bronze backing and are available as bushes, flanged bushes, thrust washers and strips, see Figure 130 and Figure 131. Designs with a bronze backing are highly resistant to corrosion, have very good thermal conductivity and are antimagnetic.

Figure 130
Plain bearing types and designs

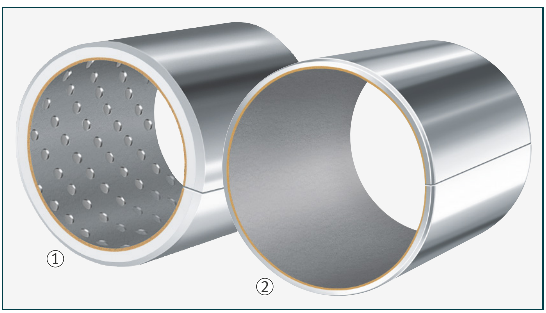


Materials E40, E40-B, E50

The material used in metal/polymer composite plain bearings is the maintenance-free E40 (with steel backing), E40-B (with bronze backing) or the low-maintenance E50. The dry lubricant is based on polytetrafluoroethylene PTFE with embedded chemically non-reactive additives. The materials fulfil the regulations on lead-free plain bearings and thus the Directive 2000/53/EC (End-Of-Life Vehicles Directive) and the Directive 2011/65/EU (RoHS-II).

Figure 131
Metal/polymer composite plain bearings (bush)

① E50, low-maintenance
② E40, maintenance-free



Maintenance-free plain bearing material

The maintenance-free plain bearing material E40, see Figure 132, is intended for dry running due to the use of PTFE as a dry lubricant. These bearings are thus particularly suitable where the bearing position must be maintenance-free, there is a risk of lubricant starvation or where lubricant is unacceptable or undesirable. The material E40 can be used not only for rotary and oscillating motion but also for short stroke linear motions. Typical areas of application can be found, for example, in fluid technology, in sports gear, in medical or electrical equipment as well as in automotive engineering.

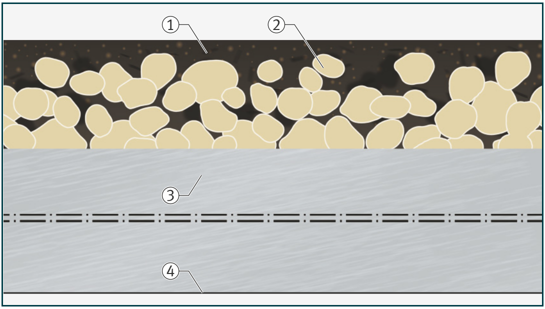


Figure 132
Structure of maintenance-free plain bearing material E40 – three-layered, with steel backing

① Running-in layer
② Sliding layer
③ Steel backing
④ Tin layer as surface protection

Low-maintenance plain bearing material

The low-maintenance plain bearing material E50, see Figure 133, is a low-wear material with good damping characteristics and long relubrication intervals. The sliding layer is made from polyoxymethylene POM. E50 can be used for rotary and oscillating motion and is recommended for long stroke linear motions. It is only slightly sensitive to edge loads and is insensitive to shocks. Application examples can be found in particular in the area of production machinery, construction and agricultural equipment as well as commercial vehicles.

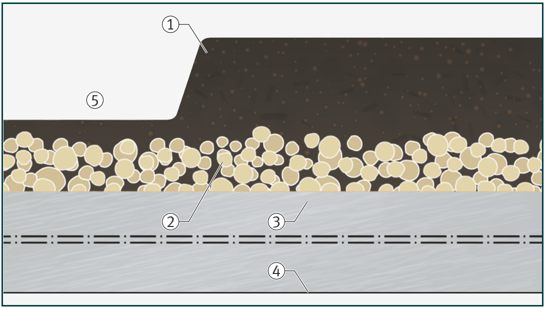


Figure 133
Structure of low-maintenance plain bearing material E50 – three-layered, with steel backing

① Sliding layer
② Intermediate layer
③ Steel backing
④ Tin layer as surface protection
⑤ Lubrication pocket

Plain bearing range from Schaeffler

The complete range is described in the Schaeffler catalogue HG 1, Plain Bearings, in Technical Product Information TPI 211 and in the online version ***medias*** *professional*: *http://medias.schaeffler.com*.

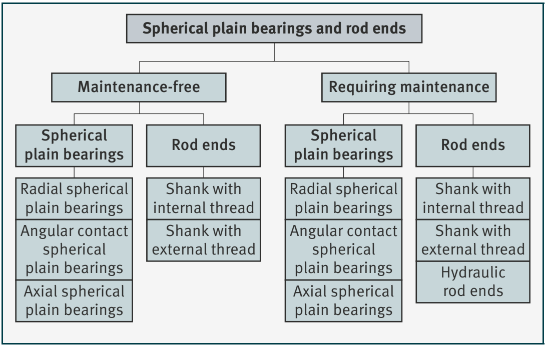
Guiding elements in a rotary direction – spherical plain bearings

Spherical plain bearings

Spherical plain bearings are ready-to-fit, standardised machine elements. Due to the concave outer ring bore and the curved inner ring geometry, they allow spatial adjustment motions. The bearings can support static loads and are suitable for tilting and swivel motion. They can compensate for shaft misalignment, are not subject to edge stresses under misalignment and permit substantial manufacturing tolerances in the adjacent construction.

An overview of the important types and designs is shown in Figure 134:

Figure 134
Types and designs of spherical plain bearings



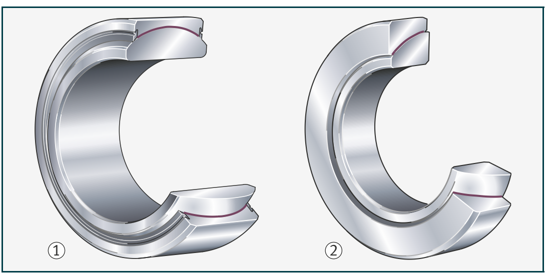
Maintenance-free spherical plain bearings

Maintenance-free spherical plain bearings are available as radial, axial and angular contact spherical plain bearings, see Figure 135. The sliding layer between the inner ring and outer ring is ELGOGLIDE, PTFE composite or PTFE-bronze film.

Radial spherical plain bearings are preferably used to support radial forces. Certain series are also suitable for alternating loads up to a contact pressure of p = 300 N/mm^2. The bearings are used where particular requirements for operating life apply in conjunction with maintenance-free operation or where, for reasons of lubrication, bearings with metallic sliding contact surfaces are not suitable, for example under unilateral load. Radial spherical plain bearings are available in open and sealed designs.

Figure 135
Maintenance-free spherical plain bearings, sliding layer ELGOGLIDE

① Radial spherical plain bearing, lip seal on both sides
② Angular contact spherical plain bearing, open design



Angular contact spherical plain bearings conform to ISO 12240-2. They have inner rings with a curved outer slideway and outer rings with a concave inner slideway to which an ELGOGLIDE sliding layer is attached by adhesive. The bearings can support radial and axial forces and are suitable for alternating dynamic loads. Preloaded units can be achieved using paired arrangements. Angular contact spherical plain bearings are used to support high loads in conjunction with small motions. In this case, they are an alternative to tapered roller bearings.

Axial spherical plain bearings conform to ISO 12240-3. In these units, the shaft locating washer is supported in the ball socket-shaped sliding zone of the housing locating washer. The sliding material on the housing locating washer is ELGOGLIDE. The bearings are preferably used to support axial forces. They are suitable as support or base bearings and can also be combined with radial spherical plain bearings of dimension series E to DIN ISO 12240-1.

Spherical plain bearings requiring maintenance

Spherical plain bearings requiring maintenance can be designed as radial, axial and angular contact spherical plain bearings and must be lubricated using oil or grease via the outer or inner ring or the housing locating washer as appropriate. The bearings comprise inner rings and outer rings with a steel/steel or steel/bronze sliding contact surface. The inner rings have a cylindrical bore and a curved outer slideway. The outer rings have a cylindrical outside surface and a concave inner slideway.

Radial spherical plain bearings can support radial forces, transmit motion and loads with low moment levels and thus keep bending stresses away from the construction elements. They are particularly suitable for alternating loads with impact and shock type stresses and support axial loads in both directions. The bearings are available in open and sealed designs.

Angular contact spherical plain bearings conform to DIN ISO 12240-2. The sliding contact surface is steel/steel. For a further description, see section Maintenance-free spherical plain bearings, Page 640, and Schaeffler catalogue HG 1, Plain Bearings.

Axial spherical plain bearings conform to ISO 12240-3. The sliding contact surface is steel/steel. For a further description, see section Maintenance-free spherical plain bearings, Page 640, and Schaeffler catalogue HG 1, Plain Bearings.

Rod ends

Rod ends are spherical plain bearing units. They comprise a housing and integral shank, into which a spherical plain bearing is integrated, and have an external or internal thread. They are used as connecting levers and connecting rods and as connecting elements between cylinders and their adjacent parts in hydraulic and pneumatic cylinders.

Spherical plain bearing range from Schaeffler

The complete range is described in the Schaeffler catalogue HG 1, Plain Bearings, and in the online version ***medias*** *professional*: *http://medias.schaeffler.com*.

Guiding elements in a translational direction – linear rolling element guidance systems

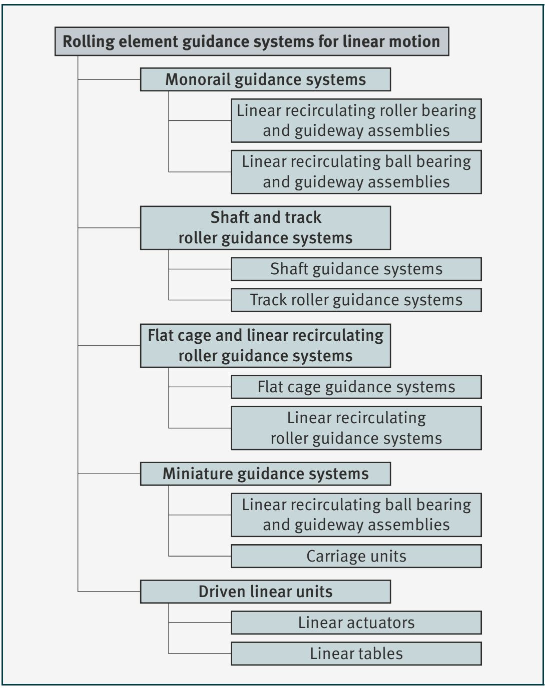
Linear rolling element guidance systems, also known as linear guidance systems, are translational guidance systems. They are based on the principle of the rolling of rolling elements (balls, rollers, needle rollers) between moving guidance elements.

Such guidance systems ensure the translation with particularly low friction of one or more movable subassemblies while maintaining a direction of motion on a linear track (profiled guideway, guideway, cylindrical shaft).

Linear guidance systems are responsible for the guidance and transmission of force between machine parts moving in a translational direction and exert a substantial influence on the performance capability and accuracy of a machine.

An overview of common linear rolling element guidance systems is shown in Figure 136:

Figure 136
Rolling element guidance systems for linear motion – overview



Differentiation of linear rolling element guidance systems

Linear rolling element guidance systems are available in numerous designs, for example:

- flat cage guidance system
- linear roller bearing
- monorail guidance system (linear recirculating ball bearing unit, linear recirculating roller or ball guidance system)
- shaft guidance system
- track roller guidance system
- miniature guidance system
- driven linear unit.

Guidance systems with balls or rollers contain balls or rollers running between the moving component and stationary component, while track roller guidance systems contain profiled track rollers supported by rolling bearings.

Linear rolling element guidance systems can be differentiated in terms of:

- the type of rolling element motion (with/without recirculation of the rolling elements)
- the type of rolling contact on the raceways (point contact or line contact).

Linear rolling element guidance systems without rolling element recirculation

In the case of linear rolling element guidance systems without rolling element recirculation, the rolling elements are guided between the moving table and the stationary guideway in a normally rigid cage. The stroke length of the moving component is therefore restricted by the difference in length between the table and the rolling element cage (based on the motion of the cage relative to the two raceways).

Flat cage guidance systems

Flat cage guidance systems are linear bearings without rolling element recirculation. The rolling elements move at half the velocity of the table and thus cover only half the distance.

Areas of application

Due to their design, flat cage guidance systems are particularly suitable for oscillating motion and where linear locating or non-locating bearings with extremely high load carrying capacity with restricted stroke length and very smooth running are required. Bearing arrangements with these guidance elements have high rigidity, high accuracy, low friction and a smaller design envelope relative to comparable linear guidance systems.

Guideway/cage combinations

Flat cage guidance systems comprise a pair of guideways between which angled needle roller flat cages, angled cylindrical roller flat cages or needle roller flat cages are arranged. The guideways are available in various profiles, examples of which are shown in M/V and J/S combinations in Figure 137.

Guideways with an adjusting gib are suitable for setting of preload, and guideways with an integral toothed rack for positive control of the angled flat cage are suitable where there is a risk of cage creep.

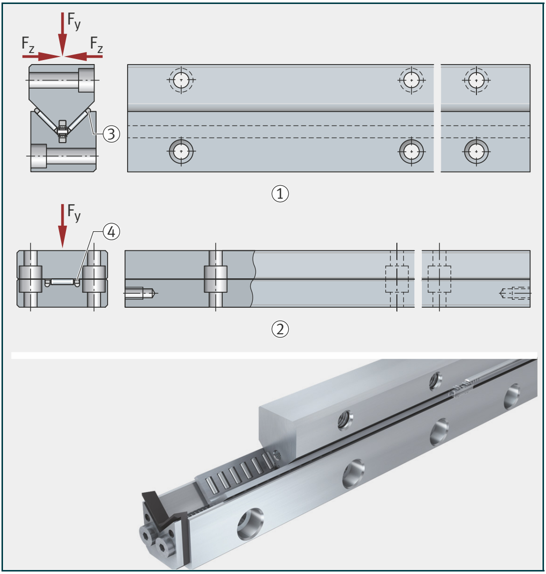
End pieces at the ends of the guideways hold the cage in its nominal position at the ends of the stroke length to prevent the cage from creeping out of the load zone.

The cages have a basic component made from light metal, steel, brass or plastic. A large number of rolling elements are guided by precise cage pockets.

Figure 137
Flat cage guidance systems

F_y, F_z = load directions

① M/V guideways
② J/S guideways
③ Angled needle roller flat cage
④ Needle roller flat cage



Linear rolling element guidance systems with rolling element recirculation

In the case of guidance systems with rolling element recirculation, the rolling elements in the carriage are recirculated by means of channels and special return elements, see Figure 138. This does not restrict the stroke length of these guidance systems, which is generally limited by the length of the guideways.

Areas of application

Such guidance systems are designed on the basis of balls or rollers and intended for applications with unlimited stroke length. This functional principle applies to linear roller bearings and monorail guidance systems.

Linear roller bearings with guideways

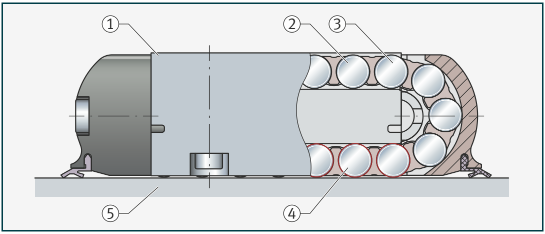
These linear guidance systems with very high load carrying capacity for linear locating/non-locating bearing arrangements comprise linear roller bearings with cylindrical rollers and guideways that have up to four raceways for the linear roller bearings, see Figure 138. Guideways with four raceways can support forces in the main load direction, together with forces in the opposing direction if a counterstay is fitted, as well as lateral forces in two directions.

In a closed arrangement, they can support forces from all directions and moments about the axis. They run with high precision, have low friction and allow compact designs.

In a preloaded design, guidance systems with linear roller bearings achieve extremely high rigidity values. Adjusting gibs are suitable for preloading. These transmit the defined values uniformly to the whole length of the linear roller bearing.

Figure 138
Linear roller bearing with rolling element recirculation

① Carriage
② Rolling element
③ Roller in return zone
④ Roller in load zone
⑤ Guideway



Monorail guidance systems

Monorail guidance systems are among the most important designs of linear rolling element guidance system. As high performance components, they are almost indispensable in general machine building.

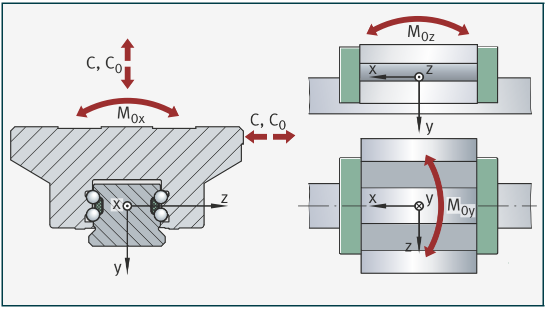
Dimensioning – load carrying capacity and life

The size of a monorail guidance system is determined by the demands made on its load carrying capacity, life and operational security.

Load carrying capacity

The load carrying capacity is described in terms of the basic dynamic load rating C, the basic static load rating C_0 and the static moment ratings M_{0x}, M_{0y} and M_{0z}, see Figure 139.

Figure 139
Load carrying capacity and load directions



Calculation of basic load ratings according to DIN

The calculation of the basic dynamic and static load ratings given in the product tables of Schaeffler catalogues is based on DIN ISO 14728-1 and 2. The information on the basic dynamic load rating C in the product tables corresponds to the basic dynamic load rating C_{100}.

Suppliers from the Far East frequently calculate basic load ratings using a basic rating life based on a distance of only 50 km in contrast to 100 km in accordance with DIN.

The basic load ratings C_{50}, C_{100} for linear recirculating ball bearing and guideway assemblies can be converted using the following equations:

Equation 75

$$C_{50} = 1{,}26 \cdot C_{100}$$

Equation 76

$$C_{100} = 0{,}79 \cdot C_{50}$$

For linear recirculating roller bearing and guideway assemblies, conversion is as follows:

Equation 77

$$C_{50} = 1{,}23 \cdot C_{100}$$

Equation 78

$$C_{100} = 0{,}81 \cdot C_{50}$$

Legend

C_{100} N Basic dynamic load rating C for distance of 100 km – definition according to DIN ISO 14728-1	C_{50} N Basic dynamic load rating C for distance of 50 km.

Dynamic load carrying capacity and life

The dynamic load carrying capacity is described in terms of the basic dynamic load rating and the basic rating life. The basic dynamic load rating is the load in N at which the guidance system achieves a distance of 100 km (C_{100}) with a survival probability of 90%.

Basic rating life

The basic rating life L and L_h is achieved or exceeded by 90% of a sufficiently large group of apparently identical bearings before the first evidence of material fatigue occurs (for an explanation of the symbols used, see Page 649).

Equation 79

$$L = \left(\frac{C_{100}}{P}\right)^p \cdot 100$$

Equation 80

$$L_h = \frac{833}{H \cdot n_{osc}} \cdot \left(\frac{C_{100}}{P}\right)^p$$

Equation 81

$$L_h = \frac{1666}{v_m} \cdot \left(\frac{C_{100}}{P}\right)^p$$

According to DIN ISO 14728-1, the equivalent dynamic load P should not exceed the value 0,5 · C.

Equivalent load and velocity

The equations for calculating the basic rating life assume that the load P and the velocity v_m are constant. Non-constant operating conditions can be taken into consideration by means of equivalent operating values. These have the same effect as the loads occurring in practice.

Equivalent dynamic load

Where the load varies in steps and the velocity varies in steps, the equivalent dynamic load is calculated as follows:

Equation 82

$$P = \sqrt[p]{\frac{q_1 \cdot v_1 \cdot F_1^p + q_2 \cdot v_2 \cdot F_2^p + \ldots + q_z \cdot v_z \cdot F_z^p}{q_1 \cdot v_1 + q_2 \cdot v_2 + \ldots + q_z \cdot v_z}}$$

Mean velocity

Where the velocity varies in steps, the mean velocity is calculated as follows:

Equation 83

$$v_m = \frac{q_1 \cdot v_1 + q_2 \cdot v_2 + \ldots + q_z \cdot v_z}{100}$$

Combined load

If the direction of the load acting on an element does not coincide with one of the main load directions, an approximate value for the equivalent load is calculated as follows:

Equation 84

$$P = |F_y| + |F_z|$$

If an element is simultaneously subjected to a load F and a moment M, an approximate value for the equivalent dynamic load is calculated as follows:

Equation 85

$$P = |F| + |M| \cdot \frac{C_0}{M_0}$$

Symbols, units and definitions

The following values are used in calculation of the equivalent load and velocity as well as the basic rating life:

Legend

C_{100} N Basic dynamic load rating C for distance of 100 km – definition according to DIN ISO 14728-1	M_0 Nm Static moment rating
C_0 N Basic static load rating in the direction of the force acting on the element	n_{osc} min^{-1} Number of return strokes per minute
F N Force acting on the element	P N Equivalent dynamic bearing load
F_y N Vertical component	p – Life exponent: Monorail guidance systems based on balls: p = 3 Monorail guidance systems based on rollers: p = 10/3
F_z N Horizontal component	
H m Single stroke length for oscillating motion	q_z % Duration as a proportion of the total operating time
L, L_h km, h Basic rating life for a distance of 100 km or in operating hours	v_z m/min Variable velocity
M Nm Acting moment	v_m m/min Mean velocity.

Operating life

The operating life is defined as the life actually achieved by monorail guidance systems. It may differ significantly from the calculated life.

The following influences can lead to premature failure through wear or fatigue:

- excess load due to misalignment as a result of temperature differences and manufacturing tolerances (elasticity of the adjacent construction)
- contamination of the guidance systems
- inadequate lubrication
- oscillating motion with very small stroke length (false brinelling)
- vibration while stationary (false brinelling)
- overloading of the guidance system (even for short periods)
- plastic deformation.

Static load carrying capacity

The static load carrying capacity of the monorail guidance system is limited by:

- the permissible load on the monorail guidance system
- the load carrying capacity of the raceway
- the permissible load on the screw connection between the upper and lower components
- the permissible load on the adjacent construction.

For design purposes, the static load safety factor S_0 required for the application must be observed, see tables starting Page 651.

Basic static load ratings and moment ratings

The basic static load ratings and moment ratings are those loads under which the raceways and rolling elements undergo permanent overall deformation equivalent to one tenth of a thousandth of the rolling element diameter.

Static load safety factor

The static load safety factor S_0 is the security against permanent deformation at the rolling contact:

Equation 86

$$S_0 = \frac{C_0}{P_0}$$

Equation 87

$$S_0 = \frac{M_0}{M}$$

Legend

S_0	–	M_0	Nm
Static load safety factor		Static moment rating in the load direction (M_{0x}, M_{0y}, M_{0z}) according to the dimension tables	
C_0	N	M	Nm
Basic static load rating in the load direction		Equivalent static moment rating in the load direction.	
P_0	N		
Equivalent static bearing load in the load direction			

The equivalent static bearing load is determined in approximate terms from the maximum loads:

Equation 88

$$P_0 = F_{max}$$

Equation 89

$$M_0 = M_{max}$$

Application-oriented static load safety factor

In machine tool applications, the static load safety factor S_0 is in accordance with the following tables. The data given in the tables are subject to the precondition that the specifications for the strength of the connection as given in Schaeffler Group catalogues are fulfilled.

Preconditions	S_0
Critical case ■ High dynamic loading with one axis stationary ■ Severe contamination ■ Actual load parameters are not defined	8 to 12
Normal case ■ Not all load parameters are completely known or: ■ Loads are estimated from the performance data of the machine	5 to 8
■ All load parameters are known	4 to 5
■ All load parameters are known (and definitely correspond to reality)	3 to 4

In machine tool applications, safety factors of $S_0 > 10$ are normally required for reasons of rigidity. For the precise design of the guidance system, Schaeffler offers BEARINX-online or design by the "Schaeffler Technology Center" in conjunction with Application Engineering facilities. In the precise design process, the displacement of the tool point can also be analysed.

In general applications, the static load safety factor S_0 is in accordance with the following table:

Preconditions	S_0
■ Predominantly oscillating load with stationary guidance system	20
■ All load parameters are completely known, running is smooth and free from vibrations	3 to 4

In general applications with a suspended arrangement (where a suspended arrangement is present, a drop guard is recommended), S_0 is in accordance with the following table:

Preconditions	S_0
■ Not all load parameters are known and a coherent weight is supported by fewer than 4 carriages	20
■ Not all load parameters are known and a coherent weight is supported by at least 4 carriages or: ■ All load parameters are known and a coherent weight is supported by fewer than 4 carriages	8 to 12
■ All load parameters are known and a coherent weight is supported by at least 4 carriages	5 to 8

Fracture strength of guidance systems

If the fixing screw threads are of a sufficient size, monorail guidance systems can be subjected to loads up to the static load carrying capacity C_0 and M_0 according to the product tables.

The load must be transmitted via locating surfaces. The basic load ratings can only be achieved when utilising the full thread lengths.

Lubrication – oil or grease lubrication

Monorail guidance systems must be lubricated. Technical, economic and ecological factors will determine whether oil or grease should be used and which lubrication method should be applied.

A significant factor in selecting the type of lubrication is the environmental conditions (such as contamination) acting on the guidance system.

Suitable lubricants

Monorail guidance systems are supplied with preservative or with initial greasing. The guidance systems run exclusively under mixed friction conditions. As a result, doped lubricants (type P to DIN 51502) should be used in preference.

Design of a monorail guidance system

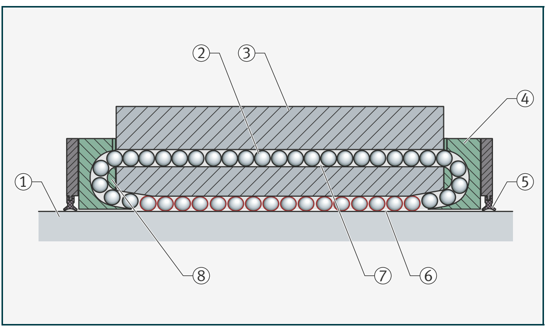
Monorail guidance systems comprise one or more carriages running on a profiled guideway. Depending on the type of rolling element, monorail guidance systems are differentiated into linear recirculating ball bearing and guideway assemblies or linear recirculating roller bearing and guideway assemblies.

The technical characteristics and area of application of the monorail guidance system are determined by the number, arrangement and contact geometry of the rolling element rows.

The carriage comprises the functional components of the saddle plate, end pieces, rolling elements, end seals and sealing strips, see Figure 140. The rolling elements in the carriage are guided by a rolling element recirculation system comprising a forward section and reverse section.

Figure 140
Monorail guidance system (linear recirculating ball guidance system)

① Guideway with raceway profile
② Rolling elements (balls)
③ Saddle plate (steel part)
④ End piece with outer return device
⑤ End seal
⑥ Load zone (raceway)
⑦ Recirculation channel
⑧ Inner return device



The saddle plates and guideways are made from hardened rolling bearing steel and have ground raceways. The rolling elements are in point or line contact with the guideway and carriage (depending on the rolling element type).

The end pieces of the carriage contain return devices that direct the rolling elements from the forward section into the reverse section. They also support the end seals.

The rolling element system is protected against contamination by sealing elements. The guidance systems are lubricated via lubrication connectors in the end pieces and/or lubrication pockets within the carriages.

Nominal contact angle

The load carrying capacity and rigidity of the monorail guidance system are influenced by the arrangement of the rolling element raceways. The raceways and contact points are therefore arranged at a specific angle, the nominal contact angle. The nominal contact angle specifies the direction of force flow relative to the horizontal plane of the guidance system and is stated for loads in the main load direction. In two-row and four-row linear recirculating ball bearing and guideway assemblies, it is normally 45°.

Arrangement of rolling element rows – X or O arrangement

Linear recirculating ball bearing and guideway assemblies can be designed with 2, 4 or 6 rows of rolling elements. Linear recirculating roller bearing and guideway assemblies predominantly have 4 rows of rolling elements.

Four-row systems are constructed in an X or O arrangement, see Figure 141, and table Monorail guidance system, Page 654. Guidance systems in an O arrangement have a higher moment rigidity about the guidance system axis than systems in an X arrangement.

In the case of linear recirculating roller bearing and guideway assemblies, the cylindrical rollers are in an X or O arrangement on the raceways.

Two-row linear recirculating ball bearing and guideway assemblies have an O arrangement, four-row systems an X or O arrangement and six-row systems an X and O arrangement.

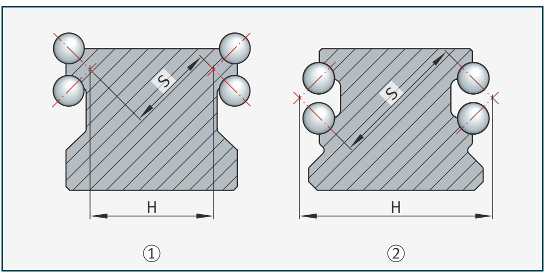
The higher the rigidity of the guidance system, the greater the effect of very small tilting as a result of inaccuracies in the adjacent construction in exerting very high internal constraining forces on the rolling elements.

Units in an X arrangement permit greater skewing about the guidance system axis. The rigidity in a compressive, tensile and lateral direction is not influenced by the X or O arrangement.

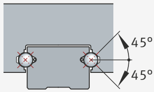
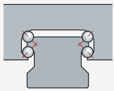
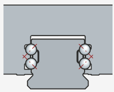
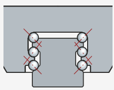
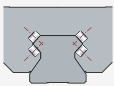
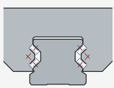
Figure 141
X or O arrangement

H = support distance
S = lever arm

① X arrangement
② O arrangement



Typical arrangements of rolling elements and contact geometries in monorail guidance systems are shown in the following table. This presents a selection of commonly available monorail guidance systems.

Monorail guidance system	Rolling contact	Number of rolling element rows	Arrangement
	Ball, 4 point	2	O
	Ball, 2 point	4	X
	Ball, 2 point	4	O
	Ball, 2 point	6	X and O
	Roller, line	4	X
	Roller, line	4	O

Running accuracy/ accuracy classes

Monorail guidance systems are divided into various accuracy classes. The specific class is defined by the different tolerances for the maximum deviations in height and lateral dimensions.

The accuracy requirements placed on the linear elements used increase with the requirements for the accuracy of an application. The accuracy class is determined by the application conditions of the guidance system.

Preload

Monorail guidance systems are predominantly preloaded. As a result, significantly greater rigidity is achieved than in clearance-free systems (increasing preload leads to an increase in static rigidity). The preload is induced by specific sorting of the rolling elements with oversize and the spring rate at the rolling contact is set.

When selecting the preload class, it must be taken into consideration that a high preload induces additional loads on the rolling element set and leads to a reduction in the basic rating life. It must therefore be critically assessed whether it is always advisable to select very high preload classes.

Rigidity of monorail guidance systems

Rigidity is an important characteristic of monorail guidance systems. The rigidity is dependent on the type and size of the guidance system.

Rigidity is defined as the ratio of load to deflection. A distinction is drawn between the compressive, tensile and lateral rigidity of a guidance system.

Factors influencing the rigidity

The rigidity is influenced by:

- the rolling element type (ball or roller)
- the arrangement of the rolling elements (number of rows, nominal contact angle)
- the osculation
- the carriage design (normal, long, low, narrow, high)
- the size of the guidance system (size 5 to 100)
- the load direction (compressive, tensile, lateral load)
- the preload class.

Friction

The frictional force F_R is influenced by the load, preload, travel velocity, design of the rolling element recirculation channels, lubricant (quantity and viscosity), temperature, misalignment and sliding motion components of the seals. It is the product of the normal force F_N and the friction coefficient μ. The friction coefficient is dependent on the system used (ball or roller guidance system) and differs in magnitude.

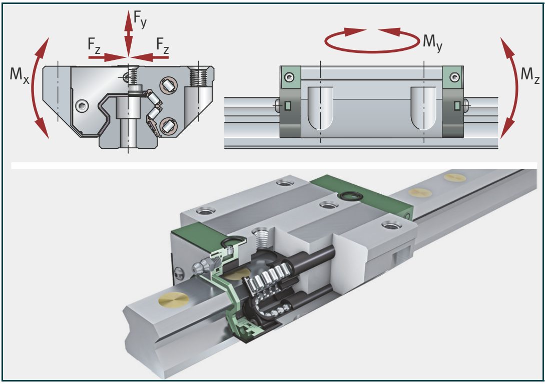
Linear recirculating roller bearing and guideway assemblies

The units have a full complement roller set, see Figure 142, Page 656. Since they have the maximum possible number of rolling elements, bearings with a full complement of rollers have extremely high load carrying capacity and particularly high rigidity.

Linear recirculating roller bearing and guideway assemblies are used wherever linear guidance systems must support extremely heavy loads, where particularly high rigidity is required and where very precise travel is also necessary. In preloaded form, they are particularly suitable for machine tools.

Figure 142
Full complement linear recirculating roller bearing and guideway assembly

F_y, F_z = load-bearing component in y and z direction
M_x, M_y, M_z = moment about x, y and z axis

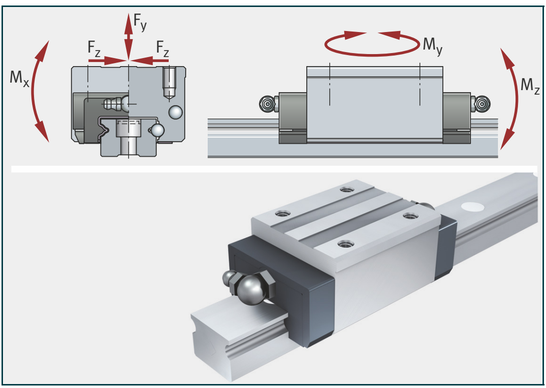


Two-row linear recirculating ball bearing and guideway assemblies

These linear recirculating ball bearing and guideway assemblies have two rows of balls and a full complement rolling element system, see Figure 143. The rolling elements are in four point contact with the raceways.

Since the load carrying capacity and rigidity are lower than that of the other linear recirculating ball bearing and guideway assemblies, they are used where there are lower requirements for load carrying capacity and rigidity of the guidance system. Two-row units can be used to achieve economical linear guidance systems in the lower and medium range of load carrying capacity.

Figure 143
Two-row linear recirculating ball bearing and guideway assembly

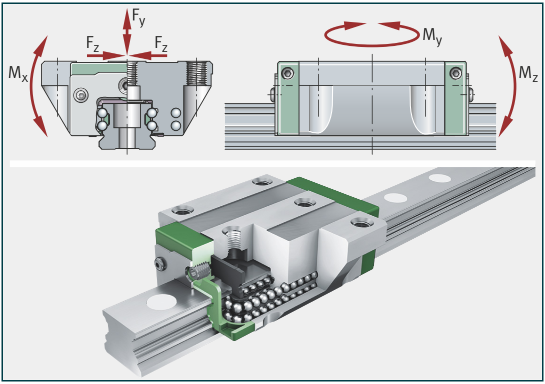


Four-row linear recirculating ball bearing and guideway assemblies

Four-row linear recirculating ball bearing and guideway assemblies with a full complement ball set represent the most extensive and complex group within the range of monorail guidance systems, see Figure 144. Since they have the maximum possible number of rolling elements, bearings with a full complement of balls have extremely high load carrying capacity and particularly high rigidity.

Linear recirculating ball bearing and guideway assemblies are used where linear guidance systems with high load carrying capacity and rigidity must move heavy loads with high running and positional accuracy as well as low friction. The guidance systems are preloaded and – depending on the application – can be used at accelerations up to 150 m/s^2 and velocities up to 360 m/min.

Figure 144
Four-row linear ball bearing and guideway assembly



Six-row linear recirculating ball bearing and guideway assemblies

The rolling elements are in two point contact with the raceways, see Figure 145, Page 658. Four outer rows of balls support compressive loads while the inner rows of balls support tensile loads and all the rows support lateral loads. The guidance systems are preloaded in order to increase their rigidity.

Due to their six rows of balls, these recirculating guidance systems are the ball monorail guidance systems with the highest load carrying capacity and rigidity.

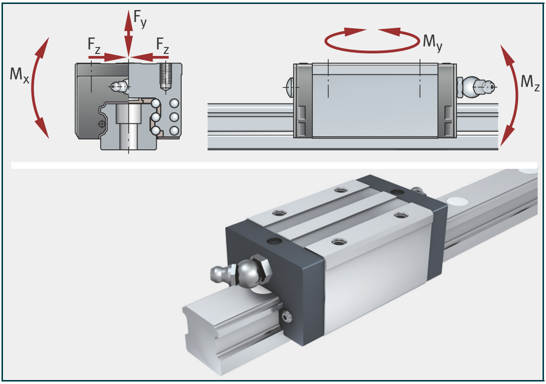


Figure 145
Six-row linear recirculating ball bearing and guideway assembly

Track roller guidance systems

Track roller guidance systems in accordance with Figure 146 are highly suitable, due to their lightweight construction, for applications in handling systems where low-noise running, high velocities and long travel distances are required together with low, uniform displacement resistance.
The guidance systems can support forces from all directions, except in the direction of motion, and moments about all axes (in the case of track roller guidance systems with non-locating bearing carriages, the load directions are restricted).

The system elements, namely carriages, composite guideways and track rollers can be combined to achieve economical designs that are precisely matched to the application.

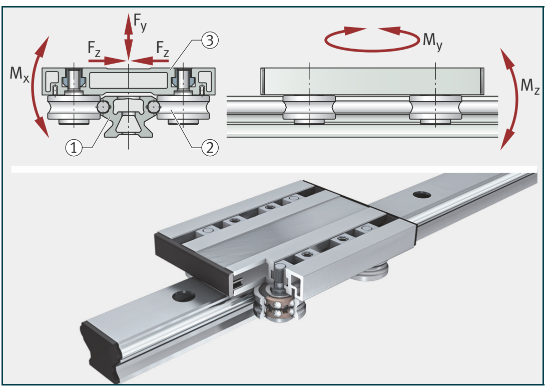


Figure 146
Track roller guidance system

F_y, F_z = load-bearing component in y and z direction
M_x, M_y, M_z = moment about x, y and z axis

1 Guideway with hollow section profile
2 Profiled track roller
3 Hollow section carriage

The carriages are available as lightweight hollow section carriages, open carriages for high performance guidance systems of a simple design, closed carriages for guidance systems in contaminated environments, non-locating bearing carriages for locating and non-locating bearing applications with two guideways in parallel and as bogie carriages for curved tracks or guidance systems in the form of closed oval or circular tracks.

Composite guideways are available as solid and hollow section guideways, with a support rail of high bending rigidity, as a half guideway, a curved guideway element or a flat type. Guideways are also available with slots for toothed racks or toothed belts.

Profiled track rollers are used for the guidance of carriages and support of forces. These double row angular contact ball bearings have an outer ring with a gothic arch profile raceway, are sealed on both sides and are greased for life. They can support axial loads from both sides and high radial forces due to the thick-walled outer ring.

Under static loading, both the permissible radial load of the bearing and the permissible radial load of the mating track must be taken into consideration.

In order to ensure that the outer ring is driven, that no slippage occurs and that the track roller does not lift from the mating track, the track rollers must be subjected to a minimum load in dynamic operation. Values are indicated in the Schaeffler product catalogues.

Dimensioning – load carrying capacity and life

The loads present in track roller guidance systems differ from those in supported, rotary rolling bearings. When calculating their load carrying capacity, additional parameters must therefore be taken into consideration.

Permissible radial loads

The thick-walled outer rings of the track rollers can support high radial loads. If these track rollers are used against a shaft as a raceway, the outer rings undergo elastic deformation.

Compared to rolling bearings supported in a housing bore, track rollers have the following characteristics:

- modified load distribution in the bearing.
  This is taken into consideration by means of the basic load ratings C_{rw} und C_{0rw} that play a definitive role in rating life calculation
- bending stress in the outer ring.
  This is taken into consideration by means of the permissible radial loads $F_{r\ per}$ and $F_{0r\ per}$. The bending stresses must not exceed the permissible strength values of the material (due to the risk of fracture).

Permissible radial load under dynamic loading

For rotating bearings under dynamic load, the effective dynamic load rating C_{rw} is used. C_{rw} is used to calculate the basic rating life. The permissible dynamic radial load $F_{r\ per}$ must not be exceeded. If the basic static load rating C_{0rw} is lower than the basic dynamic load rating C_{rw}, then C_{0rw} applies.

Permissible radial load under static loading

For bearings under static load, when stationary or with only infrequent motion, the effective static load rating C_{0rw} is used. The value C_{0rw} is used to calculate the static load safety factor S_0. The permissible static radial load $F_{0r\ per}$ must not be exceeded. In addition to the permissible radial load of the bearing, the permissible radial load of the mating track must also be taken into consideration. The basic load ratings stated are valid only in conjunction with a shaft as a mating track that is hardened (to at least 670 HV) and ground (Ra = 0,3 μm).

Fatigue limit load

The fatigue limit load C_{urw} is defined as the load below which – under laboratory conditions – no fatigue occurs in the material.

Calculation of the rating life

The general methods for calculating the rating life are:

- the basic rating life in accordance with DIN ISO 281
- the adjusted rating life in accordance with DIN ISO 281
- the expanded calculation of the adjusted reference rating life in accordance with DIN ISO 281-4.

These methods are described in the Schaeffler catalogues and the Technical Pocket Guide STT.

Rating life of track rollers

In comparison with the Schaeffler catalogue data, the following values must be replaced:

- $C_r = C_{rw}$
- $C_{0r} = C_{0rw}$
- $C_{ur} = C_{urw}$.

The carriages LFCL, LFL..-SF, LFLL, LFKL and the bogie carriage LFDL contain four track rollers LFR.

The equivalent principle applies here. The corresponding parameters are taken into consideration in the basic load ratings C_y, C_{0y}, C_z, C_{0z} and the permissible moment ratings M_{0x}, M_{0y} and M_{0z}.

Legend

C_y N
Basic dynamic load rating in y direction

C_{0y} N
Basic static load rating in y direction

C_z N
Basic dynamic load rating in z direction

C_{0z} N
Basic static load rating in z direction

M_{0x} Nm
Static moment rating about x axis

M_{0y} Nm
Static moment rating about y axis

M_{0z} Nm
Static moment rating about z axis.

In the case of track rollers with a profiled outer ring, calculation is carried out exclusively by means of the basic rating life in accordance with DIN ISO 281.

Equations for the basic rating life

Equation 90

$$L_s = 0{,}0314 \cdot D_a \left(\frac{C_{rw}}{P_r}\right)^p$$

Equation 91

$$L_h = 26{,}18 \cdot \frac{D_a}{H \cdot n_{osc}} \left(\frac{C_{rw}}{P_r}\right)^p \qquad L_h = 52{,}36 \cdot \frac{D_a}{v_m} \left(\frac{C_{rw}}{P_r}\right)^p$$

Rating life for carriages with four track rollers

Equation 92

$$L_s = \left(\frac{C_y, C_z}{P}\right)^p$$

Equation 93

$$L_h = \frac{1666}{v_m} \cdot \left(\frac{C_y, C_z}{P}\right)^p \qquad L_h = \frac{833}{H \cdot n_{osc}} \cdot \left(\frac{C_y, C_z}{P}\right)^p$$

Legend

L_s	10^5 m	Basic rating life in 10^5 metres
L_h	h	Basic rating life in operating hours
C_{rw}, C_y, C_z	N	Effective dynamic load rating
P_r	N	Equivalent dynamic bearing load (radial load)
P	N	Equivalent dynamic load in corresponding load direction
D_a	mm	Rolling contact diameter of track roller, see product table in Schaeffler catalogue LF 1, Track Roller Guidance Systems
H	m	Single stroke length for oscillating motion
n_{osc}	min^{-1}	Number of return strokes per minute
v_m	m/min	Mean travel velocity
p	–	Ball: p = 3; Needle roller (non-locating track roller or carriage): p = 10/3.

Operating life

The operating life is defined as the life actually achieved by a track roller. It may differ significantly from the calculated life.

This may be due to wear or fatigue as a result of:

- deviations in the operating data
- insufficient or excessive operating clearance (roller, guideway)
- contamination, inadequate lubrication, operating temperature too high or too low
- overloading of the guidance system
- vibration stress – false brinelling; oscillating motion with very small stroke length, which can lead to false brinelling
- very high shock loads
- prior damage during installation.

Due to the variety of mounting and operating conditions, the operating life cannot be precisely calculated in advance. The most reliable way of arriving at a close estimate is by comparison with similar applications.

Static load safety factor

The parameter for the static load is the static load safety factor S_0. This indicates the security against impermissible permanent deformations in the bearing and is determined by means of the following equation:

Equation 94

$$S_0 = \frac{C_{0rw}}{F_{0r}}$$

Static load safety factor for carriages with four track rollers

Equation 95

$$S_0 = \frac{C_{0r}}{F_0} \qquad S_0 = \frac{M_0}{M}$$

Legend

S_0 N
Static load safety factor

C_{0rw} N
Effective radial static load rating of track roller, in accordance with product table in Schaeffler catalogue LF 1, Track Roller Guidance Systems

F_{0r} N
Static force acting in a radial direction

C_{0r} N
Basic static load rating in accordance with product table in Schaeffler catalogue LF 1, Track Roller Guidance Systems

F_0 N
Components of static force in y and z direction

M_0 Nm
Permissible static moment in x, y, z direction

M Nm
Moment acting in load direction (M_x, M_y, M_z).

At a static load safety factor of $S_0 < 4$, track rollers are regarded as highly loaded. For applications with normal operating conditions, a value $S_0 > 4$ is required.

When using individual track rollers, for example in conjunction with guideways, the permissible load of the guideway should be taken as decisive where necessary.

Static load safety factors $S_0 < 1$ cause plastic deformation of the rolling elements and the raceway, which can impair smooth running. This is only permissible for bearings with small rotary motions or in secondary applications.

Minimum load

In order to ensure that the outer ring is driven, that no slippage occurs and that the track roller does not lift from the mating track, the track rollers must be subjected to a minimum load in dynamic operation.

In general, the ratio for the minimum load is $C_{0rw}/F_r < 60$.

Lower hardness of raceway

If shafts with a lower surface hardness are used (such as X46, X90), a hardness factor must be applied, see following equations and Figure 147, Page 664.

Equation 96

$$C_H = f_H \cdot C$$

Equation 97

$$C_{0H} = f_{H0} \cdot C_0$$

Legend

C N
Basic dynamic load rating

C_0 N
Basic static load rating

C_H N
Effective dynamic load rating

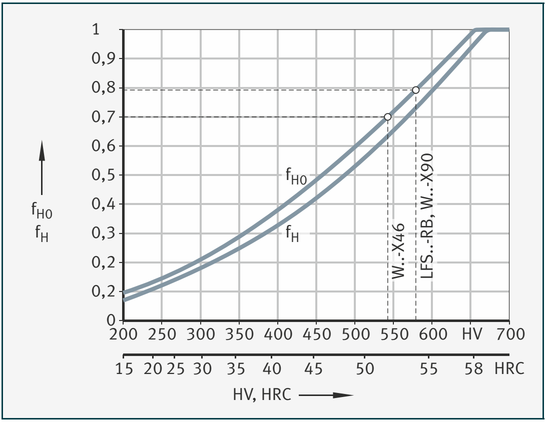
C_{0H} N
Effective static load rating

f_H –
Dynamic hardness factor (reduction factor), see Figure 147, Page 664

f_{H0} –
Static hardness factor (reduction factor), see Figure 147, Page 664.

Figure 147
Static and dynamic hardness factors for lower hardness of raceways

f_{H0} = static hardness factor
f_H = dynamic hardness factor
HV, HRC = surface hardness



Lubrication of raceways

The guideway raceways must be lubricated (even before first use). Lubrication can be carried out by means of lubrication and wiper units.

These units are already integrated in the compact carriage LFKL..-SF. For carriages LFL..-SF and LFCL, the lubrication and wiper unit AB is available as an accessory.

The guideway raceway is lubricated by an oil-soaked felt insert. Oil can be fed to the felt inserts via lubrication nipples in the end faces. At delivery, the felt inserts are already soaked with oil (H1 approval for the food industry). For relubrication, an oil of viscosity $\nu = 460 \text{ mm}^2/\text{s}$ is recommended.

Lubrication intervals

The lubrication intervals for guideway raceways are dependent on the environmental influences. The cleaner the environment, the smaller the quantity of lubricant required. The time and quantity can only be determined precisely under operating conditions since it is not possible to determine all the influences by calculation. An observation period of adequate length must be allowed.

Fretting corrosion is a consequence of inadequate lubrication and is visible as a reddish discolouration of the mating track or outer ring. Inadequate lubrication can lead to permanent system damage and therefore to failure of the linear unit. It must be ensured that the lubrication intervals are reduced accordingly in order to prevent fretting corrosion.

In general, a thin film of oil should always be present on the guideway.

Lubrication of track rollers

Track rollers LFR have an initial greasing of a high quality lithium soap grease. Track rollers of smaller diameters are lubricated for life.

Design of bearing arrangements

The accuracy of the guidance system as achieved by the manufacturer can only be properly utilised if the adjacent construction fulfils certain requirements.

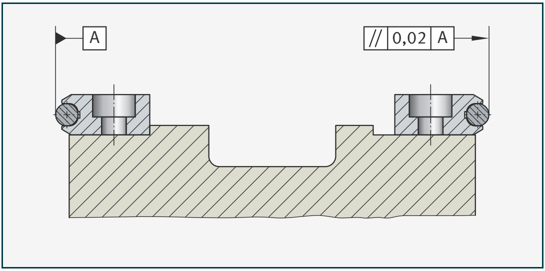
Requirements for the adjacent construction

The running accuracy of the linear guidance system is essentially dependent on the straightness, accuracy and rigidity of the mounting surfaces.

The higher the requirements for accuracy and smooth running of a track roller guidance system, the more attention must be paid to the geometrical and positional accuracy of the adjacent construction. The adjacent surfaces should be flat and have parallel faces.

If two guideways are present, parallelism in accordance with Figure 148 is recommended.

Figure 148
Parallelism of guideways



Displacement force

The displacement force is dependent on the preload, the lubrication and the particular application.

Location of carriages and guideways

If lateral loads are present, it is recommended that the guideways and carriages should be located against locating surfaces. In the case of guideways comprising multiple sections joined together, it is recommended that the guideways should be aligned by means of the shaft. If necessary, the shafts should be located on the adjacent construction by means of dowels.

If two guideways are arranged in parallel, the first guideway should be clamped against a stop, see Figure 148. The second guideway should then be aligned accordingly. Any gaps between the guideway and the adjacent construction should be filled with resin.

Shaft guidance systems, linear ball bearings

Shaft and round guidance systems with linear ball bearings are among the oldest guidance systems based on rolling elements. Such guidance systems comprise a hardened and ground shaft and one or more low-friction linear bearings, see Figure 149.

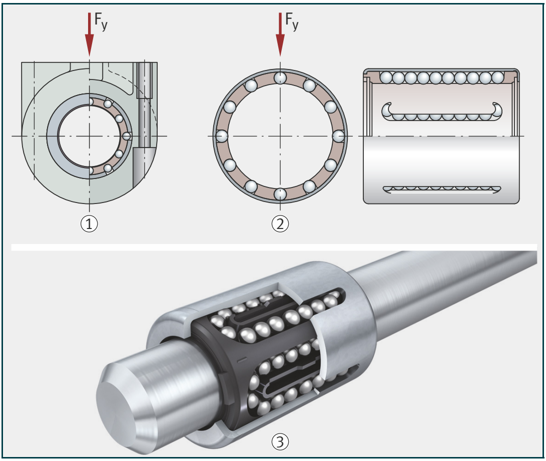
In shaft guidance systems, linear ball bearings are used for the support and transmission of forces. These bearings can support high radial loads while having a relatively low mass and allow the construction of linear guidance systems with unlimited travel.

The shaft is generally mounted on a support rail. The shafts can be either solid or hollow shafts, while the support rails are solid. Bearings and units are available in various series (light, heavy duty, compact etc.).

Figure 149
Shaft guidance system, closed housing

F_y = load-bearing component in y direction

① Linear ball bearing in closed housing
② Closed linear ball bearing
③ Shaft guidance system with linear ball bearing and solid shaft



Linear ball bearings

The structure of linear ball bearings is shown in Figure 150. While rotary ball bearings perform rotary motion, linear ball bearings run back and forth on the shaft as linear motion elements. The unlimited stroke length and return of the balls from the loaded zone to the unloaded zone is facilitated by the cage.

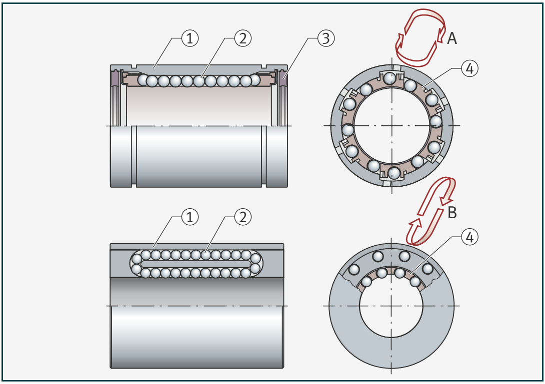


Figure 150
Linear ball bearings with tangential and radial return

A = tangential return
B = radial return

1 Steel sleeve or load plates
2 Balls made from rolling bearing steel
3 Sealing rings
4 Plastic or steel cage

Linear ball bearings without compensation of shaft deflection and misalignments

Linear ball bearings of series 1 in accordance with ISO 10285 are also designated, due to the very small radial section height, as sleeve type linear motion ball bearings, see Figure 151 1, Page 668. They comprise a formed and hardened outer sleeve with an integrated plastic cage. The balls undergo return travel along the openings in the outer sleeve.

Linear ball bearings of series 3 in accordance with ISO 10285 have larger radial section heights than series 1. The best known is the machined series, the so-called linear ball bearing with a solid housing, see Figure 151 2, Page 668.

Linear ball bearings comprise solid, ground and hardened outer rings with a plastic cage in which the rolling elements circulate.

Sleeve type linear ball bearings and machined linear ball bearings of these designs cannot compensate for deflections and misalignments of the shaft or misalignments of the bearings. This must be taken into consideration in the design of bearing arrangements.

Linear ball bearings with compensation of shaft deflection and misalignments

Linear ball bearings of ISO series 3 (linear ball bearings with single row adjustable load plate) have several load plate segments distributed around the circumference arranged such that they can swivel in an axial direction, see Figure 151 ③. This facilitates self-alignment of the bearing by up to ±30 angular minutes. Each load plate has a ball raceway that is deflected in each case by a return channel made from plastic in the stationary part of the housing.

The segments are supported centrally on a retaining ring. Their common contact point is also the centre point of the rocking motion.

Linear ball bearings of this design are intended for moderate loads.

The linear ball bearing design in ISO series 3 with the highest load carrying capacity allows self-alignment by up to ±40 angular minutes by means of several segments arranged around the circumference that themselves constitute independent linear bearings, see Figure 152, Page 669. Each segment has its own housing sections, return areas and recirculation channels. In contrast to the design described above for moderate loads, the segments in this case have two rows of balls, see Figure 151 ④.

In contrast to conventional linear ball bearings, the self-alignment function has the advantage that, even if misalignments are present, constraining forces are prevented from acting on the bearing and thus reducing the rating life. Due to the self-alignment function, the balls run without difficulty into the load zone. At the same time, load is more uniformly distributed over the entire ball row. This leads to smoother running, allows higher accelerations and prevents overloading of the individual balls.

Figure 151
Cross-sections of the type series in accordance with ISO 10285

① Series 1, sleeve type linear ball bearing
② Series 3, machined linear ball bearing
③ Series 3, linear ball bearing for moderate load and self-alignment
④ Series 3, linear ball bearing for very high load carrying capacity and self-alignment

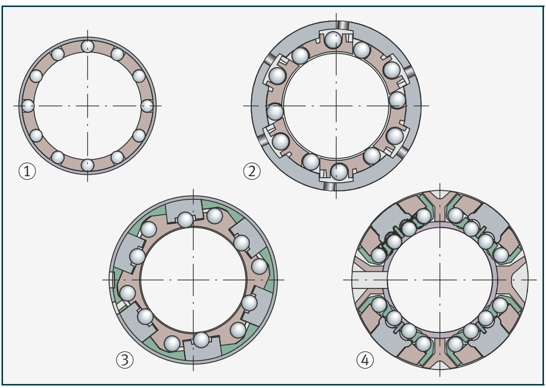
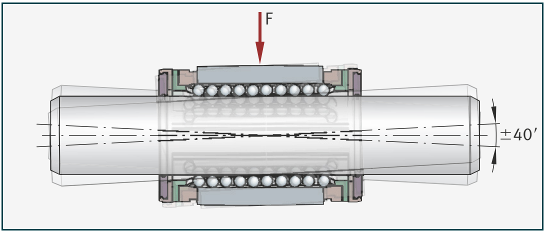


Figure 152
Angular compensation (self-alignment) in linear ball bearing, ISO series 3

Angular compensation
±40 angular minutes
F = load



Open and closed design of linear ball bearings
Linear ball bearings of all type series are available in open and closed designs. While the closed variant completely encloses the circumference of the shaft, the open design covers only a part thereof. In the case of the open design, a recess (segment cutout) allows underpinning or support of the shaft in this area. With the aid of this support, it is possible to prevent – particularly at higher operating loads – sagging of the shaft. Open bearings are designed for applications incorporating a supported shaft.

In addition to the designs for supported shafts, complete housing units are also available for open and closed bearings. In this case, the bearing is integrated in a strong, rigid housing. The housings are available in closed, open, slotted and tandem versions. Due to their low total mass, the units are particularly suitable for reduced mass designs with high loads and where higher accelerations and travel velocities are required.

Dimensioning – load carrying capacity and life

The size of a linear ball bearing is determined by the demands made in terms of load carrying capacity, rating life and operational security.

The load carrying capacity is described in terms of:
- the basic dynamic load rating C
- the basic static load rating C_0.

The calculation of the basic dynamic and static load ratings given in the dimension tables is based on DIN 636-1.

Basic rating life
The basic rating life L is reached or exceeded by 90% of a sufficiently large group of apparently identical bearings before the first evidence of material fatigue occurs.

Equation 98

$$L = \left(\frac{C}{P}\right)^3$$

Equation 99

$$L_h = \frac{833}{H \cdot n_{osc}} \cdot \left(\frac{C}{P}\right)^3$$

Equation 100

$$L_h = \frac{1666}{v_m} \cdot \left(\frac{C}{P}\right)^3$$

Legend

L 10^5 m
Basic rating life L in 10^5 m

L_h h
Basic rating life in operating hours

C N
Basic dynamic load rating

P N
Equivalent dynamic bearing load

H m
Single stroke length

n_{osc} min^{-1}
Number of return strokes per minute

v_m m/min
Mean travel velocity.

Operating life

The operating life is defined as the life actually achieved by a shaft guidance system. It may differ significantly from the calculated life.

The following influences can lead to premature failure through wear or fatigue:

- misalignment between the shafts or guidance elements
- contamination
- inadequate lubrication
- oscillating motion with very small stroke length (false brinelling)
- vibration during stoppage (false brinelling).

Due to the wide variety of mounting and operating conditions, it is not possible to precisely predetermine the operating life of a shaft guidance system. The safest way to arrive at an appropriate estimate of the operating life is comparison with similar applications.

Static load safety factor

The static load safety factor S_0 indicates the security against impermissible permanent deformations in the bearing and is determined in accordance with the following equation:

Equation 101

$$S_0 = \frac{C_0}{P_0}$$

Legend

S_0 –
Static load safety factor

P_0 N
Equivalent static load

C_0 N
Basic static load rating.

For linear ball bearings KH and KN..-B, the value must be $S_0 \geqq 4$.

In relation to guidance accuracy and smooth running, the value $S_0 \geqq 2$ is regarded as permissible. If $S_0 < 2$, please contact us.

Influence of the shaft raceway on basic load ratings

The basic load ratings are only valid if a ground (Ra = 0,3 μm) and hardened shaft (at least 670 HV) is provided as a raceway.

Lower hardness of raceway

If shafts with a surface hardness lower than 670 HV are used (such as shafts made from X46 or X90), a hardness factor (reduction factor) must be applied, see Figure 147, Page 664:

Equation 102

$$C_H = f_H \cdot C$$

Equation 103

$$C_{0H} = f_{H0} \cdot C_0$$

Legend

C N
Basic dynamic load rating

C_0 N
Basic static load rating

C_H N
Effective dynamic load rating

C_{0H} N
Effective static load rating

f_H –
Dynamic hardness factor (reduction factor), see Figure 147, Page 664

f_{H0} –
Static hardness factor (reduction factor), see Figure 147, Page 664.

Load direction and orientation of the ball rows

The effective load rating of a linear ball bearing is dependent on the position of the load direction in relation to the orientation of the ball rows:

- The lowest basic load rating C_{min} and $C_{0\,min}$ will occur in the apex orientation, see Figure 153
- The highest basic load rating C_{max} and $C_{0\,max}$ will occur in the symmetrical orientation, see Figure 153.

If the bearings are mounted in correct alignment, the maximum basic load rating can be used. If mounting in correct alignment is not possible or the load direction is not defined, the minimum basic load ratings must be assumed.

Main load direction

For linear ball bearings and linear ball bearing and housing units, where the fitting position of the ball rows is defined, the basic load ratings C and C_0 in the main load direction are stated, see Figure 154. For other load directions, the effective load ratings can be determined using the load direction factors in the Schaeffler catalogue WF 1, Shaft Guidance Systems.

If the mounting position of the ball rows is not defined, the minimum basic load ratings are stated.

Figure 153
Load carrying capacity as a function of the orientation of the ball rows

C_{min}, $C_{0\,min}$ = lowest basic dynamic or static load rating
C_{max}, $C_{0\,max}$ = highest basic dynamic or static load rating

1 Apex orientation
2 Symmetrical orientation

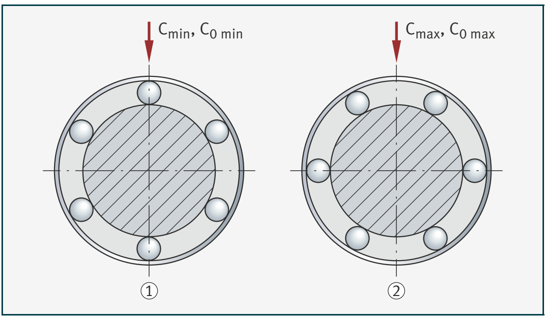
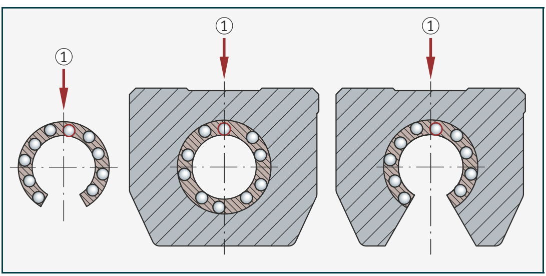


Figure 154
Main load direction for bearings and units

1 Main load direction



Lubrication

Open linear ball bearings are supplied with a wet or dry preservative and can be lubricated using either grease or oil. The oil-based preservative is compatible and miscible with lubricants with a mineral oil base, which means that it is not generally necessary to wash out the bearings before mounting. Bearings with a dry preservative must be greased or oiled immediately after they are removed from the packaging (due to the risk of corrosion).

Grease lubrication

Grease lubrication should be used in preference to oil lubrication, since the grease adheres to the inside of the bearing and thus prevents the ingress of contamination. This sealing effect protects the rolling elements against corrosion.

In addition, the design work involved in providing grease lubrication is less than that for providing oil, since design of the sealing arrangement is less demanding.

Composition of suitable greases

The greases for linear ball bearings have the following composition:

- lithium or lithium complex soap
- base oil: mineral oil or poly-alpha-olefin (PAO)
- special anti-wear additives for loads $C/P < 8$, indicated by "P" in the DIN designation KP2K–30
- consistency to NLGI grade 2 in accordance with DIN 51818.

Initial greasing and operating life

Based on experience, the operating life is achieved with the initial greasing when bearings are operated in normal ambient conditions ($C/P > 10$), at room temperature and with $v \leqq 0,6 \cdot v_{max}$. If it is not possible to achieve these conditions, relubrication must be carried out.

Sealed linear ball bearings are already adequately greased when delivered and are therefore maintenance-free in many applications.

Influence of the adjacent construction on the running accuracy of the guidance system

The good running characteristics of shaft guidance systems are dependent not only on the bearings. They are also influenced to a large extent by the geometrical and positional tolerances of the adjacent construction as well as the mounting of the guidance systems. The higher the accuracy to which the adjacent construction is produced and assembled, the better the running characteristics.

Applications

Examples of the design of bearing arrangements The design and layout of bearing arrangements for widely varying areas of engineering requires fundamental and extensive knowledge and experience of the application of the corresponding bearings. This section describes applications as examples to show designers and students how bearing arrangement tasks can be realised.

Bearing selection The issues to be clarified in preparing a design solution for a bearing arrangement are derived from the type of machine and its working conditions. The designers must therefore clarify first the tasks for the bearings as a result of the operational function of the machine, plant etc. and the operating conditions. They must therefore know the direction and magnitude of the forces to be supported. This will then give reference points to indicate the type of bearing.

Dimensioning The correct bearing size is generally defined by the calculation of the fatigue life L_{10h}. Initially, the focus here is on bearing arrangements that have already proved successful in practice.

Internal clearance and running accuracy The design of the bearing in relation to radial or axial internal clearance and the running accuracy is dependent not only on the operating conditions but also on how closely the bearing arrangement should guide the rotating part.

Design of the adjacent construction Once the bearing type, bearing size and bearing design have been defined, the next task is the design of the bearing position. This includes the design of the adjacent parts and the selection of the fits.

Lubrication, sealing The operating conditions determine the type of lubrication and sealing. Since a high percentage of bearing failures can be attributed to defects in lubrication and sealing, these components must be selected with particular care.

Mounting and dismounting When considering all the aspects described above, attention must also be paid to the correct mounting and dismounting of the bearing.

Special requirements There are cases in which certain requirements acquire particular importance and therefore move to being the starting point for the considerations. In this way, a decisive influence from the very beginning may be exerted by high speeds or temperatures, unusual load conditions or accuracy requirements etc.

Electronic selection system As a means of support to designers, Schaeffler offers a bearing selection system in the form of the software: ***medias*** *professional*: *http://medias.schaeffler.com*.

Examples describing typical bearing arrangement issues

The following examples are taken from various areas of application. The selection was made in such a way that typical issues relating to bearing arrangements can be discussed. The examples are not provided for the purpose of giving the designer specific advice for his own area of work beyond these issues; they are intended rather to convey knowledge of bearing technology in numerous areas, in order that designers can draw on these to develop suggestions for their own work.

Equations for calculation

The equations for calculating the rating life of rotational rolling bearings are given in the section Dimensioning – load carrying capacity and life, starting Page 529.

The rating life equations for translational rolling bearings are given in the section Guiding elements in a translational direction – linear rolling element guidance systems, starting Page 642.

Attention

The applications provide example solutions only. Their content is exclusively for the purposes of information and cannot replace technical advice on the application of bearings in individual cases. In practice, the specifications of the actual application and the information provided by bearing manufacturers must always be observed.

The publisher assumes no liability for compliance between the content and any legal regulations.

Bearing arrangement for the rotor shaft in a threephase motor

Electric motors transform electrical energy into mechanical energy. They convert the force that is exerted by a magnetic field on the wire in a coil carrying current into rotary motion.

The quality of the motor is assessed to a significant extent on the basis of its quiet running in operation. For electrical machinery, the noise limit values are defined in VDE 0530 and the maximum permissible mechanical vibrations are specified in DIN ISO 2373.

Noise and vibration are influenced principally by the rolling bearings fitted. In addition to the geometrical accuracy of the rolling bearing raceways once mounted, the radial internal clearance of the bearings has a decisive effect on the running noise. The best results are achieved with bearing arrangements that are almost clearance-free when warm from operation. Due to the tolerances of the joined parts, however, this is only possible with considerable effort. Such bearing arrangements are therefore frequently fitted with an axial spring element to achieve clearance-free adjustment. This is situated between the housing cover and the outer ring of the non-locating bearing.

Application data

The technical data of the motor are as follows:

- drive power of the belt drive 3 kW
- mass of the rotor $m_L = 8$ kg
- nominal speed $n = 2\,800$ min^{-1}
- size 100L
- surface cooling in accordance with 42673, page 1, type B3, protection class IP44, insulation class F
- requisite rating life 20 000 h.

Bearing selection

The bearing arrangement should be simple, economical, maintenance-free and quiet. These requirements are best fulfilled by deep groove ball bearings. In DIN 42673, size 100L is defined as having a shaft end diameter of 28 mm. A bore diameter of 30 mm is thus specified.

For the bearing arrangement, a bearing of series 62 is suitable for both bearing positions (bearing position A and B). These bearings guide the rotor shaft on the output side and ventilation side. The spring element on the ventilation side (bearing position B) gives clearance-free adjustment of the bearing arrangement and also provides the axial counter-guidance of the rotor shaft. The clearance-free adjustment of the bearings prevents the internal clearance from having a deleterious effect on the noise behaviour.

Dimensioning

Further calculation of the bearing arrangement is carried out in a slightly different way from normal practice. Since the manufacturer of the motors does not know the magnitude of the load on the shaft end, he indicates the permissible radial load in his catalogues. In order to determine the radial load carrying capacity, the deep groove ball bearing on the output side is considered. The calculation is based on a requisite rating life L_{10h} of 20 000 hours. The rotor mass, the unilateral magnetic attraction and the imbalance must also be taken into consideration.

Since the latter two criteria are not known, the rotor mass is multiplied by a safety factor of 1,5. As a result, a permissible radial load of 1 kN is calculated for the centre of the shaft end.

Since the operating load is less than the permissible load in most cases, this gives an achievable rating life of more than 20 000 hours: $L_{10h} = (16\,666/n) \cdot (C/P)^3$. The operating life is thus normally determined by the grease operating life and not by the material fatigue.

Machining tolerances The inner rings are subjected to circumferential load. They therefore have a tight fit on the shaft seats machined to k5. The outer rings are fitted in end cap bores machined to H6. The outer ring on the non-locating bearing side (bearing position B) can therefore be axially displaced in the bore and compensate thermal expansion under spring preload.

Internal clearance Since the inner ring has a tight fit on the shaft, the radial clearance is reduced. Due to heating of the rotor, the inner ring is at a higher temperature during operation than the outer ring. This also gives a reduction in the radial internal clearance. In order to prevent distortion of the bearings, the radial internal clearance C3 is selected.

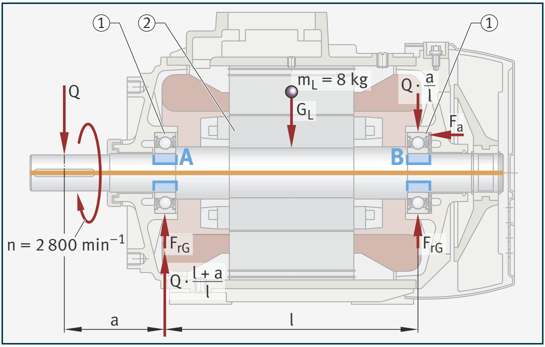
Lubrication The grease filling as supplied is sufficient for the entire operating life of the bearings. Since increased temperatures must be taken into consideration as a result of the insulation class F, a high temperature grease is used.

Sealing In small to medium-sized electric motors, bearings with sealing shields (non-contact seals) on both sides have proved effective. The sealing shields prevent the escape of grease and also protect the rolling element system from foreign bodies in the motor body. In order to prevent the ingress of dust and moisture, the shaft opening on the drive side is designed as an extended gap and covered by a protective cap. This fulfils the requirements of protection class IP44.

Figure 155
Load diagram and bearing arrangement

A, B = bearing positions

① Deep groove ball bearing 6206-2Z
② Rotor



Bearing arrangement for the main spindle in a CNC lathe

The heart of a machine tool is the main or work spindle and its bearing arrangement. The quality of the main spindle bearing arrangement is measured in terms of the cutting volume and machining precision.

The only bearings used as main spindle bearings are rolling bearings with increased accuracy; these are mainly angular contact ball bearings or spindle bearings (radial angular contact ball bearings with contact angles of 15° and 25°), double direction axial angular contact ball bearings, radial and axial cylindrical roller bearings and occasionally tapered roller bearings.

Application data

The technical data of the application are as follows:

- drive power P = 25 kW
- maximum spindle speed n = 5 000 min^{-1}.

Bearing selection

Depending on the requisite performance data of the machine tool, the spindle bearing arrangement is designed in accordance with the criteria of rigidity, friction behaviour, accuracy, speed capacity, lubrication and sealing.

The bearings must give precise radial and axial guidance of the spindle and must exhibit high rigidity. This is achieved by using the largest possible shaft diameter and a corresponding bearing arrangement, see Figure 156, Page 679. The bearings are additionally preloaded to increase the rigidity and have increased accuracy.

Dimensioning

The bearing is determined principally by the requisite rigidity of the spindle. While the fatigue life is included as a factor in dimensioning, it plays only a subordinate role in practice.

The decisive factors for the operating duration of the bearings are the Hertzian pressure p_0 and the grease operating life t_{fG}. For deep groove ball bearings with rolling elements made from the rolling bearing steel 100Cr6, the fatigue strength for example at p_0 is $\leqq$ 2 000 N/mm^2.

Main spindle bearings generally fail not as a result of material fatigue but of wear. The objective of the design is therefore to give rolling bearings that are fatigue-resistant at very high cleanliness and with a hydrodynamic lubricant film capable of supporting load at the contact points of the rolling contact parts (wear-free running).

Fatigue resistance can be ensured with a ratio $S_0^* = (C_0/P_0^*) \geqq 8$. P_0^* is calculated from the dynamic load forces in accordance with the equation for the equivalent static load: $P_0 = F_{0r}$ at $F_{0a}/F_{0r} \leqq 1{,}3$. Since the ratio is $F_a/F_r \leqq 1{,}3$, P_0^* is assigned the value for F_r. Calculation shows that the bearings are fatigue-resistant under the stated operating conditions ($S_0^* \geqq 8$).

Design of the adjacent construction, machining tolerances

The table gives the machining tolerances of the bearing seats for the spindle bearings and cylindrical roller bearings in this application.

Bearing	Seat	Diameter tolerance	Cylindricity tolerance (DIN EN ISO 1101) µm	Total axial run-out tolerance of abutment shoulder µm
Spindle bearing	Shaft	+5/−5 µm	1,5	2,5
	Housing	−4/+8 µm	3,5	5
Cylindrical roller bearing	Shaft, tapered	Taper 1:12	1,5	2,5
	Housing	−15/+3 µm	3,5	5

Bearing arrangement, preload

The work side is fitted with a locating bearing comprising a spindle bearing set in a tandem O arrangement with slight preload. This preload fulfils the normal requirements.

A single row cylindrical roller bearing is fitted as a non-locating bearing on the drive side. Due to its conical inner ring bore, the bearing is set almost clearance-free when it is pressed axially onto the spindle.

The bearing combination and arrangement ensure the high speeds and cutting performance values required.

Lubrication

The spindle bearings and cylindrical roller bearings are greased for life with a high quality rolling bearing grease. The bearing interiors are filled with grease to approx. 35% in the case of the spindle bearings and approx. 20% in the case of the cylindrical roller bearings. Since the fatigue strength is ensured, the grease operating life t_{fG} is the decisive factor for the operating duration of the bearings.

Sealing

The bearings are protected against contamination by a labyrinth with defined narrow radial gaps.

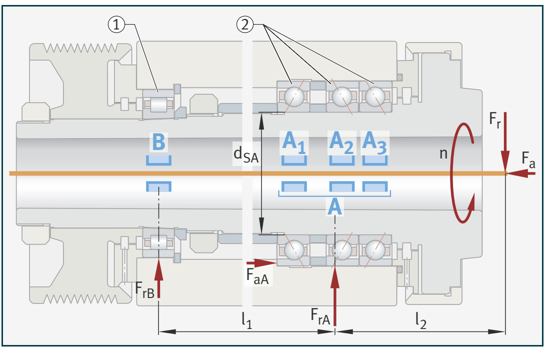
Figure 156

Load diagram and bearing arrangement

A, B = bearing positions

(1) Cylindrical roller bearing N1016-K-M1-SP

(2) Spindle bearing set in tandem O arrangement B7018E-T-P4S-TB-TL



Bearing arrangement for radial support rollers in a rotary kiln

Rotary kilns are manufactured in versions with direct and indirect heating and are used for process plant applications. Materials transport is carried out – in a longitudinal direction through the kiln with rotation of the drum – from the inlet side to the outlet side. The drum body is therefore slightly inclined in the longitudinal direction. The feed materials (such as solids, dusts, slurries) may vary widely in consistency and granularity.

Two opposing radial support rollers form a support angle φ and are incorporated in one station. Depending on the length of the drum body, support is provided using two or more stations. At each station, a support ring is arranged around the drum body that rolls on the support rollers as the drum rotates.

For axial guidance of the drum and support of the axial loads, the normal arrangement comprises two axial support rollers on one support ring.

Application data

Rotary kiln:

- total weight (drum plus charge) $G = 5\,100$ kN
- number of stations $Z = 2$
- support angle of station $\varphi = 60°$
- inclination angle of drum $\beta = 2°$
- dimensions of support ring $D \times b = 6\,600$ mm $\times$ 600 mm.

Radial support rollers:

- dimensions of radial support rollers $D \times b = 1\,400$ mm $\times$ 630 mm
- speed of radial support rollers $n = 10$ min^{-1}
- diameter of bearing seat for bearings in radial support rollers $d = 320$ mm
- requisite rating life 50 000 h to 110 000 h.

Bearing selection

For the bearing arrangement of the radial support rollers, spherical roller bearings 24164-E1 are selected, whose basic rating life (load carrying capacity) must be checked as a function of the bearing diameter. Spherical roller bearings are selected since, in this application, shaft deflections can occur and misalignments due to the adjacent construction must be anticipated. The bearings selected are highly suitable for compensation of these influences. In addition, they can support combined loads, since not only are high radial loads present (at low speed) but axial loads can also occur due to displacement of the kiln.

The spherical roller bearing series 241 has high radial and axial load carrying capacity. Shocks and vibration are effectively compensated by the double row line contact in conjunction with a suitable grease. The bearings are catalogue products and thus available rapidly worldwide.

Dimensioning

The strength specifications for the axis determine the shaft and journal diameters (and therefore the bearing sizes) for the bearings. However, the requisite rating life L_{10h} of the rolling bearings must be checked for the specified diameter d.

The bearing arrangement of the radial support rollers requires a basic rating life of 50 000 h to 110 000 h. After calculation of the radial, axial and tilting moment load F_r, F_a and F_k and the equivalent dynamic bearing load P, the rating life equation is used to check whether the spherical roller bearings defined by the strength specification for the shaft are sufficiently well dimensioned. The rating life is calculated: $L_{10h} = (16\,666/n) \cdot (C/P)^{10/3}$ and compared with the above value. With a value of 144 900 h, the bearings significantly exceed the requirement. This is due to the specified shaft diameter.

Design of the adjacent construction, machining tolerances

The bearing arrangement used is a floating bearing arrangement. The outer rings undergo point load, so the housing bores can have a loose fit (machining tolerance H7, clearance fit). Due to the spacing between the housing cover and the bearing outer ring, the outer rings of bearing A and B are not in contact with each other. The bearing rings can therefore be axially displaced by 2 mm to 3 mm and are abutted on the relevant inner shoulders of the housings. The inner rings undergo circumferential load. They therefore have a tight fit (interference fit) on the axis machined to n6 and are clamped in place using caps. The housings for the bearings are made from flake graphite cast iron.

Lubrication

The bearings in the radial support rollers are subjected to heavy load at low speeds. They are therefore lubricated using greases of high base oil viscosity and containing EP additives. Based on empirical values for such bearing arrangements in rotary kilns, the spherical roller bearings should be relubricated after no more than 2 500 h. The precise lubrication intervals must be determined by lubrication interval calculation.

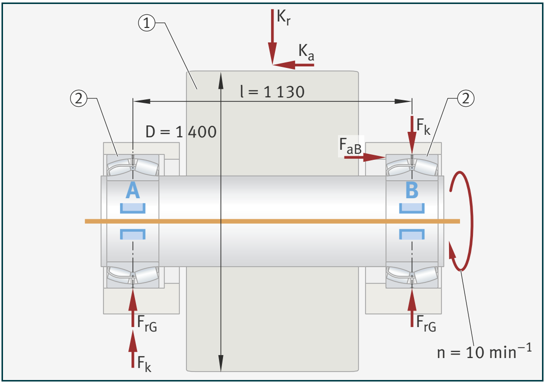
Sealing

The bearing arrangement is sealed by means of felt rings, in front of which are labyrinths with a relubrication facility.

Figure 157
Load diagram and bearing arrangement

A, B = bearing positions

1 Radial support roller
2 Spherical roller bearing 24164-E1



Bearing arrangement for paper guide rolls in web offset printing machines

In web offset printing machines, the paper webs are fed – in contrast to the sheet offset process – through a system of rotating cylinders. The blanket cylinder transfers the image to the paper. This process is stabilised by an impression cylinder that is pressed against the paper and the blanket cylinder. Modern web offset printing machines run at more than 40 000 revolutions of the cylinders per hour.

Paper guide rolls guide the paper web on its path through the machine. The bearing arrangement must have low frictional torque, since there is a risk of slippage, especially at a small wrap angle. In this case, the paper web would slide over the roll surface to a greater or lesser degree and could no longer drive the paper guide roll by friction. This can in turn impair the quality of the print results or even lead to tearing of the paper web.

In order to prevent contamination of the paper by lubricant, the bearing arrangement must be well seated and maintenance-free. A printing machine contains numerous paper guide rolls. The bearing arrangement must therefore be easy to mount and economical.

Application data

The technical data of the example are as follows:

- operating speed (nominal speed) $n = 2\,000\ \text{min}^{-1}$
- total radial load $F_r = 800$ N
- diameter of the shaft stubs $d = 40$ mm
- operating temperature $\vartheta = +120$ °C
- requisite rating life of the bearings 40 000 h.

Bearing selection

The requirement for the smallest possible frictional torque of the bearing can be fulfilled by the selection of a ball bearing with a small grease quantity and the use of a free-running grease. Since the sealing arrangement of the bearing has a considerable influence on the bearing frictional torque, a bearing with non-contact seals is selected. With this information and the application data, preliminary selection of a ball bearing can be made. As a result, the radial insert ball bearing with locking collar E40-KLL and integrated gap/labyrinth seals was selected.

Dimensioning

The axial load can be disregarded, since the paper web generates no axial load and the inherent mass of the paper guide roll acts only in a radial direction. The radial load F_r comprises the inherent mass of the roll and the maximum web tension in downward guidance of the web. It acts equally on both bearings.

The equivalent dynamic load P is determined according to the same method as for deep groove ball bearings, since radial insert ball bearings are based on the same construction as deep groove ball bearings (series 60, 62 and 63). Since no axial load is present, P corresponds to the highest radial load F_r (F_{rA}, F_{rB}) for each bearing.

The values determined can now be used to calculate the basic rating life: $L_{10h} = (16\,666/n) \cdot (C/P)^3$. The result ($>$ 40 000 h) fulfils the requirement for basic rating life. The radial insert ball bearing is thus suitable for the bearing arrangement for the paper guide rolls.

Design of the adjacent construction, machining tolerances

The selected radial insert ball bearing gives very easy mounting and simple location and places only slight requirements on the adjacent construction. Due to the circumferential load on the outer ring, this bearing ring must have a tight fit (transition fit). The bore in the cylinder is therefore produced in accordance with K7. For the shaft stubs, drawn shafts of the tolerance class h6 to h9 can be used.

On the locating bearing side, the inner ring is located on the shaft by means of the locking collar and secured by means of grub screws. The bearing mounted in this way now acts as a locating bearing. In order to support the thermal expansion of the cylinder, the opposing bearing must be mounted as a non-locating bearing. The locking collar is therefore loosened from the inner ring by the amount of the cylinder expansion and not clamped on the shaft but instead fixed in place only by means of the grub screws. In this way, the inner ring can be displaced on the shaft if there is thermal expansion of the cylinder.

Lubrication

The bearings are greased with a grease optimised for friction (free-running grease). The grease quantity is selected such that maintenance-free operation is ensured for the entire operating life of the bearings. The grease operating life is 40 000 h.

Sealing

The non-contact seal in the bearing causes no seal friction and thus supports the requirement for a low-friction bearing arrangement. It reliably prevents the ingress of contamination and foreign matter into the bearing and the escape of lubricant from the bearing.

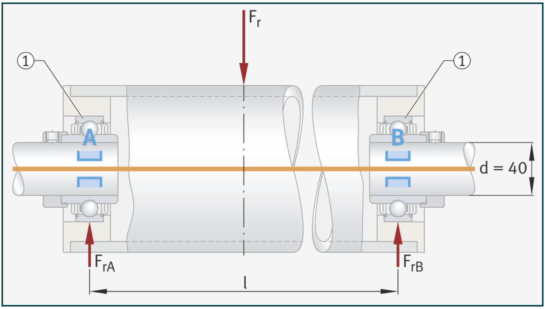


Figure 158
Load diagram and bearing arrangement

A, B = bearing positions

1 Radial insert ball bearing with locking collar E40-KLL

Bearing arrangement for connecting rods (crank pins) in piston compressors

Piston compressors operate on the displacement principle. Gases such as air or nitrogen can be used as the compression medium. The gas to be compressed is enclosed in a chamber where it is then compressed. Compression is carried out by means of pistons lubricated by oil or grease.

Piston pins supported by means of bearings connect the pistons to the connecting rods. The latter are supported on the crankshaft by means of the crank pin bearing arrangement. The connecting rods support the forces occurring and transmit these to the crankshaft in order to build up the torque. The crankshaft converts the oscillating linear motion of the piston into rotary motion with the aid of connecting rods and thus transmits the motor torque generated from the piston force to the drive.

Application data

The technical data of the crank pin bearing arrangement are as follows:

- offset of the crank pins = 180°
- piston diameter D_K = 60 mm
- nominal pressure p = 15 bar
- operating speed (nominal speed) n = 660 min^{-1}
- requisite rating life of the bearings 5 000 h to 35 000 h.

Bearing selection

Connecting rod bearing arrangements are, due to their unusual force, motion and lubrication conditions, some of the most challenging bearing positions in automotive engineering and machine building. Due to the inertia forces, the mass of the connecting rod with the bearings and the piston mass must be as small as possible. The design envelope for the bearings is thus very small. For this reason, and due to the non-uniform rotary and swivel motion, crank pin and piston pin bearing arrangements in piston compressors are realised predominantly by means of needle roller bearings, in which the rolling elements run directly on the hardened crank pin or piston pin and in the hardened connecting rod eyes. In the piston compressor described here, the crankshaft and connecting rod are not necessarily hardened. A direct bearing arrangement is therefore not possible for the crank pin bearing arrangement. For this reason, ribless needle roller bearings NAO 30×47×16 with an inner ring are selected.

The connecting rod is to be guided axially on the bearing (crank end guidance). The outer ring is somewhat narrower than the inner ring. The outer ring, needle roller and cage assembly and the connecting rod eye run between hardened axial contact washers.

Dimensioning

For the compressor, the size and rating life of the crank pin bearings A should be checked. First, the nominal pressure p is used to determine the maximum piston force F_K. The radial load F_{rA} (bearing A) corresponds to the maximum piston force F_K ($F_{rA} = F_K$). The piston force F_K can vary with time.

The load pattern varying with time can be summarised in its effect on the dynamic bearing load for bearing A in the equivalent dynamic load P. Since no axial load is present, the following applies to bearing A: $P = 0{,}55 \cdot F_{rA}$.

In the low-speed piston compressor described here, the inertia forces can be disregarded; this means that calculation of the bearing load must only take account of the gas forces due to the equivalent dynamic load P. The needle roller bearing is thus subjected to a load P = 2,33 kN.

The calculation results are used to check whether the needle roller bearing will achieve the requisite rating life. Checking is carried out by comparing the requisite rating life with the basic rating life: $L_{10h} = (16\,666/n) \cdot (C/P)^{10/3}$. The bearing size is suitable for the application, since the calculation gives 98 697 h.

Machining tolerances

The crank pin bearings are subjected to high levels of shocks and non-uniform loads. The inner and outer rings therefore have a tight fit. The bores in the connecting rod have a fit in accordance with N6, while the bearing seats on the crank pin have a fit in accordance with k5.

Internal clearance

A tight fit on the shaft and in the housing will reduce the radial internal clearance of the bearings. The needle roller bearings therefore have the internal clearance C3.

Lubrication

The open bearings are lubricated by the motor lubrication system. In order that the spray oil enters the bearings and reaches the lateral connecting rod surfaces, the axial contact washers have bowstring-shaped sections.

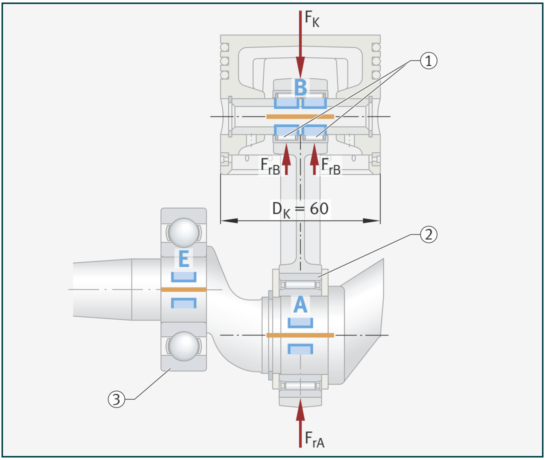
Sealing

The crank pin bearings are sealed from the outside by the compressor housing.

Figure 159
Load diagram and bearing arrangement

A, B = bearing positions

1 Drawn cup needle roller bearings with open ends
2 Needle roller bearing NAO 30×47×16
3 Deep groove ball bearing



Bearing arrangement for a wheelset in rail vehicles

Vehicles that run or are guided on one or more rails are known as rail vehicles (for example trains or trams). The rail and vehicle are closely matched to each other and are normally described as a wheel/rail system.

Application data

The technical data of the wheelset bearing arrangement are as follows:

- proportion of the vehicle mass acting on the wheelset bearing $m_A = 21\,800$ kg
- mass of the wheelset $m_R = 2\,465$ kg
- wheel diameter (rolling diameter) $D_R = 1\,250$ mm
- diameter of the shaft journal $d_{WS} = 130$ mm
- number of bearings per axle $i_R = 2$
- maximum travel velocity $v = 200$ km/h
- ambient temperature $\vartheta = -50$ °C to +50 °C
- requisite rating life of the bearings in travel kilometres 5 000 000 km.

Bearing selection

Wheelset bearings are components with safety implications; the operational security of rail vehicles depends to a large extent on them.

Wheelset bearing units comprise a housing with integrated rolling bearings. Since the bearings are subjected to high loads and high operational security is required, the only bearings used are radial roller bearings (cylindrical and tapered roller bearings). The bearings transmit the forces from the vehicle or bogie frame to the wheelset and thus to the rail.

For the 6-axle locomotive with three bogies in this example, wheelset bearing units with cylindrical roller bearings, plastic cages, sheet metal caps and preset radial internal clearance are used. Cylindrical roller bearings are radial bearings but they can also support the axial forces present by means of the ribs. The greased and sealed units are ready-to-mount and simply require pressing onto the wheelset shaft.

Dimensioning

The guidelines from the international railways association UIC envisage shaft journals from d = 120 mm to d = 130 mm. The bearing size is to be checked for wheelset bearing units with cylindrical roller bearings d = 130 mm. The axle load is taken as the basis for determining the bearing size. This is the proportion of the vehicle mass that is transmitted per wheelset to the rails. The static load on the wheelset bearings is calculated from the axle load m_A reduced by the wheelset mass m_R.

In addition to the static load, the roller bearings must also support dynamic forces occurring during travel, for example when passing over points, crossings etc. Additional, radial dynamic loads are taken into consideration by means of the safety factor f_z. This is now normally between 1,2 and 1,5. The radial load F_r on a roller bearing is calculated by splitting the total load uniformly over the number of roller bearings i_R per axle.

In order to calculate the rating life L_{km}, the equivalent dynamic load P must be determined. The equivalent dynamic load P corresponds to the radial load F_r, since the factor f_a for the axial load is 1 (the axial forces have a not inconsiderable effect on the bearings).

The basic rating life in travel kilometres is determined from the equivalent dynamic load P, the basic dynamic load rating C and the wheel diameter D_R: $L_{km} = (C/P)^{10/3} \cdot D_R \cdot \pi$. The calculation gives a value of $5{,}44 \cdot 10^6$ km. Since the application requires a nominal operating life of 5 million kilometres, the wheelset bearing unit is adequately dimensioned.

Design of the adjacent construction, machining tolerances

The housing must, as a connecting part between the vehicle bogie frame and the wheelset, reliably transmit the forces present. The material for the housings is dependent essentially on the operating and application conditions. Vehicles for commuter transport are subjected to frequent acceleration and braking intervals. As a result, preference is given to the use of light metal housings. In other cases, the predominant choice is spheroidal graphite cast iron.

Since the inner rings of the roller bearings are subjected to circumferential load, they have a tight fit on the shaft journals. For the shaft diameter $>$ 105 mm, the tolerance p6 has proved successful. The outer rings are subjected to point load. The housing bore normally has the tolerance H7 or H6.

Lubrication

The wheelset bearing units are supplied already greased. The lubricants used are lithium soap greases of consistency grade (NLGI grade) 2 or 3 with suitable additives. The grease filling is normally renewed at the time of general inspection of the vehicle. The good running characteristics of the plastic cages increase the grease operating life and thus the maintenance intervals.

Sealing

In addition to contact seals, the use of non-contact seals has proved effective for higher velocities and speeds. An arrangement of several gaps in the form of a labyrinth is also possible. In many cases – such as this application – gap seals by means of sheet metal caps are implemented.

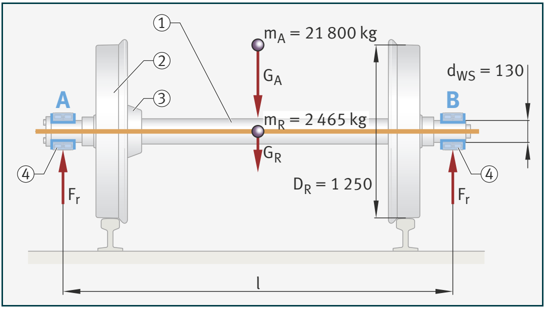


Figure 160
Load diagram and bearing arrangement

A, B = bearing positions

① Wheelset shaft
② Wheel disc
③ Hub seat
④ Wheelset bearing unit F-801804.ZL-L055-M32AX

Linear bearing arrangement for an automated connecting rod marking system

The automated connecting rod marking system reduces throughput times in the production chain and ensures automatic (contactless) measurement and marking of connecting rods in the process flow. The connecting rods are deposited on workpiece carriers and fed to the measurement and marking station. In the integrated measurement station, the connecting rod is measured, classified and then inked accordingly in the subsequent station, with markings applied to ensure clear identification of the connecting rod eye. The connecting rods are then removed from the workpiece carrier or subjected to further processing and placed into storage.
The machine is designed as a "rotary indexing system".

Application data

The technical data of the connecting rod marking system are as follows:

- force due to connecting rod mass and contact pressure in marking of the connecting rod $F_m = 90$ N
- requisite rating life of the carriages 60 000 km.

Selection of the guidance system

In design of the oval guidance system, a track roller curved guidance system with bogie carriages having four concentric bolts is used.
The system has the advantage that the carriage can move from a straight guideway to a curved track and vice versa.

The carriage is of a clearance-free design, comprising a steel carriage plate and two aluminium pivoting carriers fitted with track rollers. The straight guideways comprise a profiled aluminium rail with hardened steel shafts, while the curved guideways are made from steel with a hardened raceway.

Dimensioning

The force of the connecting rod mass and the contact force in the marking process ($F_m = 90$ N) are used in conjunction with permissible catalogue values to give the size of bogie carriage that is probably required. It is taken into consideration here that, under "normal" operating conditions, a minimum static load safety factor $S_0 > 4$ should be achieved.

Checking of the static load safety factor for the most heavily loaded track roller

For calculation of the axial force acting on the most heavily loaded track roller, the active spacing a_2 (support basis) is derived first. The acting point of the moment and the resulting contact points (track roller outer ring to shaft surface) are taken into consideration, giving the active spacing as a centre distance a_2 of the two shafts. Once the forces are released, the axial load on the most heavily loaded track roller can be determined.

The static load safety factor is determined by the equation $S_0 = C_0/P$.
The basic static load rating C_0 of the carriage (four track rollers) is $C_0 = 5\,200$ N (per track roller $C_0 = 1\,300$ N). The equivalent dynamic load P is calculated on the basis of the most heavily loaded track roller F_{y3}:
$F_{y3,y4} = F_m \cdot (l + a_2/2)/a_2$ and thus $F_{y3} = F_{y3,y4}/2 = 344$ N (P = 344 N).

S_0 should be > 4. Since S_0 < 4 (S_0 = 1 300 N/344 N = 3,78) in this case, a modification of the design (lever arm shortened from l = 300 mm to l = 250 mm) was subjected to recalculation. With l = 250 mm, P = 291 N. This gives a static load safety factor of S_0 = 4,47.

Calculation of carriage rating life
The basic dynamic load rating C per track roller is C = 2 500 N (10 000 N for the carriage corresponding to the number of track rollers). The equivalent dynamic load P = 291 N. Based on the assumption that the most heavily loaded track roller restricts the rating life of the carriage, the rating life calculation was carried out using the equation $L_s = (C/P)^3 \cdot 10^5$ m; $L_s = (2\,500\text{ N}/291\text{ N})^3 \cdot 10^5$ m. The calculated rating life for the carriage gives L_s = 63 400 km. The manual calculation does not take account of the deflection of the system and the associated displacements of the contact geometries in the bearing or any forces from the possible preload of the system. Preload increases the rigidity and reduces the rating life.

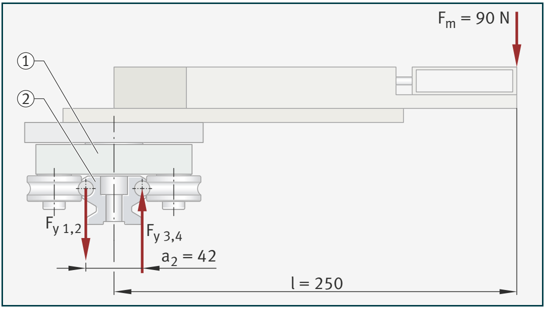
Lubrication The contact area between the track roller and shaft is lubricated with oil by means of lubrication and wiper units. They contain felt lubrication inserts. The felt inserts are supplied soaked with oil and can be replenished with oil if necessary. The track rollers are greased for life and are therefore maintenance-free.

Sealing Gap seals (non-contact seals) on both sides of the track rollers protect the rolling element system against contamination.

Mounting Guideway connectors lock the straight guideways securely to the curved elements. In order to achieve the quietest possible running behaviour when passing over the guideway joints, the guideways are precisely aligned to each other. The width tolerances are already matched to each other and marked accordingly, the height tolerances of the guideways are compensated by means of feeler gauges during mounting.

Figure 161
Load diagram and bearing arrangement

1 Bogie carriage LFDL52-SF, four clearance-free concentric bolts
2 Guideway LFS52



medias professional
Electronic selection and information system

***medias** professional*, the proven selection and information system from Schaeffler, presents the Schaeffler catalogue products in electronic form. As with the printed catalogue, Schaeffler customers thus receive product information on the two brands INA and FAG from a single data source. This saves time in selection of bearings and gives simpler handling.

***medias** professional* is available online in several languages, is easy to navigate and is particularly clear thank to the use of numerous images, diagrams and models. There are also highly representative application examples with bearings from Schaeffler. The applications are classified by market sector.

Datasheets on the bearing series can be outputted as a PDF file. There is also a lubricant database and the web2CAD link for direct download and inclusion of 3D models.

The scope of ***medias** professional* extends to single bearings. The complete shaft can be simulated and any influences arising from its deformation on the bearings can be determined using the calculation program BEARINX, see Page 691 to Page 695.

In conclusion, ***medias** professional* is a comprehensive, reliable system to help you help yourself in answering many questions on rolling bearing technology by electronic means, quickly and at any location. The following link will take you to ***medias***:
http://medias.schaeffler.com.

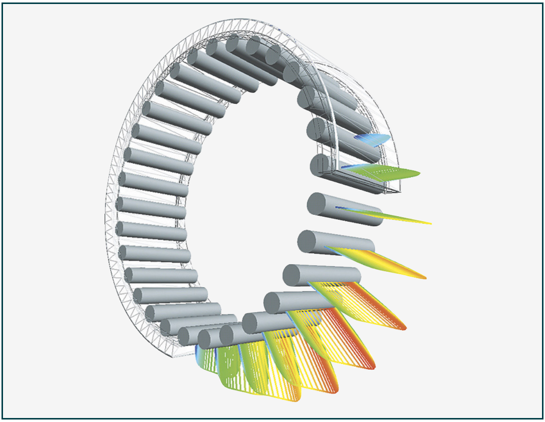
Figure 162
***medias** professional – calculation instead of estimation*

BEARINX calculation software from Schaeffler

BEARINX is one of the leading programs for the calculation of rolling bearings. It facilitates the detailed analysis of rolling bearing arrangements – from the individual bearing through complex shaft and linear guidance systems to complex transmissions. The complete calculation is carried out in a comprehensive calculation model. Even in the case of complex transmissions, the contact pressure on each individual rolling element is included in the calculation. BEARINX takes account of factors including:

- the non-linear elastic deflection behaviour of the bearings
- the elasticity of shafts and housings
- the influences of fit, temperature and speed on the operating clearance or preload of the bearings and on their contact angle
- the profiling of rollers and raceways as well as raceway osculations
- the displacement of contact angles as a function of load in ball bearings
- the actual contact pressure taking account of the misalignment and profiling of rolling elements
- the influence of lubrication conditions, contamination and actual contact pressure on the fatigue life.

Figure 163
Precise detail: even the contact pressure on each individual rolling element is included in the calculation



The analysis of design variants is aided by the transparent documentation of results and the graphical representation of shaft reactions and the internal load distribution of bearings. Thanks to an online tutorial and a detailed help system, it is possible to utilise the full potential of BEARINX-online Shaft Calculation.

The uniform calculation of the fatigue life using computer-aided calculation methods corresponding to the state of the art is defined in DIN 26281. This calculation method is of course also included in the online version.

Bearinx-online Shaft Calculation

Shaft calculation online – customer version

Commonly available calculation tools normally use heavily simplified calculation methods. In these cases, the misalignment of bearings as a result of shaft deflection and the different deflection behaviour of various bearing types is largely disregarded. In general, the internal load distribution of bearings – which is decisive for the fatigue life – is generally determined only by means of approximation methods.

It is possible using Bearinx-online Shaft Calculation to determine the actual load taking account of shaft deflection and the deflection behaviour of the rolling bearings. The internal load distribution of the bearings is of course also calculated precisely – to the level of the contact pressure taking account of the actual rolling element profile.

The algorithms in Bearinx-online Shaft Calculation are identical with those applied in Bearinx, which is used at Schaeffler. Bearinx-online Shaft Calculation facilitates calculation at the workstation of single axis shaft systems supported at several points. Input of data is supported by a Java-aided user interface. Checking of data is aided by graphical representation of the design.

An integrated database facilitates retrieval of data and the geometry of rolling bearings in the Schaeffler catalogue range. The actual calculation is then performed by the powerful Schaeffler calculation servers. The input files created using Bearinx-online Shaft Calculation are compatible with Bearinx. As a result, further communication with Schaeffler advisory engineers is easier and duplicate work is avoided.

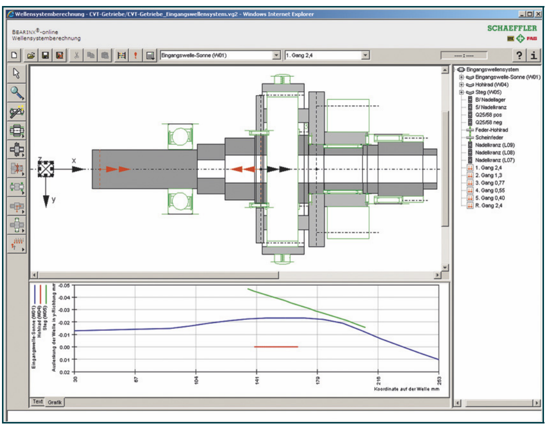


Figure 164
Bearinx-online Shaft Calculation, shaft reactions presented in graphical form

The software is not provided with the intention of transferring advice and calculation services from Schaeffler to customers. The reality is the opposite: with this arrangement, Schaeffler wishes to work even more closely with its customers. The objective is to make a suitable preliminary selection of rolling bearings in the early design phase, in order to reduce development times at the customer.

BEARINX-online Shaft Calculation – an overview

- Calculation of bearing rigidity at the operating point taking account of all relevant influences
- Graphical representation of shaft reactions (shaft deflection and shaft inclination)
- Rigid and elastic adjustment of bearings in the relevant shaft system
- Calculation of fatigue life in accordance with DIN 26281
- Simple modelling of shaft systems by means of integrated wizards.

The terms of use of the software and the access to additional necessary services such as training and support are defined in a mutual contractual agreement.

Training and access to BEARINX-online is available on payment of a fee. In the case of technical colleges, the introduction of software usage is free of charge.

Information about the customer version and the possibility of applying for registration/usage can be found on the Schaeffler Internet portal at: *http://www.schaeffler.de/Calculation*.

BEARINX-online Easy Friction

Friction calculation online – customer version

The BEARINX-online module Easy Friction can be used to determine the friction values of Schaeffler rolling bearings using a detailed method. The internal load distribution of the bearings and the contact pressures on the raceways and ribs together with the actual rolling element profile is of course taken into consideration. The new module is based on a theory of friction calculation that employs physical algorithms and has been confirmed by means of extensive test values. The bearing rating life is calculated in accordance with ISO/TS 16281.

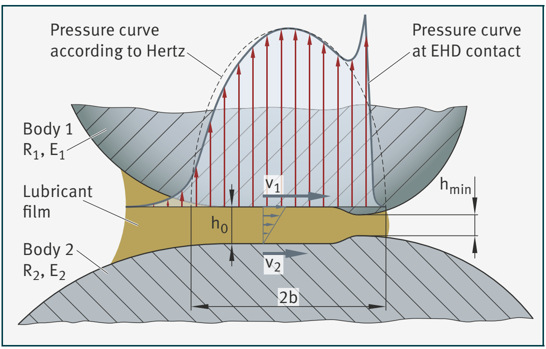
The algorithms in BEARINX-online Easy Friction take account in particular of the following parameters:

- losses in rolling and sliding contacts
- losses in the load-free zone
- splash losses
- seal friction.

Since the module is seamlessly incorporated in the "parent software" BEARINX used at Schaeffler, other typical influencing factors are also taken into consideration:

- radial and axial load
- tilting of bearing rings
- lubricant (viscosity class)
- temperature
- precise internal bearing geometry
- bearing clearance
- profiling of the bearing components
- rib geometry.

Figure 165
Elastohydrodynamic contact (EHD contact)



Rapid comparison of friction between various bearing arrangement concepts

Since bearings can be exchanged, different bearing arrangement concepts can be quickly and easily compared with each other. This makes it possible to achieve a bearing arrangement that is efficient and has optimised friction. All input data can be stored locally. As a result, modifications can be quickly made to an existing application case without the need for duplicated input of data. Furthermore, the stored file can be exchanged with the Schaeffler engineering service in order to achieve an optimum bearing design.

The most important results are displayed directly in a results window. In addition, the input data and the calculation results can be documented in a PDF file.

Bearing types suitable for calculation
BEARINX-online Easy Friction can be used for calculation of the following types:

- deep groove ball bearings
- angular contact ball bearings
- tapered roller bearings
- spherical roller bearings
- needle roller bearings
- cylindrical roller bearings.

The calculation software is available only in an online format. Initial registration takes only a short period of time.

Information about the customer version and the possibility of applying for registration/usage can be found on the Schaeffler Internet portal at: *http://www.schaeffler.de/Calculation*.

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Schaeffler Technical Pocket Guide

Contents

- Units of measurement and scripts
- Mathematics
- Technical statistics
- Chemistry
- Physics
- Mechanics
- Mechanical vibrations and acoustics
- Hydraulics and pneumatics
- Mechatronics
- Strength of materials
- Construction materials
- Technical drawing guideline
- Tolerances and fits
- Design elements

In line with the philosophy of a structured reference work, the chapters are composed in a comprehensive yet concise form and are oriented towards direct, practical usability. With its compact and succinct manner of presentation, the Schaeffler Technical Pocket Guide thus successfully bridges the gap between a comprehensive textbook and purely tabular work.

Target groups

On the basis of its concept, the STT is aimed primarily at students attending technical universities, colleges and technical colleges as well as students at technical institutes specialising in mechanical engineering. Due to its didactic concept, the book is suitable both for teaching purposes and for independent study. In addition, the compendium is a proven and reliable companion for experienced designers in the fields of vehicle, mechanical, instrument and plant engineering.

Publisher

Schaeffler is a leading global manufacturer of rolling bearings and a renowned supplier to the automotive industry. This preeminent position is also evidenced by the large number of patent applications filed each year.

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